ENVIRONMENTAL SYSTEMATICS AND ASTRONOMICAL REFRACTION II.

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ABSTRACT

The role of urban heat islands in producing systematic isopycnic tilts is explored in more detail, and with greater rigor, than in Part I of this series. (Perth, 1974).

Specifically, a three dimensional integration is carried out, and light rays are, in effect, "traced" through the resulting perturbation field by evaluating the integral of anomalous refraction. This is done for various values of the parameters, viz., wind direction and observatory location relative to the heat island, strength of the central perturbation, zenith distance of the observed object, etc.

It is stressed that heat islands are not the only source of such systematic effects.

Finally, a brief discussion of some possible methods of determining observationally the effects here treated theoretically, as well as other site dependent effects, is appended.

INTRODUCTION

The effects which an urban heat island can have upon astronomical refraction were briefly discussed in an essentially qualitative way in a previous paper by the author (1). That paper will be referred to as I.

In the present work a more quantitative approach is taken. The new elements which are introduced to make this possible are: (1) a three dimensional heat island is used, (2) the perturbation field is used to calculate the isopycnic tilt, which in turn is used to evaluate the integral of anomalous refraction, and (3) the anomalous refraction is evaluated for non-zenith observations.

The point of view taken here is that of an astrometrist engaged in

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fundamental meridian observations which ultimately lead to fundamental reference systems such as the FK4 catalog.

Certainly no one engaged in such work would locate, initially, an astrometric observatory within the sphere of influence of such a perturbing factor as an urban heat island. However, such islands have a way of growing, and in the process they often engulf long established observatories. Among several possible examples of such cases, two are: the Cerro Calan Observatory in Santiago, Chile, and my own institution, the U. S. Naval Observatory in Washington, D. C. When the latter was established at its present location in 1895 it was remarked that it would be, "one-hundred years", before any deleterious effects could be expected from the capital city. The Observatory is located approximately 6 km. from the U. S. Capitol, and I, together with both preceeding and contemporary colleagues, daily travel "downtown" to reach the Observatory from home.

On the other hand, before one rushes off to locate an astrometric observatory in some remote mountain fastness, or perhaps upon some island in a gentle clime, one ought to be aware that the mathematics governing, e.g., mountain lee waves, have an uncanny resemblance to those concerning urban heat islands.

Over one-half century ago, Emden (2) summed up one of his works by writing, "... a favorable location of the station, namely, at a site at which experience shows that no sloping of layers in the atmosphere sets in, even if only transient. Whether broad plains or mountain summits are to be preferred must be learned from experience".

Unfortunately, the fundamental astrometric quality of a site can be determined, usually, only a posteriori, after a considerable investment for installation and for perhaps five or ten years of observing. Thus the economics of the situation do not permit much in the way of "site testing", and so the fundamental astrometrist must investigate the site with which he may be blessed, or more likely, cursed, with any tools at hand, theoretical or observational.

THEORETICAL BACKGROUND

The basis for the present calculations is contained in an excellent paper by Olfe and Lee (3). The reader is referred to their work for the somewhat lengthy details.

In brief, their governing two-dimensional equation for the temperature perturbation field caused by the heat island's conduction and convection effects is:

$$\left[\frac{\partial^2}{\partial Z^2}\left(\frac{\partial}{\partial X} - \frac{\partial}{\partial Z^2}\right) + \tfrac{1}{4}\gamma \frac{\partial}{\partial X}\right]\psi = 0, \qquad (1)$$

where γ is a non-dimensional parameter depending upon various meteorlogical quantities, X and Z are non-dimensionalized horizontal and vertical coordinates, and ψ is the ratio of the perturbation at (X, Y) to the perturbation at the center of the heat island (0,0).

That is,

$$\psi = \frac{T'}{T'_o}, \qquad (2)$$

where primes indicate perturbation quantities.

Elementary solutions of the form,

$$\mathrm{RE}\left[\exp(\sigma Z + ikX)\right], \qquad (3)$$

lead to the total solution,

$$\psi\ (X,Z) = \int_0^\infty g(k)\,\mathrm{RE}\left\{\left[C_1\exp(\sigma_1 Z) + C_2\exp(\sigma_2 Z)\right]\exp(ikX)\right\} dk. \qquad (4)$$

The C's and σ's are complex, functions of κ and γ, and g (κ) is the Fourier cosine transform of the assumed (symmetric) surface temperature distribution.

