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Pricing of a resettable guarantee of a salary-connected individual pension account

CHAO-LIANG CHEN

Department of Economics, Tamkang University, 151 Ying-chuan Road, Tamsui, New Taipei City, Taiwan 251, R.O.C.
(*e-mail:* chen@mail.tku.edu.tw)

Abstract

The present paper numerically prices a resettable guarantee of a salary-connected individual pension account (IPA). The results indicate that a principal guarantee without a reset feature is not worth much unless the volatility of assets in the IPA is huge, while the death benefit contributes a very small proportion to the guarantee value. Deferred proportional funding is an alternative to reducing problems from the difficulty in modeling salary behavior, because the required deferred proportional cost is impacted less by the salary behavior. Moreover, if the lapse from a guarantee is possible, then the guarantee is not necessarily more valuable for a younger individual.

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Introduction

As demographic aging becomes a global trend, workers everywhere are realizing that they are exposed to the uncertainty that arises from unfunded social security systems. This recognition is prompting a global wave of social security reforms, resulting in a larger role for defined contribution (DC) pension plans and raising the question regarding how workers should be protected from capital market volatility since a notable feature of DC plans is that workers take charge of their own investment decisions and therefore bear the investment risk. Concern over this question has consequently prompted policymakers to provide or propose some forms of guarantee for DC pension accumulations (see Lachance and Mitchell, 2003; Walliser, 2003). Aside from mandatory DC plans, a wide range of voluntary DC plans have also offered retirement guarantees, as discussed by Turner and Rajnes (2003).

Although one can find attractive provisions in guarantees on DC plans, there remains the important issue of how to finance and price the costs of offering various guarantees. The pension literature presents contingent claims pricing techniques along the line of the Black and Scholes (1973) option price formula to value pension guarantees. Marcus (1987), Hsieh *et al.* (1994), and Pennacchi and Lewis (1994) used the contingent claims methodology to price the value of Pension Benefit Guarantee

Corporation (PBGC) insurance. They focused on the defined benefit (DB) pension guarantee and chose different termination provisions. In their models, the guaranteed portfolio consists of a single contribution that grows with returns following a lognormal distribution, in which the Black–Scholes formula provides a closed-form solution for the guarantee values. Our study focuses efforts on a more complicated DC pension guarantee that provides the death benefit and a reset provision. The 'death benefit' causes the guarantee to be paid off and terminated immediately upon the death of the worker. The reset provision allows the holder to reset the guarantee level in order to lock in market gains, but not to extend the ultimate maturity time.

Windcliff *et al.* (2001, 2002) indicated that Canada's segregated fund guarantees, which embed a death benefit and a reset feature in the contract, can be very valuable. Although previous studies showed the segregated funds to be invested in by a group of investors, each guarantee priced by this paper herein is provided for an individual pension account (IPA). Moreover, while the guaranteed asset consists of a single contribution in the segregated fund guarantee, it is periodically contributed during the life of the participant's working career in the case of IPA. The guarantee on an IPA should be salary-connected when the contributions of an IPA are connected with individual salaries, and therefore the accumulation of an IPA is a function of the entire historical salaries.

This paper takes the salary-connected issue as an important consideration compared to previous studies in the literature that price the segregated fund guarantee that embeds a death benefit and a reset feature in the contract. Although this IPA framework assumes the value of each contribution follows a lognormal distribution, the accumulation of the contributions is not lognormal. The valuation of the guarantee on an IPA is therefore more complicated than simple applications of Margrabe's (1978) option pricing model where two assets to be exchanged are required to grow according to a lognormal distribution in order to obtain a closed-form solution of the option's value.

In addition to the path-dependent feature given above, both the reset provision and the death benefit also increase the complexity of the valuation of a guarantee on an IPA. The reset provision allows the guarantee level to rise not only by the new contribution, but also by the current profit of an IPA. The death benefit makes the guarantee be like time life insurance, where the policy allows only one claim to be made within a contract period. Shimko (1992) provided a solution of a time integral of European option prices for the value of a multiple claim insurance contract where idiosyncratic risk affects policy valuation. One can apply Shimko's (1992) approach to the case of a claim-terminating policy using the intensity-adjusted risk-free interest rate to replace the market risk-free interest rate. Shimko's insurance pricing technique cannot help generate a closed-form solution for the value of a guarantee on an IPA due to the salary-connected feature, but it does help to construct an appropriate equivalent security for the guarantee on an IPA. The value of the guarantee on an IPA can then be numerically priced by evaluating its equivalent security. Chen (2006) applied contingent claims pricing on the claim-terminating insurance contract to value the guarantee on a new DC pension plan to exchange back an old defined benefit. Although the guarantee priced herein has claim-terminating and salary-connected features similar to Chen (2006), this paper focuses on pricing a resettable principal guarantee and extends beyond Chen (2006) by modeling the practice of deferred funding. Based upon a variety of scenarios this paper further numerically estimates the required deferred funding costs as well as up-front values for guarantees on an IPA.

The remaining work is as follows: Section 1 provides the theoretical analysis of a resettable guarantee on an IPA. Section 2 describes the numerical approach. Section 3 presents results and discussions. Section 4 is a summary conclusion for this paper.

Theoretic analysis of a resettable guarantee on an IPA

Assume each contract priced here is written for a principal guarantee on the accumulation of an IPA with a death benefit and a reset provision. The guarantee level, or the principal of an IPA, ascends by every contribution to the IPA during the life of the contract as a proportion of the corresponding salary. Moreover, the reset provision allows the holder to lock up the market gains of the IPA in the principal so as to increase the guarantee level. The guarantee has an ultimate maturity time conditional on the individual's age and cannot be extended even if the guarantee level is reset. Early termination can occur under a holder's death or if the holder lapses before the ultimate maturity. While the death benefit is provided, where the guarantee is paid off immediately upon death, there is no payoff from the guarantee if the holder lapses from the guarantee contract. To reduce moral hazard and basis risk for the contract provider, each contract requires the asset of the IPA to be connected with a specific traded portfolio or mutual fund.

The guarantee on an IPA is priced as a claim-terminating insurance contract. The effect of the death benefit on the payoff of the guarantee is similar to one of life insurance. The reset feature is treated as an option provision of a structured insurance. The approach from Shimko (1992) helps construct the appropriate equivalent security to replicate the payoff of the guarantee.

While every contract is assumed to be written at time (t) 0, the ultimate maturity time, T, is a function of the individual age at time 0, y. The value of an individual guarantee $V(C, K, y, t)$ is a function of the accumulation of an IPA, $C(y,t)$, and the guarantee level, $K(y,t)$. The $C(y,t)$ is determined from the salary history as well as the rate of return on the portfolio connected to the IPA. Valuing $C(y,t)$ is equivalent to valuing a security with negative dividends and whose capital value grows like a market index. The dividends are not proportional to the $C(y,t)$ itself, but fixed proportional to the worker's salary level $S(y,t)$. While the guarantee level, $K(y,t)$, is set at the level of the current accumulation of the IPA, $C(y,t)$, if the reset is executed, then it also automatically ascends by new contributions. We model the stochastic differential equations for $C(y,t)$ and $S(y,t)$ as:

$$dC(y, t) = [\pi(C, t) \cdot C(y, t) + g \cdot S(y, t)] \cdot dt + \sigma_C(C, t) \cdot C(y, t) \cdot dZ_C, \tag{1}$$

$$dS(y, t) = w(S, t) \cdot S(y, t) \cdot dt + \sigma_S(S, t) \cdot S(y, t) \cdot dZ_S, \tag{2}$$

where $\pi(C,t)$ is the instantaneous expected rate of return on the connected portfolio, g is the fixed contribution rate as a proportion of the salary, $\sigma_C(C,t)$ is the standard deviation of the rate of return, and $W(S,t)$ and $\sigma_S(S,t)$ are the salary's instantaneous average growth rate and its volatility, respectively. Terms dZ_C and dZ_S are the standardized Wiener increments.

The net cash flow of the guarantee is a terminating type and equals the guarantee payoff minus the value of the guarantee at the time of payment to surrender the contract. Since there is no payoff from the guarantee for the holder's lapse, the payoff of the guarantee for this case is zero. Therefore, the net cash flow conditional on the holder's lapse, CF_l, satisfies $CF_l = -V(C, K, y, t)$, where the holder gets nothing, but surrenders the value of the contract. In the other case, the 100% guarantee is paid off immediately upon death or ultimate maturity, where $G(y, t) = \max\{[K(y, t) - C(y, t)], 0\}$, representing the payoff of the guarantee for this case. The net cash flow conditional on death or ultimate maturity, CF_d, therefore satisfies that $CF_d = G(y, t) - V(C, K, y, t)$, where the holder gets the 100% guarantee payoff and surrenders the value of the contract.

While the intensity rate for individual lapse from the guarantee contract, $E_l(y, t)$, and the intensity rate for individual death or ultimate maturity, $E_d(y, t)$, are assumed to be deterministic conditional functions of time, the terminating rate $E(y, t) = E_d(y, t) + E_l(y, t)$ also is a deterministic conditional function of time. The value of an individual guarantee involves a minimum value constraint, $V^{\min} = V(C, C, y, t)$, which is the value of the contract if the reset is executed and the guarantee level is reset at the level of the current accumulation of the IPA. The value of an individual guarantee also must satisfy the boundary condition of being worthless at ultimate maturity, $V(C, K, y, T) = 0$.

Subject to the boundary condition, the guarantee is equivalent to a risky security that has an ultimate maturity value equal to 0 and dividends equal to $G(y,t)$ $E_d(y,t) \times \Delta t$, and its value could jump to zero with an intensity of $E(y,t) \times \Delta t$. One may substitute the intensity-adjusted interest rate, $r + E(y, t)$, for the risk-free interest rate in order to automatically include the terminating payment in the price of this risky security. Merton (1976) and Shimko (1989, 1992) found a similar conclusion about the pricing of the terminating payment. An up-front value of the guarantee on an IPA can then be solved by numerically pricing its equivalent security.

We consider the Cox *et al.* (1985) single factor term structure model as an example for stochastic short-term risk-free interest rate $r(t)$.[1] Let $H(t, \tau)$ be the risk-neutral price at time t for a pure discount bond that pays off US$1 at maturity time $t + \tau$ and has a deterministic average intensity of $\bar{E} \cdot dt$ to jump instantaneously to zero before time $t + \tau$. $H(t, \tau)$ has the form:[2]

$$H(t, \tau) = P(t, \tau) \cdot \exp(-\bar{E} \cdot \tau), \tag{3}$$

Where $P(t, \tau) = A(t, \tau) \exp[-B(t, \tau) \cdot r(t)]$,

$$A(t, \tau) = \left[\frac{2\gamma \cdot \exp[(\alpha + \gamma)\tau/2]}{(\alpha + \gamma)[\exp(\gamma \cdot \tau) - 1] + 2\gamma}\right]^{2\alpha \cdot \beta/\sigma_r^2},$$

1 The risk-neutral process for r in their model is $dr = \alpha(\beta - r)\,dt + \sigma_r \sqrt{r}\,dZ_r$.
2 Please refer to Shimko (1989) and Merton (1976) for this intensity-adjusted bond-pricing formula.

$$B(t,\tau) = \frac{2[\exp(\gamma \cdot \tau) - 1]}{(\alpha + \gamma)[\exp(\gamma \cdot \tau) - 1] + 2\gamma},$$

and $\gamma = \sqrt{\alpha^2 + 2\sigma_r^2}$. Parameter β in equation (3) is the mean-reverting drift, α is the reversion rate, and σ_r is the volatility of changes in the interest rate. The claim-terminating type of payment is automatically priced in the intensity-adjusted bond pricing equation. Since the terminating rate of the guarantee on an IPA is assumed to be a deterministic function of time, then term $\bar{E}$ in equation (3), applied to a guarantee valuation, should be the average terminating rate between time t and time $t + \tau$.

Modeling the deferred funding, one can assume that the deferred proportional costs are extracted from the accumulation of the IPA. The required deferred cost rate charged for the guarantee is then determined so that the net residual value of the guarantee is zero.

Numerical approach

The value of an equivalent security of the guarantee just described in the last section will be numerically estimated by Monte Carlo simulation in a risk-neutral world. In that, paths for all variables are simulated from their risk-neutral processes and each payoff of the equivalent securities is discounted by multiplying the risk-neutral price of a risky zero-coupon-bond as shown in equation (3).

The risk-neutral stochastic difference equation for salary level (S), the accumulation of an IPA (C), and short-term risk-free interest rate (r) are set by equation (4), (5), and (6), respectively:

$$S_{t+\Delta t} = S_t + (w - \lambda_S \cdot \sigma_S) \cdot S_t \cdot \Delta t + \sigma_S \cdot S_t \cdot \sqrt{\Delta t} \cdot Z_S, \tag{4}$$

$$C_{t+\Delta t} = C_t + (r_t \cdot C_t + g \cdot S_t) \cdot \Delta t + \sigma_C \cdot C_t \cdot \sqrt{\Delta t} \cdot Z_C, \tag{5}$$

$$r_{t+\Delta t} = r_t + \alpha \cdot (\beta - r_t) \cdot \Delta t + \sigma_r \cdot \sqrt{r_t \cdot \Delta t} \cdot Z_r, \tag{6}$$

where the terms Z_S, Z_C, and Z_r are stochastic variables of a standardized trivariate normal distribution. Generating the samples Z_S, Z_C, and Z_r requires that independent samples x_1, x_2, and x_3 from a univariate standardized normal distribution are obtained first. The samples Z_S, Z_C, and Z_r are then calculated one after another as follows:

$$Z_S = x_1, Z_C = \rho_{SC} \cdot x_1 + \sqrt{1 - \rho_{SC}^2} \cdot x_2, \quad Z_r = \rho_{Cr} \cdot Z_C + \sqrt{1 - \rho_{Cr}^2} \cdot x_3,$$

where ρ_{SC} is the correlation between Z_S and Z_C, and ρ_{Cr} is the one between Z_C and Z_r. This scheme was suggested by Hull (1997) to avoid an impossible correlation structure where the Cholesky decomposition of the correlation matrix for Z_S, Z_C, and Z_r does not exhibit real solutions. The feasible correlations between Z_S, Z_C, and Z_r in this scheme are required to satisfy $\rho_{Sr} = \rho_{SC}\,\rho_{Cr}$, where ρ_{Sr} is the correlation between Z_S and Z_r.

Equation (4) indicates that the unconditional distribution of salary is lognormal. Following Chen (2006), this description of salary allows the possibility for negative wage growth and is based on the fact that negative wage growth could happen during an economic slump. In equation (4), the term $w - \lambda_S\,\sigma_S$ is the risk-adjusted expected growth rate of salary, whereby λ_S and σ_S are respectively the market price of salary risk and the volatility of salary.

Equation (5) presents that the risk-neutral expected rate of return on the IPA is assumed to be the short-term risk-free interest rate, r_t. This assumption is based on a result for the equilibrium pricing theory, as the asset of the IPA is connected with a specific traded portfolio. In equation (5), the term g is the fixed contribution rate as a proportion of salary and σ_C is the volatility of the accumulation of an IPA.