Following Scorer (4), it is possible to superpose two-dimensional solutions at varying angles, α, to the uniform flow direction, and thus generate a three-dimensional solution. In simple dimensional heat island cylindrical coordinates (where ϕ=0 denotes the free flow direction) this has the form,

$$\psi(r,z,\phi) = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{+\pi/2}\int_0^\infty g(k)\,\mathrm{RE}\left\{\left[C_1\exp(\sigma_1 Z') + C_2\exp(\sigma_2 Z')\right]\exp(ikX')\right\} dk\,d\alpha \qquad (5)$$

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$$X' = \frac{r}{L_o}\cos(\phi+\alpha), \tag{6}$$

where

$$Z' = (\cos\alpha)^{\frac{1}{2}} Z,$$

with L_o the half-width of the island.

THE PRESENT CASE

Referring to Figure 1, if $\overline{AB}$ is a tilted isopycnic induced by the perturbation, then for the density, ρ, one has,

$$\Delta\rho = -\left[\frac{\partial\rho'}{\partial q}\Delta q + \frac{\partial\rho'}{\partial z}\Delta z\right], \tag{7}$$

where q is any horizontal coordinate, ρ_a is the unperturbed density along $\overline{AC}$, and $\Delta\rho$ is the normal change of density between A and B in the absence of any perturbation.

So,

$$\beta \simeq \frac{\Delta z}{\Delta q} = \frac{-\dfrac{\partial\rho'}{\partial q}}{\dfrac{\Delta\rho}{\Delta z} + \dfrac{\partial\rho'}{\partial z}}. \tag{8}$$

Using the well known result that,

$$\rho(x) = \rho_o \exp\left(\frac{-gz}{RT}\right) = \rho_o \exp\left(\frac{3.4\text{x}10^{-2}z}{T}\right) \simeq \rho_o\left(1 - \frac{3.4\text{x}10^{-2}z}{T} + \ldots\right), \tag{9}$$

with z in meters and T in ^{o}K,

one has,

$$\Delta\rho = \rho_B - \rho_A = \rho_o\left(\frac{-3.4\text{x}10^{-2}\Delta z}{\overline{T}}\right) \tag{10}$$

In order to work with a definite number, $\overline{T}$ will be taken as constant throughout the domain of the perturbation. This approximation is not critical in the present case since a $\pm 10^o$ swing of $\overline{T}$ can only change the results by less than $\pm 4\%$. Also, we are concerned with heights of less than one kilometer.

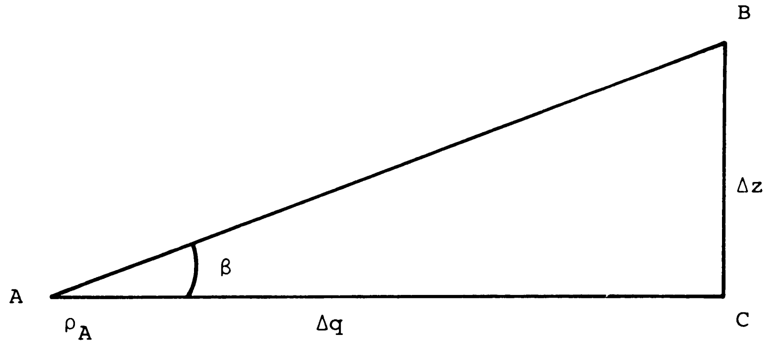

Figure 1

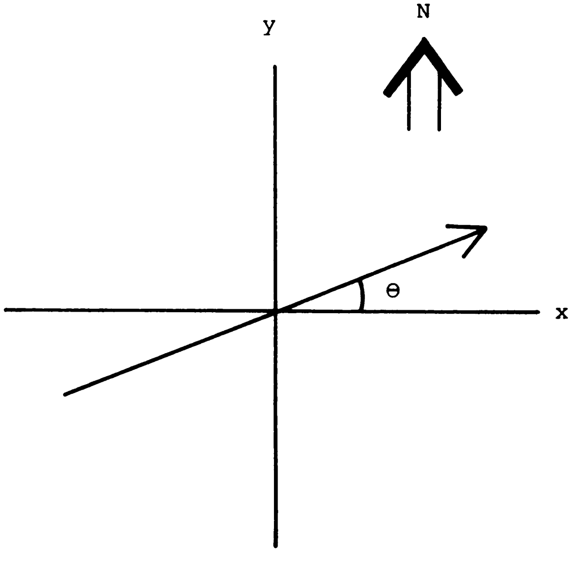

Figure 2

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Taking $\overline{T} = 285^{o}$, then,

$$\beta = \frac{-\ \partial\rho'/\partial q}{-1.2\mathrm{x}10^{-4}\rho_o + \partial\rho'/\partial z.} . \tag{11}$$