Equation (6) indicates that the risk-neutral short-term risk-free interest rate is generated using the Euler scheme to discrete-time approximate the stochastic interest rate modeled in Cox *et al.* (1985).[3] In that, β is the mean-reverting drift, α is the reversion rate, and $\sigma_r\sqrt{r_t}$ is the volatility of the interest rate. One problem with this scheme is that the discrete process for the interest rate could become negative with non-zero probability, which makes the computation of $\sqrt{r_t}$ impossible, if the size of the time step is not enough small. To handle this problem and to consider both simplicity and efficiency simultaneously, Lord *et al.* (2010) proposed a Full Truncation fix as in equation (7):

$$r_{t+\Delta t} = r_t + \alpha \cdot (\beta - r_t{}^{+}) \cdot \Delta t + \sigma_r \cdot \sqrt{r_t{}^{+} \cdot \Delta t} \cdot Z_r, \tag{7}$$

where $r_t^+ = \max(r_t, 0)$. This scheme allows the process for the interest rate to go below zero while avoiding making the computation of $\sqrt{r_t}$ impossible. A full truncation fix to prevent the interest rate from going to become negative involves taking the maximum of the value generated from equation (6) and zero as equation (8):

$$r_{t+\Delta t} = \max\{[r_t + \alpha \cdot (\beta - r_t) \cdot \Delta t + \sigma_r \cdot \sqrt{r_t \cdot \Delta t} \cdot Z_r], 0\}. \tag{8}$$

The fix schemes of equations (7) and (8) are not to be used before a known failure of the time stepping from the original Euler scheme as in equation (6).[4]

When modeling the deferred funding, the risk-neutral stochastic difference equation for the accumulation of an IPA, equation (5), is reset by equation (9):

$$C_{t+\Delta t} = C_t + [r_t \cdot C_t + (g - p) \cdot S_t] \cdot \Delta t + \sigma_C \cdot C_t \cdot \sqrt{\Delta t} \cdot Z_C, \tag{9}$$

where p is the deferred proportional cost on salary. In order to maintain the central theme as being salary-related throughout this paper, we only model the cost proportional to salary and leave the alternative of the cost proportional to the value of the IPA aside here.[5]

3 According to Hull (1997), equation (6) leads directly to the risk-neutral process for interest rates, and the necessary risk-adjustments are incorporated in equation (6), so that the assumption about the market price of interest rate risk does not matter in the current valuation.

4 One can also review other discrete-time approximate schemes for square root diffusions in the studies of Andersen (2008) and van Haastrecht and Pelsser (2010). They also proposed some new algorithms for time discretization of the Heston stochastic volatility model, in that the volatility process has the same form as the interest rate process of Cox *et al.* (1985).

5 While the proportional deferred cost (p) is charged and increasing, the net residual up-front value of the guarantee should be decreasing to zero when it reaches a required charge fee. Herein, we exclude the case when the deferred fee is deeply overcharged, where the term $(g - p)$ in equation (9) could become a minus.

Regarding the expected terminating rate, we suppose that the guarantee should ultimately mature when the individual reaches 60 years old and assume two types, 5% and no lapse, for the deterministic annual lapse intensity. Table 1 shows the female mortality rates on an annual basis based on the fourth and third experience mortality tables provided by the Life Insurance Association of R.O.C.,[6] which are used respectively as the low and high types of expected intensity of individual death (EIID). The ratio of the EIID of the low type to the high type is about 50%–60%. For monthly calculations, the early terminations are assumed to be uniformly distributed over the year of age. The two-type lapse and mortality purposes not only mitigate the question from the arbitrary choice of the rate, but also mainly help analyze the sensitivity of the guarantee value to early terminating intensity.

We assume the reset provision is ignored or used optimally at any time a new contribution is made. If resets are ignored, then the guarantee level at time $t + \Delta t$, $K_{t+\Delta t}$, satisfies $K_{t+\Delta t} = K_t + g \times St \times \Delta t$. If the resets are used optimally, then $K_{t+\Delta t} = \max\{C_t, K_t\} + g \times S_t \times \Delta t$. The value of a reset feature is observed from the difference in results between the two types of reset.[7]

Results and Discussion

The results and discussion are followed by carrying out Monte Carlo simulations based on 11 scenario assumptions about the parameter values of the stochastic variables as summarized in Table 2. Scenario 1 is the base scenario, and each of the latter eight scenarios adjusts one of the parameter values to see its impact on the results. Although none of the scenarios are factual enough for the real world, the differences in results among the scenarios contribute to the discussion on the sensitivities of the value of the guarantee and the values of a death benefit and a reset feature attached to the guarantee on the parameter values.

Within Scenario 1, assume that both coefficients of correlations ρ_{SC} and ρ_{Cr} are equal to zero. The annual average growth rate of salary (w) and its volatility (σ_S) are respectively 0.04 and 0.03, the market price of salary risk is zero, and the volatility of the return on the asset of an IPA, σ_C, is conservatively assumed to be 0.04. We take the parameter values for the process of the risk-free interest rate from Maurer and Schlag (2003) as a benchmark, where the mean-reverting drift (representing the long-run average level) β, the reversion rate α, and the volatility σ_r are respectively equal to 0.05, 0.15, and 0.05.[8] We set the initial risk-free interest rate or the risk-free interest rate at the pricing time, r_0, at 0.03, reflecting a low interest rate relative to its long-run average level.

Following Scenario 1, the volatility of the return on the asset of an IPA, σ_C, is doubled to 0.08 in Scenario 2. The parameter values for the risk-free interest rate

6 The Life Insurance Association of R.O.C. develops a new mortality table every 8 years, with each mortality table developed based on Taiwan's life insurance experiences of the preceding 4 years.

7 One could treat an optimal reset herein as an automatic reset conditional on the value of the current accumulation of the IPA, C_t, being greater than the current guarantee level K_t.

8 The original estimates in Maurer and Schlag (2003) are 0.0539, 0.1494, and 0.0511 for the mean-reverting drift, reversion rate, and the volatility, respectively.

Table 1. *The EIID*

Age	EIID (low type)	EIID (high type)
20	0.000530	0.000838
21	0.000536	0.000849
22	0.000533	0.000855
23	0.000525	0.000860
24	0.000515	0.000870
25	0.000507	0.000890
26	0.000504	0.000926
27	0.000510	0.000982
28	0.000527	0.001063
29	0.000556	0.001159
30	0.000593	0.001259
31	0.000638	0.001353
32	0.000688	0.001428
33	0.000743	0.001479
34	0.000802	0.001516
35	0.000865	0.001551
36	0.000931	0.001599
37	0.001001	0.001675
38	0.001074	0.001789
39	0.001153	0.001944
40	0.001240	0.002138
41	0.001336	0.002371
42	0.001445	0.002641
43	0.001567	0.002947
44	0.001707	0.003280
45	0.001867	0.003633
46	0.002049	0.003997
47	0.002257	0.004362
48	0.002491	0.004723
49	0.002747	0.005090
50	0.003017	0.005474
51	0.003294	0.005889
52	0.003572	0.006346
53	0.003848	0.006852
54	0.004140	0.007393
55	0.004469	0.007949
56	0.004858	0.008499
57	0.005327	0.009024
58	0.005895	0.009521
59	0.006554	0.010064

Note: These are the female mortality rates on an annual basis based on the fourth and third experience mortality tables provided by the Life Insurance Association of R.O.C. The former is used as the low type of EIID and the latter is used as the high type of EIID.

as σ_r decrease to be 0.025 in Scenario 3, as β increases to be 7% in Scenario 4, as r_0 increases to be 5% in Scenario 5, and as α increases to be 0.3 in Scenario 6. The average salary growth rate (w) is increased to 7% in Scenario 7, and the market price of