Since the Boussinesq approximation is valid in this case (5),

$$\frac{T'}{T_o} = -\frac{\rho'}{\rho_o} \tag{12}$$

and

$$\psi = \frac{T'}{T'_o} = \frac{T'}{cT_o} = -\frac{\rho'}{c\rho_o} , \tag{13}$$

where c (<<1) is used to represent the central perturbation as a fraction of the unperturbed central temperature or density.

Then, e.g.,

$$\frac{\partial\psi}{\partial q} = -\frac{1}{c\rho_o}\frac{\partial\rho'}{\partial q} \tag{14}$$

and

$$\beta = -\frac{c\ \partial\psi/\partial q}{1.2\mathrm{x}10^{-4} + c\partial\psi/\partial z} \tag{15}$$

Figure 2 illustrates the coordinate system used for the calculations. The positive y-axis is directed towards the north. The computations are carried out in a coordinate system which rotates in accordance with the azimuth of the wind, and hence which differs from the (x,y,z) system by a single rotation about z of amount θ. For example, $\theta = -45^{o}$ indicates a NW wind.

The procedure consisted of choosing values for the parameters (which amounts to setting a value for γ), and then varying the location of the observatory, the wind direction and the zenith distance of the observations. Values of (x,y,z) were chosen and converted by Eq. (6) to X', Z' for use in Eq. (5).

The integration with respect to k is of the form,

$$\int_{0}^{\infty} e^{-x} f(x)\, dx, \tag{16}$$

and the integration with respect to α can be brought to the form,

$$\int_{-1}^{+1} F(y)\, dy. \tag{17}$$

The former is suitable for Laguerre integration and the latter for Gaussian integration. A cartesian product was formed so that,

$$\psi = \sum_{j} \sum_{i} L_i(x) G_j(y) f(x_i) F(y_j), \tag{18}$$

where L and G are the appropriate weighting factors.

The partial derivatives could be expressed analytically by differentiation under the integral sign, however, since the values of ψ itself were of interest, the partials were evaluated by

$$\frac{\partial \psi}{\partial q} \simeq \frac{\psi_{+\Delta q} - \psi_{-\Delta q}}{2\Delta q}, \tag{19}$$

which for present purposes has no essential effect upon the numerical results. The quantity Δq was 25 meters for q = x or y, and 10 meters for q=z.

The tilt angle was computed using Eq. (15), for vertical steps of 25 meters, up to a maximum height, z_{max}, of 0.8 or 1.0 km. That is;

$$\begin{aligned} \Delta z &= 25\text{m}, \\ \Delta y &= \tan \zeta \Delta z, \\ \Delta x &= 0, \end{aligned} \tag{20}$$

where ζ is the zenith distance (positive to the north), and Δx is zero for meridian observations in the (x, y, z) system.

The tilt in right ascension is given by taking q = x, and the tilt in declination by taking q = y.

The integral of anomalous refraction,

$$R_a = \int_{n1}^{n2} \frac{\beta dn}{n\left[1 - \left(\frac{r_o n_o}{r\, n} \sin\zeta\right)^2\right]}, \tag{21}$$

can be considerably simplified in the present case by setting

$$n = 1 \quad \text{and} \quad r_o n_o = r\, n. \tag{22}$$

Since $\frac{dn}{dz} \simeq 3 \times 10^{-8}\ m^{-1}$,

$$R''_a = \frac{1.1\times10^{-4}}{\cos^2\zeta} \int_0^{z_{max}} \beta(z)dz \quad \text{sec. of arc.} \tag{23}$$

The angles given by Eq. (15) were very closely fitted by a polynomial in z and the integration carried out term by term to yield R".