Table 2. *The assumptions of stochastic scenarios*

Scenario	w	σ_S	λ_S	σ_C	α	β	σ_r	r_0
1	0.04	0.03	0	0.04	0.15	0.05	0.05	0.03
2	–	–	–	0.08	–	–	–	–
3	–	–	–	–	–	–	0.025	–
4	–	–	–	–	–	0.07	–	–
5	–	–	–	–	–	–		0.05
6	–	–	–	–	0.30	–	–	–
7	0.07	–	–	–	–	–	–	–
8	–		−0.1	–	–	–	–	–
9	–	0.06	−0.1	–	–	–	–	–

Note: This table presents the assumptions of the main parameter values for equations (4)–(6). Here, r_0 is the interest rate at pricing time. The short dotted line (–) hints that the referred value is assumed to be the same as the one in Scenario 1 and therefore omitted.

salary risk (λ_S), is adjusted to −0.1 in both Scenarios 8 and 9. Scenario 9 also simultaneously adjusts the volatility of salary σ_S to 0.06 in order to see the effect of an increase in the volatility of salary with a negative market price of salary risk.[9] Lastly, we respectively adjust the coefficients of correlations ρ_{SC} and ρ_{Cr} to 0.5 in Scenarios 10 and 11.

For all scenarios, we assume that the initial yearly salary is US$240,000, the initial accumulation of an IPA is zero, and the contribution rate of an IPA is 6%, which is the current mandatory contribution rate for Taiwan's labor IPAs. We use a monthly time interval, setting the size of the time step $\Delta t = 1/12$. Every final valuation is calculated by the arithmetic average of the valuations from the 30,000 paths. All simulations are executed by the version 6.0.0.1 of 'The GAUSS Mathematical and Statistical System'. Since the time stepping scheme does not fail in executing the discrete process of the interest rate of equation (6), we do not use the Full Truncation fix of Lord *et al.* (2010) as in equation (7). Before determining the number of simulation paths to be 30,000, we first execute 20,000 paths and then increase the number to 30,000. The simulation's estimates of the up-front guarantee values remain almost the same, while the standard errors of the simulations drop to a reasonably small level.[10]

The up-front guarantee values

Tables 3 and 4 present the up-front guarantee values as a fraction of the initial contribution of the IPA for individuals whose ages are 20, 25, 30, 35, 40, and 45. The standard errors for every valuation are in the parentheses. Values in Table 4 are the results when ignoring the reset feature. We omit the results for Scenarios 10

9 The negative market price of salary risk is considered, as salary growth and the return on the market portfolio could be negatively correlated.

10 One can refer to chapter 15 of Hull (1997) for some variance reduction procedures for Monte Carlo simulation when the strict accuracy of a simulation is required with a limited cost of computation time.

Table 3. *The up-front guarantee values with a death benefit and a reset feature as a fraction of the initial monthly contribution*

Age	Type of EIID	Annual lapse intensity	Scenario 1	2	3	4	5	6	7	8	9
20	High	0.05	0.3550	1.5602	0.2988	0.1741	0.3197	0.2968	0.6159	0.3733	0.3931
			(0.0022)	(0.0078)	(0.0018)	(0.0012)	(0.0020)	(0.0018)	(0.0040)	(0.0023)	(0.0025)
		0	2.2875	10.1273	1.9143	1.1013	2.0658	1.9071	4.0708	2.4115	2.5468
			(0.0161)	(0.0569)	(0.0130)	(0.0090)	(0.0146)	(0.0131)	(0.0291)	(0.0170)	(0.0185)
	Low	0.05	0.3196	1.4121	0.2678	0.1547	0.2883	0.2664	0.5664	0.3368	0.3554
			(0.0023)	(0.0081)	(0.0019)	(0.0013)	(0.0021)	(0.0019)	(0.0042)	(0.0024)	(0.0026)
		0	2.1768	9.6610	1.8170	1.0413	1.9667	1.8109	3.9190	2.2973	2.4283
			(0.0170)	(0.0594)	(0.0137)	(0.0095)	(0.0154)	(0.0138)	(0.0306)	(0.0179)	(0.0194)
25	High	0.05	0.3887	1.6990	0.3320	0.1954	0.3499	0.3301	0.6293	0.4063	0.4251
			(0.0024)	(0.0085)	(0.0020)	(0.0014)	(0.0022)	(0.0020)	(0.0041)	(0.0026)	(0.0027)
		0	1.9888	8.7561	1.6921	0.9838	1.7963	1.6874	3.2837	2.0833	2.1834
			(0.0139)	(0.0487)	(0.0114)	(0.0079)	(0.0126)	(0.0116)	(0.0233)	(0.0146)	(0.0157)
	Low	0.05	0.3539	1.5562	0.3013	0.1756	0.3193	0.3002	0.5834	0.3706	0.3884
			(0.0026)	(0.0089)	(0.0021)	(0.0014)	(0.0023)	(0.0021)	(0.0043)	(0.0027)	(0.0029)
		0	1.8998	8.3904	1.6131	0.9332	1.7172	1.6103	3.1696	1.9921	2.0897
			(0.0148)	(0.0508)	(0.0121)	(0.0083)	(0.0133)	(0.0122)	(0.0245)	(0.0153)	(0.0164)
30	High	0.05	0.4204	1.8097	0.3634	0.2187	0.3780	0.3603	0.6354	0.4370	0.4544
			(0.0027)	(0.0092)	(0.0023)	(0.0016)	(0.0025)	(0.0023)	(0.0042)	(0.0028)	(0.0030)
		0	1.7093	7.4032	1.4730	0.8772	1.5418	1.4648	2.6205	1.7791	1.8528
			(0.0121)	(0.0411)	(0.0101)	(0.0070)	(0.0110)	(0.0102)	(0.0187)	(0.0126)	(0.0134)
	Low	0.05	0.3878	1.6780	0.3342	0.1993	0.3495	0.3322	0.5946	0.4036	0.4203
			(0.0028)	(0.0096)	(0.0024)	(0.0017)	(0.0026)	(0.0024)	(0.0044)	(0.0030)	(0.0031)
		0	1.6407	7.1267	1.4113	0.8362	1.4817	1.4055	2.5381	1.7091	1.7814
			(0.0126)	(0.0428)	(0.0106)	(0.0074)	(0.0115)	(0.0107)	(0.0197)	(0.0132)	(0.0140)

35	High	0.05	0.4421	1.8701	0.3868	0.2388	0.3956	0.3804	0.6232	0.4567	0.4719
			(0.0029)	(0.0096)	(0.0025)	(0.0018)	(0.0026)	(0.0025)	(0.0042)	(0.0030)	(0.0032)
		0	1.4272	6.0699	1.2458	0.7624	1.2815	1.2286	2.0317	1.4757	1.5262
			(0.0101)	(0.0334)	(0.0086)	(0.0062)	(0.0091)	(0.0086)	(0.0145)	(0.0104)	(0.0110)
	Low	0.05	0.4127	1.7553	0.3603	0.2205	0.3705	0.3553	0.5886	0.4268	0.4415
			(0.0030)	(0.0100)	(0.0026)	(0.0019)	(0.0028)	(0.0026)	(0.0044)	(0.0031)	(0.0033)
		0	1.3761	5.8721	1.1996	0.7302	1.2381	1.1851	1.9742	1.4240	1.4737
			(0.0105)	(0.0348)	(0.0091)	(0.0065)	(0.0096)	(0.0091)	(0.0152)	(0.0109)	(0.0115)
40	High	0.05	0.4419	1.8316	0.3903	0.2502	0.3911	0.3797	0.5806	0.4536	0.4661
			(0.0030)	(0.0096)	(0.0026)	(0.0019)	(0.0027)	(0.0026)	(0.0040)	(0.0031)	(0.0032)
		0	1.1319	4.7154	0.9980	0.6350	1.0053	0.9733	1.4967	1.1626	1.1954
			(0.0081)	(0.0259)	(0.0070)	(0.0051)	(0.0073)	(0.0069)	(0.0108)	(0.0083)	(0.0087
	Low	0.05	0.4180	1.7435	0.3684	0.2341	0.3715	0.3595	0.5540	0.4294	0.4416
			(0.0031)	(0.0100)	(0.0027)	(0.0020)	(0.0028)	(0.0027)	(0.0042)	(0.0032)	(0.0033)
		0	1.0978	4.5923	0.9664	0.6114	0.9775	0.9446	1.4604	1.1282	1.1607
			(0.0084)	(0.0269)	(0.0073)	(0.0053)	(0.0076)	(0.0072)	(0.0112)	(0.0086)	(0.0090)
45	High	0.05	0.4225	1.6760	0.3780	0.2538	0.3656	0.3601	0.5175	0.4309	0.4397
			(0.0030)	(0.0092)	(0.0026)	(0.0020)	(0.0027)	(0.0026)	(0.0037)	(0.0031)	(0.0032)
		0	0.8601	3.4236	0.7685	0.5134	0.7463	0.7331	1.0568	0.8775	0.8958
			(0.0063)	(0.0193)	(0.0055)	(0.0042)	(0.0056)	(0.0054)	(0.0078)	(0.0064)	(0.0066)
	Low	0.05	0.4069	1.6224	0.3633	0.2420	0.3536	0.3470	0.5011	0.4152	0.4240
			(0.0031)	(0.0095)	(0.0027)	(0.0021)	(0.0028)	(0.0027)	(0.0038)	(0.0032)	(0.0033)
		0	0.8419	3.3635	0.7511	0.4990	0.7328	0.7179	1.0387	0.8592	0.8775
			(0.0065)	(0.0199)	(0.0057)	(0.0044)	(0.0058)	(0.0056)	(0.0081)	(0.0067)	(0.0069)

Source: Author’s calculations.
Note: The standard errors for every valuation are in the parentheses (Is). A 95% confidence interval for every valuation is between every valuation ± $1.96I$.

Table 4. *The up-front guarantee values ignoring resets as a fraction of the initial monthly contribution*

Age	Type of EIID	Annual lapse intensity	Scenario 1	2	3	4	5	6	7	8	9
20	High	0.05	0.0006	0.0167	0.0003	0.0002	0.0002	0.0003	0.0007	0.0006	0.0006
			(0.0000)	(0.0005)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
		0	0.0009	0.0625	0.0004	0.0004	0.0004	0.0003	0.0015	0.0010	0.0010
			(0.0000)	(0.0035)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0003)	(0.0000)	(0.0001)
	Low	0.05	0.0003	0.0111	0.0002	0.0001	0.0001	0.0002	0.0004	0.0003	0.0003
			(0.0000)	(0.0005)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
		0	0.0005	0.0488	0.0002	0.0002	0.0002	0.0002	0.0010	0.0005	0.0006
			(0.0000)	(0.0035)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0003)	(0.0000)	(0.0001)
25	High	0.05	0.0008	0.0248	0.0004	0.0003	0.0003	0.0003	0.0011	0.0008	0.0008
			(0.0000)	(0.0008)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0000)
		0	0.0015	0.0876	0.0005	0.0007	0.0007	0.0004	0.0026	0.0015	0.0016
			(0.0002)	(0.0043)	(0.0000)	(0.0002)	(0.0002)	(0.0000)	(0.0006)	(0.0002)	(0.0002)
	Low	0.05	0.0005	0.0176	0.0002	0.0001	0.0002	0.0002	0.0007	0.0005	0.0005
			(0.0000)	(0.0008)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0000)
		0	0.0009	0.0714	0.0003	0.0002	0.0005	0.0002	0.0020	0.0010	0.0010
			(0.0002)	(0.0043)	(0.0000)	(0.0000)	(0.0002)	(0.0000)	(0.0006)	(0.0002)	(0.0003)
30	High	0.05	0.0011	0.0373	0.0005	0.0004	0.0005	0.0004	0.0014	0.0011	0.0012
			(0.0001)	(0.0012)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0001)
		0	0.0020	0.1181	0.0007	0.0009	0.0009	0.0006	0.0030	0.0020	0.0021
			(0.0003)	(0.0050)	(0.0000)	(0.0002)	(0.0002)	(0.0000)	(0.0005)	(0.0003)	(0.0003)
	Low	0.05	0.0007	0.0282	0.0003	0.0002	0.0003	0.0002	0.0009	0.0007	0.0007
			(0.0001)	(0.0012)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0001)
		0	0.0013	0.0995	0.0004	0.0003	0.0006	0.0003	0.0022	0.0013	0.0014
			(0.0003)	(0.0050)	(0.0000)	(0.0000)	(0.0002)	(0.0000)	(0.0005)	(0.0003)	(0.0003)

35	High	0.05	0.0015	0.0577	0.0007	0.0005	0.0006	0.0006	0.0018	0.0015	0.0016
			(0.0001)	(0.0017)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0001)
		0	0.0025	0.1595	0.0009	0.0003	0.0010	0.0008	0.0032	0.0025	0.0026
			(0.0002)	(0.0055)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0003)	(0.0002)	(0.0002)
	Low	0.05	0.0009	0.0463	0.0004	0.0003	0.0003	0.0003	0.0011	0.0009	0.0009
			(0.0001)	(0.0016)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0001)
		0	0.0015	0.1387	0.0005	0.0004	0.0006	0.0004	0.0020	0.0015	0.0016
			(0.0002)	(0.0055)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0003)	(0.0002)	(0.0002)
40	High	0.05	0.0028	0.0912	0.0011	0.0008	0.0012	0.0009	0.0036	0.0029	0.0030
			(0.0002)	(0.0024)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0003)	(0.0002)	(0.0002)
		0	0.0050	0.2149	0.0015	0.0022	0.0022	0.0013	0.0069	0.0052	0.0053
			(0.0005)	(0.0063)	(0.0001)	(0.0003)	(0.0003)	(0.0001)	(0.0007)	(0.0005)	(0.0005)
	Low	0.05	0.0018	0.0779	0.0006	0.0005	0.0008	0.0005	0.0025	0.0019	0.0020
			(0.0002)	(0.0024)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0003)	(0.0002)	(0.0002)
		0	0.0036	0.1941	0.0009	0.0006	0.0016	0.0007	0.0052	0.0037	0.0039
			(0.0005)	(0.0064)	(0.0001)	(0.0000)	(0.0003)	(0.0001)	(0.0007)	(0.0005)	(0.0005)
45	High	0.05	0.0057	0.1343	0.0021	0.0014	0.0023	0.0017	0.0071	0.0058	0.0060
			(0.0003)	(0.0032)	(0.0001)	(0.0001)	(0.0002)	(0.0001)	(0.0005)	(0.0004)	(0.0004)
		0	0.0099	0.2631	0.0029	0.0040	0.0040	0.0024	0.0128	0.0102	0.0105
			(0.0007)	(0.0066)	(0.0002)	(0.0005)	(0.0005)	(0.0002)	(0.0009)	(0.0007)	(0.0008)
	Low	0.05	0.0044	0.1216	0.0013	0.0008	0.0018	0.0010	0.0057	0.0045	0.0047
			(0.0003)	(0.0032)	(0.0001)	(0.0001)	(0.0002)	(0.0001)	(0.0005)	(0.0004)	(0.0004)
		0	0.0082	0.2460	0.0019	0.0012	0.0034	0.0016	0.0109	0.0084	0.0088
			(0.0007)	(0.0067)	(0.0002)	(0.0001)	(0.0005)	(0.0002)	(0.0010)	(0.0008)	(0.0008)

Source: Author's calculations.
Note: The standard errors for every valuation are in the parentheses (Is). A 95% confidence interval for every valuation is between every valuation $\pm$ 1.96I.