PARAMETERS AND RESULTS

The following parameters were used in the calculations:

Stability coefficient	$\equiv s$	=	1.0×10^{-4}	m^{-1}
Eddy diffusivity	$\equiv \kappa$	=	10	m^2/s
Wind velocity	$\equiv U_o$	=	3	m/s
Heat island half-width	$\equiv L_o$	=	3	km
Central perturbation	$\equiv c$	=	2%	

Note that;

$$Z = \left(\frac{U_o}{\kappa L_o}\right)^{\frac{1}{2}} z$$

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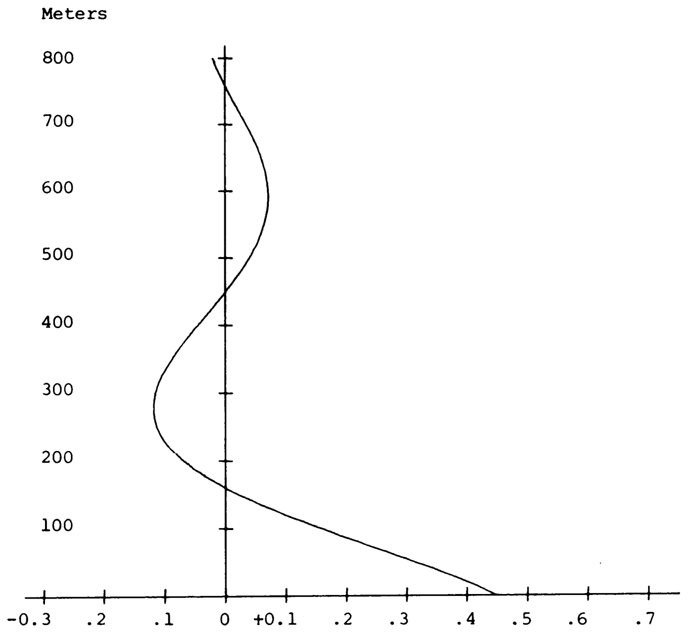

Figure 3 TILT ANGLE (DEGREES)

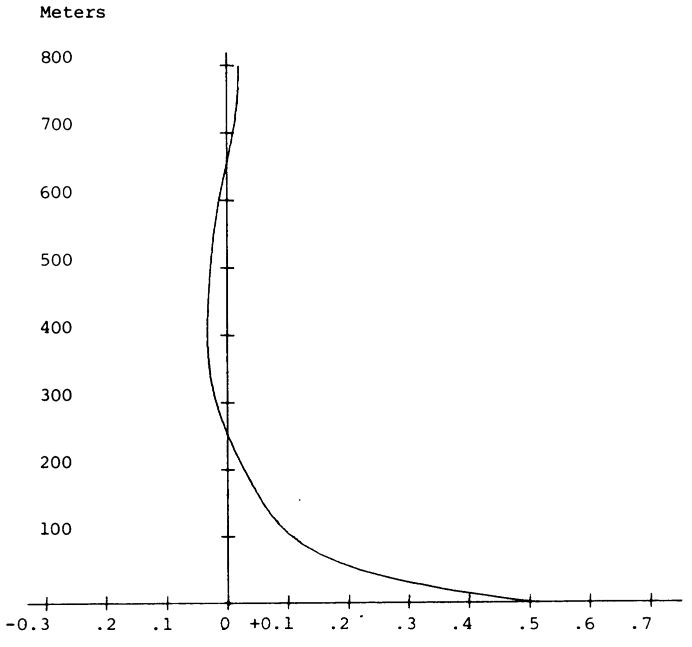

FIGURE 4 TILT ANGLE (DEGREES)

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TABLE 1

	α	δ	α	δ
N	-5	+4	-2	+14
S	-5	+5	-7	+ 5
N	-18	- 2	+ 6	- 6
S	- 7	-20	+ 9	- 7

N	-16	+ 1	+16	+ 4
S	-17	+ 1	+16	+28
N	-17	- 1	+16	-28
S	-16	-1	+16	- 4

N	- 7	+20	+ 9	+ 7
S	-18	+ 2	+ 6	+ 6
N	- 5	- 5	- 7	- 5
S	- 5	- 4	- 2	-14

TABLE 2

α	δ	ζ
+16	- 28	45
+50	- 59	60
+52	-196	70

α	δ	ζ
+ 6	- 6	45
+11	-11	60
+19	-28	70

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The quantity, γ, is given by

$$\gamma = 4 \frac{gs\kappa L_o}{(U_o \cos\alpha)^3} \tag{24}$$

where g is the acceleration of gravity.