Table 5. *The percentage reduction in the up-front guarantee values when the annual lapse intensity increases from zero to 0.05 (using the results in Table 3 for calculation)*

		Scenario								
Age	Type of EIID	1	2	3	4	5	6	7	8	9
20	High	84.48	84.59	84.39	84.19	84.52	84.44	84.87	84.52	84.56
	Low	85.32	85.38	85.26	85.14	85.34	85.29	85.55	85.34	85.36
25	High	80.46	80.60	80.38	80.14	80.52	80.44	80.84	80.50	80.53
	Low	81.37	81.45	81.32	81.18	81.41	81.36	81.59	81.40	81.41
30	High	75.41	75.56	75.33	75.07	75.48	75.40	75.75	75.44	75.48
	Low	76.36	76.45	76.32	76.17	76.41	76.36	76.57	76.39	76.41
35	High	69.02	69.19	68.95	68.68	69.13	69.04	69.33	69.05	69.08
	Low	70.01	70.11	69.97	69.80	70.08	70.02	70.19	70.03	70.04
40	High	60.96	61.16	60.89	60.60	61.10	60.99	61.21	60.98	61.01
	Low	61.92	62.03	61.88	61.71	61.99	61.94	62.07	61.94	61.95
45	High	50.88	51.05	50.81	50.56	51.01	50.88	51.03	50.89	50.92
	Low	51.67	51.76	51.63	51.50	51.75	51.66	51.76	51.68	51.68

and 11 to save space, because those results are almost the same as the results for Scenario 1. This shows that a variation in the conditional correlations between salary, the accumulation of the IPA, and the interest rate has very little effect on the guarantee's values.[11] Although each simulation proceeds by an initial yearly salary of US $240,000 and a constant contribution rate of 6%, the results are robust to a change in the initial salary or the contribution rate. That means every up-front guarantee value before being divided by the initial contribution fluctuates with the initial salary and the contribution rate on a constant scale.

Compared to Table 3, the values in Table 4 are dramatically lower, showing that a principal guarantee with a death benefit on an IPA relative to the one with both a death benefit and a reset feature is worth little. We find that the guarantee has an average 99.43% value contributed by the reset feature, which is consistent with the result by Windcliff *et al.* (2001). The rest of this subsection focuses on discussing the findings in Table 3.

Before discussing the results among different scenarios in Table 3, we summarize the main findings based on the results with different types of EIID and lapse. First, up-front guarantee values decrease by over 50% and up to 80%, while the annual lapse intensity increases from zero to 0.05. Table 5 shows the percentage reduction in the up-front guarantee values when the annual lapse intensity increases. By lapsing, the holder of the guarantee surrenders the right to the contract and the writer is not responsible for any payments, and hence lapsing from the guarantee is beneficial to the writer. Table 5 also further shows how these up-front guarantee values are reduced

11 The assumption that the coefficients of correlations ρ_{SC} and ρ_{Cr} are equal to zero only means that the conditional distributions of salary, the accumulation of the IPA, and the interest rate are independent. Based on equation (5), the accumulation of an IPA is in fact affected by the risk-free interest (r) and salary (S).

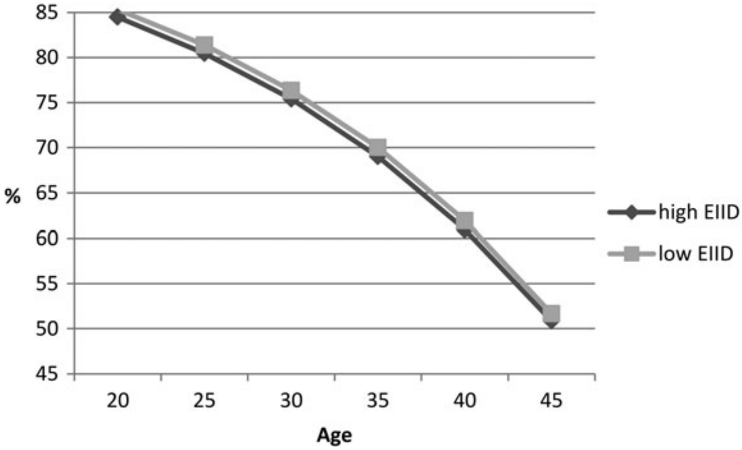

Figure 1. The percentage reduction in the up-front guarantee values when the annual lapse intensity increases from zero to 0.05 for Scenario 1 (using the results in Table 3 for calculation).

from a lapsing loss in a greater proportion for younger workers, whereas in a stable proportion among different scenarios. Figure 1 illustrates the above relationships between the percentage reduction and age for Scenario 1. Although modeling a holder's lapsing behavior is not simple work, one can infer that if lapsing from the guarantee is beneficial to the writer, then the holder remains in the contract as long as possible and lapses simply for liquidity or other non-strategic reasons, like individual unemployment.[12]

Table 6 shows the percentage increment in the up-front guarantee values when the high type of EIID is used instead of the low type of EIID, where the up-front guarantee values go up about 2% to 10%. The high type averages about 1.6 to 2 times the intensity of the low type. Since the guarantee averages a 99.43% value contributed by the reset feature, one infers that an increase in the value of a death benefit from higher death intensity is also mainly contributed by the reset feature. We find the contribution of a reset feature on the value of a death benefit by observing the spreads of figures in Tables 3 and 4 among different types of EIID. Table 6 further shows how the guarantee values increase from higher death intensity in greater proportion for younger workers and in a stable proportion among nine scenarios. Figure 2 illustrates the above relationships between the percentage increment and age for Scenario 1.

Given the type of EIID, the relationship between the up-front guarantee value and individual age depends on the holder's lapsing behavior. The results in Table 3 show that the up-front guarantee value monotonically decreases with the individual's age if there is no lapse. However, for an annual lapse intensity of 0.05, a simple monotonic relationship between the guarantee value and the individual age does not exist anymore, due to the fact that the time value of a longer contract is diluted by a greater

12 Although individual unemployment cannot cause the termination of an IPA since the IPAs are portable for individuals, it could result in a lapse from the guarantee on the IPA for the reason of individual liquidity concerns or a lapse provision of the guarantee contract.

Table 6. *The percentage increment in the up-front guarantee values when the high type of EIID is used instead of the low one (using the results in Table 3 for calculation)*

Age	Annual lapse intensity	Scenario 1	2	3	4	5	6	7	8	9
20	0.05	11.08	10.49	11.58	12.54	10.89	11.41	8.74	10.84	10.61
	0	5.09	4.83	5.36	5.76	5.04	5.31	3.87	4.97	4.88
25	0.05	9.83	9.18	10.19	11.28	9.58	9.96	7.87	9.63	9.45
	0	4.68	4.36	4.90	5.42	4.61	4.79	3.60	4.58	4.48
30	0.05	8.41	7.85	8.74	9.73	8.15	8.46	6.86	8.28	8.11
	0	4.18	3.88	4.37	4.90	4.06	4.22	3.25	4.10	4.01
35	0.05	7.12	6.54	7.36	8.30	6.77	7.06	5.88	7.01	6.89
	0	3.71	3.37	3.85	4.41	3.51	3.67	2.91	3.63	3.56
40	0.05	5.72	5.05	5.94	6.88	5.28	5.62	4.80	5.64	5.55
	0	3.11	2.68	3.27	3.86	2.84	3.04	2.49	3.05	2.99
45	0.05	3.83	3.30	4.05	4.88	3.39	3.78	3.27	3.78	3.70
	0	2.16	1.79	2.32	2.89	1.84	2.12	1.74	2.13	2.09