Physically, this represents a relatively small, but intense, heat island with a stable temperature gradient upwind.

As in (I), the "mountain function" was used to represent the surface temperature distribution. In this case,

$$g(k) = 2/\pi \quad \exp(-2k/\pi). \tag{25}$$

The integration of Eq. (5) gives a vertical perturbation which has the character of a heavily damped wave. This behavior is mirrored by the tilt angles given by Eq. (15). In the cases considered here, two dominant modes became apparent. These are illustrated in Fig. 3 and 4.

In Fig. 3 the area of the sign reversal of β (vis a vis the ground level sign) is much more pronounced than in Fig. 4. The former case adheres closely to the nature of the perturbation itself. The two cases are the result of the location of the observatory's meridian with respect to the center of the heat island, of the wind direction and of the zenith distance of the object observed. Since the anomalous refraction is proportional to the integral of these curves in the z direction, evidently the situation depicted in Fig. 3 gives, in general, a smaller effect than that in Fig. 4. It is also evident that in either case the lower levels (say $z < 250$m) are the major contributors to the effect.

Calculations were initially made for fixed zenith distances, viz., $\zeta = \pm 45^{\circ}$. The observatory was located in each quadrant of the (x,y) system in turn, for each of three wind directions. The results are shown schematically in Table 1. The unit is 0"001. In all these cases the observatory's coordinates were $|x| = |y| = 4$ km., thus placing the observatory approximately 6 km. from the center of the heat island. The wind vector is also shown in Table 1.

The assumed symmetric heat island gives rise to the "anti-symmetry" in δ and the symmetry in α.

The quantities exhibited in Table 1 are not very large, varying between essentially zero and ±0"03. However if one recalls that transit circles engaged in fundamental work must determine a celestial pole,

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and hence must work at greater zenith distances, the import becomes more serious. Extending the zenith distance to +60° and +70° for the cases $\theta = 0^{\circ}$ and $\theta = +45^{\circ}$, for x = +4km and y = -4km, gives the results shown in Table 2. The 0".03 alluded to earlier now increases as much as six-fold.

Increased zenith distances are not, however, the sole catalyst for obtaining large effects. The governing equation (1) represents the actions of two "competing" effects, conduction and a gravity wave. The latter is responsible for the negative temperature perturbation which leads to the tilt reversals. Very small changes in the perturbation profile lead to sensible changes in the tilt angles, and hence the integrated anomalous refraction.

The two most troublesome parameters are the stability coefficient and the eddy diffusivity. On the one hand, the gravity wavelength depends upon s, while the conduction profile depends upon κ. By way of experimentation, the observatory was placed on the y axis, 6 km. south of the origin. Looking downwind and upwind at $\zeta = +45^{\circ}$, with $\kappa = 10$, gave (in declination), 0".001 and 0".019, respectively. Reducing κ to 1 gave 0".031 and 0".073 for the same cases. Since the deviations from pure conduction are proportional to γ, this indicates that reducing κ (and hence γ) reduces the negative temperature perturbation and hence increases the anomalous refraction. On the other hand, it is possible to vary both s and κ in such a manner as to keep γ constant and still change the anomalous refraction. This is so since the vertical coordinate is non-dimensionalized by the factor, $(U_0/\kappa L_0)$, thus changing κ changes the metric of the entire problem.

It would appear that theory can only indicate to us that we do indeed have a problem. Worse, it now appears that we should be thankful for convection effects for reducing the anomalous refraction by inducing low level temperature "cross-over"!

CONCLUSION

It appears to the author that it is absolutely necessary to devise some means of determining these pernicious systematic effects observationally. Experiments have been carried out at the Naval Observatory in which a Raman LIDAR was successfully used to probe the atmosphere for water vapor content. It now appears that this effort should be expanded to determine systematic isopycnic tilts, and perhaps, ultimately, to real time density profiling of the atmosphere. At the same time other efforts to solve these problems should proceed apace since any single solution is apt to introduce its own systematic errors.

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