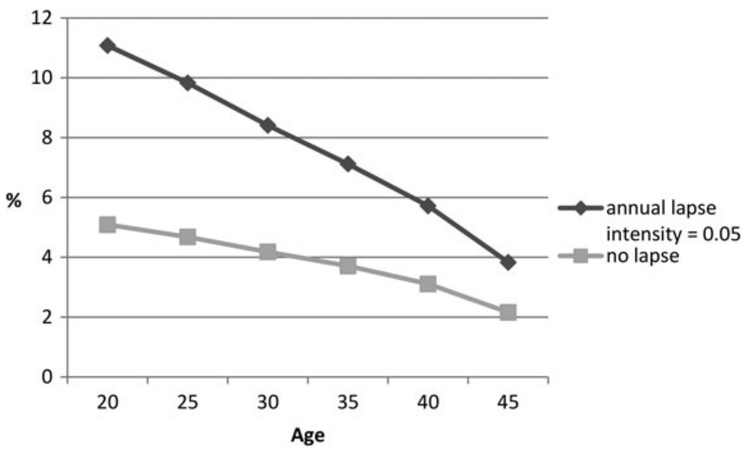

Figure 2. The percentage increment in the up-front guarantee values when the high type of EIID is used instead of the low one for Scenario 1 (using the results in Table 3 for calculation).

lapse loss. Figure 3 illustrates the above relationships between the up-front guarantee value and the individual's age using Scenario 1 with a high type of EIID.

After discussing the results of horizontal sections in Table 3, we proceed to discuss the results among different scenarios in order to analyze the sensitivities of guarantee values on parameter values. First, there is a notable spread between results in the first two scenarios. While the volatility of the asset of the IPA doubles in Scenario 2, it averages over four times the guarantee value of Scenario 1. Because the guarantee value is notably sensitive to the volatility of the IPA's assets, it is an important requirement of the guarantee contract that the asset of an IPA must be connected

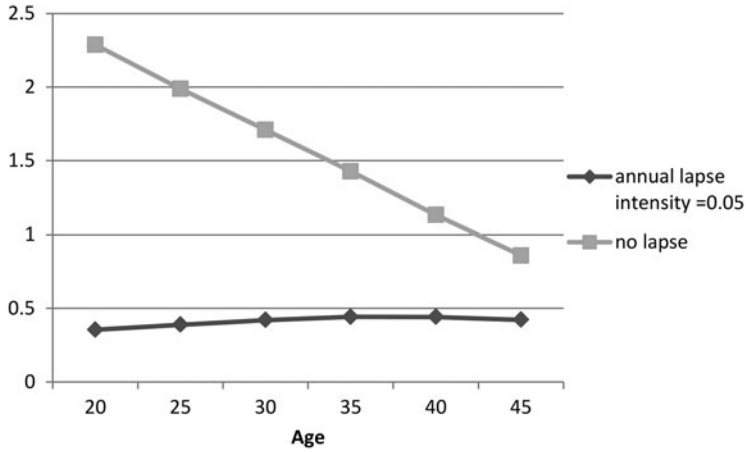

Figure 3. The up-front guarantee values with a death benefit and a reset feature as a fraction of the initial monthly contribution using a high type of EIID and Scenario 1 from the results in Table 3.

with a specific traded portfolio or fund so as to reduce both moral hazard and basis risk for the contract provider.

While we discuss the sensitivities of results on the risk-free interest rate, we are reminded that the interest rate affects the yield of the asset of an IPA, aside from impacting the present value interest factor. Those two effects are partially offset since they cause the guarantee value (especially the reset value) to change in opposite directions. Nevertheless, one still can see that the up-front guarantee value is positively related to the volatility of the interest rate (σ_r) and negatively related to the interest rate at pricing time (r_0) and the long-run average interest rate (β) based on comparing the results of Scenarios 3–5 to the results of Scenario 1 in Table 3. Nevertheless, while the guarantee value averages a 50% decline when the long-run average interest rate rises from 0.05 (in Scenario 1) to 0.07 (in Scenario 4), the sensitivities of the guarantee value on interest rate volatility and the interest rate at pricing time are not obvious.

The results in Scenario 6 additionally show that when the reversion rate of the interest rate doubles, the guarantee values decline as Scenario 3 where the volatility of the interest rate decreases in half. This is due to the fact that, once the interest rate deviates far from the long-term average, the duration of adjustment toward the long-term average shortens as its reversion rate lengthens. Thus, with a higher reversion rate, the chances for abnormally high or low interest rates are fewer. The effect of a higher reversion rate of the interest rate on the guarantee value is thus similar to the effect of a lower volatility of the interest rate.

The discussion now proceeds to the sensitivity of the guarantee value on salary. The average salary growth affects the size of the asset of an IPA and has a positive relationship with the guarantee value. Scenario 7 assumes an expected salary growth of 0.07 and averages about 1.5 times the guarantee value of Scenario 1 where the expected salary growth is 0.04. Furthermore, the sensitivity of the up-front guarantee

Table 7. *The required annual fee rate on salary: given a contribution rate of 6% (in basis points)*

Age	Type of EIID	Annual lapse intensity	Scenario 1	2	3	4	5	6	7	8	9
20	High	0.05	1.08	4.74	0.93	0.62	1.07	0.95	1.22	1.10	1.11
		0	3.19	13.92	2.73	1.97	3.19	2.85	3.06	3.17	3.16
	Low	0.05	0.97	4.25	0.83	0.55	0.96	0.85	1.11	0.98	1.02
		0	2.99	13.08	2.56	1.84	2.99	2.67	2.88	2.98	2.96
25	High	0.05	1.25	5.37	1.07	0.72	1.23	1.10	1.36	1.26	1.27
		0	3.14	13.56	2.71	1.93	3.14	2.82	3.04	3.13	3.12
	Low	0.05	1.13	4.87	0.96	0.65	1.11	0.99	1.25	1.14	1.15
		0	2.96	12.78	2.54	1.81	2.96	2.65	2.87	2.95	2.94
30	High	0.05	1.42	6.03	1.23	0.83	1.39	1.26	1.51	1.43	1.44
		0	3.08	13.14	2.67	1.88	3.07	2.77	2.99	3.08	3.07
	Low	0.05	1.29	5.52	1.12	0.75	1.27	1.15	1.39	1.30	1.31
		0	2.91	12.42	2.52	1.76	2.90	2.61	2.84	2.90	2.89
35	High	0.05	1.63	6.75	1.43	0.98	1.58	1.45	1.69	1.63	1.64
		0	3.07	12.79	2.69	1.89	3.03	2.76	3.00	3.06	3.06
	Low	0.05	1.50	6.25	1.31	0.90	1.46	1.34	1.57	1.51	1.51
		0	2.91	12.14	2.55	1.79	2.88	2.62	2.85	2.91	2.90
40	High	0.05	1.82	7.47	1.61	1.12	1.74	1.61	1.86	1.82	1.83
		0	3.00	12.36	2.66	1.87	2.90	2.68	2.94	2.99	2.99
	Low	0.05	1.70	7.00	1.50	1.03	1.63	1.51	1.74	1.70	1.71
		0	2.85	11.80	2.53	1.77	2.77	2.55	2.80	2.85	2.84
45	High	0.05	2.02	8.17	1.82	1.27	1.88	1.77	2.04	2.03	2.03
		0	2.93	11.89	2.63	1.85	2.74	2.57	2.88	2.93	2.92
	Low	0.05	1.91	7.76	1.71	1.19	1.78	1.67	1.93	1.92	1.92
		0	2.81	11.44	2.51	1.76	2.64	2.46	2.76	2.80	2.80

Source: Author's calculations.

value to the volatility of salary depends on the market price of salary risk. When the market price of salary risk is assumed to be zero, the influence of the volatility of salary on the guarantee value is not shown, because it is very small. However, in the case where the market price of salary risk is assumed to be −0.1, the guarantee values become sensitive to the volatility of salary, as seen by comparing the results in Scenario 9 with those in Scenario 8. This finding is consistent with the previous study of Chen (2006) who valued the guarantee to exchange back a DB for the DC pension plan. An accurate estimation of the market price of salary risk is thus important for valuing an IPA's guarantee.

The required deferred costs

Table 7 presents the required deferred cost proportional on salary, given a fixed contribution rate of 6%. We approximate each estimate by gradual interpolation to make the up-front guarantee value zero. Since the deferred cost is charged over the life of

Table 8. *The percentage reduction in the required annual fee rate when the annual lapse intensity increases from zero to 0.05 (using the results in Table 7 for calculation)*

		Scenario								
Age	Type of EIID	1	2	3	4	5	6	7	8	9
20	High	66.14	65.95	65.93	68.53	66.46	66.67	60.13	65.30	64.87
	Low	67.56	67.51	67.58	70.11	67.89	68.16	61.46	67.11	65.54
25	High	60.19	60.40	60.52	62.69	60.83	60.99	55.26	59.74	59.29
	Low	61.82	61.89	62.20	64.09	62.50	62.64	56.45	61.36	60.88
30	High	53.90	54.11	53.93	55.85	54.72	54.51	49.50	53.57	53.09
	Low	55.67	55.56	55.56	57.39	56.21	55.94	51.06	55.17	54.67
35	High	46.91	47.22	46.84	48.15	47.85	47.46	43.67	46.73	46.41
	Low	48.45	48.52	48.63	49.72	49.31	48.85	44.91	48.11	47.93
40	High	39.33	39.56	39.47	40.11	40.00	39.93	36.73	39.13	38.80
	Low	40.35	40.68	40.71	41.81	41.16	40.78	37.86	40.35	39.79
45	High	31.06	31.29	30.80	31.35	31.39	31.13	29.17	30.72	30.48
	Low	32.03	32.17	31.87	32.39	32.58	32.11	30.07	31.43	31.43

the guarantee contract, on a same cost rate, a young individual is expected to be charged more than an older individual. Moreover, the fact that the deferred cost is extracted from the accumulation of the IPA once it is charged overcomes the effect of the reset that pushes the guarantee level to the prevailing accumulation of an IPA. The above finding provides clues among different ages as to where, without lapse, the required deferred cost rate does not appreciably rise for younger individuals who are charged a higher up-front guarantee value. Nevertheless, a lapse intensity of 0.05 makes the required deferred cost rate monotonically increase with the individual age to reflect the lower lapse loss for older individuals.

The diversity of the required deferred costs among different types of EIID and lapse intensity appears to be similar to the diversity of the up-front values. Given the individual age, the high type of EIID with no lapse carries the heaviest required deferred cost rate to cover the guarantee value for all scenarios. Moreover, the required deferred costs do not decrease as rapidly as the up-front values to reflect a greater lapse loss when the deterministic lapse intensity increases, yet they do increase faster than the up-front values when the high type of EIID is used instead of the low one to reflect more death benefits. It is due to the fact that, given a deferred cost rate, the increase in the early terminating intensity causes the effective life of the contract to decrease, and consequently the value of the future cost drops. Tables 8 and 9 show the percentage changes of those required deferred costs when the lapse intensity and EIID increase, respectively. One can correspondingly compare figures in Tables 8 and 9 with those in Tables 5 and 6 to find the differences of a move's speed between the required deferred cost rate and the up-front value along the different types of lapse and EIID.

Among different scenarios, one finding that deserves to be mentioned is that an increase in salary growth from 4% to 7% (Scenario 7) does not cause the required

Table 9. *The percentage increment in the required annual fee rate when the high type EIID is used instead of the low type one (using the results in Table 7 for calculation)*

		Scenario							
Age	Annual lapse intensity	1	2	3	4	5	6	7	8
20	0.05	11.34	11.53	12.05	12.73	11.46	11.76	9.91	12.24
	0	6.69	6.42	6.64	7.07	6.69	6.74	6.25	6.38
25	0.05	10.62	10.27	11.46	10.77	10.81	11.11	8.80	10.53
	0	6.08	6.10	6.69	6.63	6.08	6.42	5.92	6.10
30	0.05	10.08	9.24	9.82	10.67	9.45	9.57	8.63	10.00
	0	5.84	5.80	5.95	6.82	5.86	6.13	5.28	6.21
35	0.05	8.67	8.00	9.16	8.89	8.22	8.21	7.64	7.95
	0	5.50	5.35	5.49	5.59	5.21	5.34	5.26	5.15
40	0.05	7.06	6.71	7.33	8.74	6.75	6.62	6.90	7.06
	0	5.26	4.75	5.14	5.65	4.69	5.10	5.00	4.91
45	0.05	5.76	5.28	6.43	6.72	5.62	5.99	5.70	5.73
	0	4.27	3.93	4.78	5.11	3.79	4.47	4.35	4.64

deferred cost rate to change much. It could be due to the fact that the deferred cost is modeled as a proportion of contemporaneous salary, and the required deferred cost rate therefore does not need to change much to cover the appreciation of the contract caused by the salary growth. However, one still can see that as the salary growth rises, the required deferred cost rate slightly decreases if there is no lapse and slightly increases if there is an annual lapse intensity of 0.05. Higher wage growth also brings diminutions in the percentage changes in the required deferred costs along different lapse intensities and different types of EIID. We can see this by respectively comparing the figures of Scenario 7 with the ones of Scenario 1 in Tables 8 and 9. Moreover, a negative market price of salary risk (in Scenarios 8 and 9) causes an increase in the risk-adjusted salary growth, especially when accompanied by higher salary volatility (in Scenario 9). Its effect on the required deferred cost is similar to the impact from an increase in salary growth. Other findings about the required deferred cost among the first six scenarios are limited. They are similar to findings about the up-front value among corresponding scenarios.

Conclusion

Although this paper numerically prices a guarantee of an IPA, the reset feature on the guarantee connected to the salary is the main issue herein. The results show that, without resets, a principal guarantee on an IPA is not worth much unless the volatility of assets in the IPA is huge, and the death benefit contributes a very small proportion to the guarantee value. Moreover, the reset value contributing to the guarantee on an IPA is very sensitive to the risk of the asset of the IPA. These findings are consistent with previous studies on Canada's segregated fund guarantees by Windcliff *et al.* (2001, 2002).

This paper's results also show that if a lapse from the guarantee is possible, then the time value of a longer contract is diluted by a greater expected lapse loss, such that the guarantee does not necessarily cost young individuals more. Given the individual age, the high type of EIID with no lapse carries the most expensive guarantee for all specified scenarios.

A salary-connected model requires a precise estimation of the volatility and average growth of individual salary along with the market price of salary risk in order to accurately price the up-front guarantee value. The work of modeling salary behavior is still insufficient along the scope of salary-related derivatives. However, the results of this paper show that the required deferred cost rate is affected much less by the salary behavior than the up-front guarantee value. Deferred funding is therefore an alternative to reducing problems from the difficulty in modeling salary behavior.

The practical problem of providing a guarantee on an IPA is that few structured securities with salary-linked cash flows offer providers an established hedging strategy. We advocate and expect that financial innovation will lead to the creation of new securities that provide opportunities for the providers of non-traditional pension guarantees to establish better hedging strategies.

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