

Stability of the Rayleigh-Ritz-Galerkin procedure for elliptic boundary value problems

Bernard J. Omodel

The thesis investigates the stability of the Rayleigh-Ritz-Galerkin procedure for the approximate solution of certain classes of linear and non-linear elliptic boundary value problems.

In numerical analysis literature over the last decade, piecewise Hermite and spline subspaces have often been proposed for the Rayleigh-Ritz-Galerkin procedure for the solution of elliptic boundary value problems. However, the use of piecewise polynomial subspaces has not been investigated from the point of view of Mikhlin stability, and this thesis rectifies this neglect in the literature.

In Chapter 2, we introduce the Rayleigh-Ritz, the Galerkin, the generalized Rayleigh-Ritz, and the generalized Galerkin methods for the approximate solution of linear operator equations. As well as the concept of Mikhlin stability for linear numerical processes, we also introduce Tucker stability for nonlinear numerical processes.

Chapter 3 is concerned with certain classes of linear elliptic boundary value problems, where for each class, we establish basic stability theorems and then investigate the Mikhlin stability of the Rayleigh-Ritz-Galerkin procedure when the coordinate functions are appropriately scaled B -splines or elementary Hermites. The three classes that we consider are one dimensional, two dimensional, and multidimensional elliptic boundary value problems with Dirichlet boundary conditions.

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Chapter 4 is similar to Chapter 3 except that in this case, we are concerned with nonlinear elliptic boundary value problems. The first and second classes considered are nonlinear two-point boundary value problems with Dirichlet and nonlinear boundary conditions, respectively. We also study a "model" nonlinear multidimensional problem.

In Chapter 5, we study normalized uniformly asymptotically diagonal systems from the point of view of Mikhlin stability, and illustrate the type of instability that can arise with a numerical example.

Some of the work in this thesis was done in collaboration with Dr Anderssen. In particular, §4.2 and Chapter 5 are based on results established during this collaboration. Publication details regarding this work are: Anderssen and Omodei [1] and Omodei and Anderssen [2].

References

- [1] R.S. Anderssen and B.J. Omodei, "On the stability of uniformly asymptotically diagonal systems", *Math. Comp.* 28 (1974), 719-730.
- [2] B.J. Omodei and R.S. Anderssen, "Stability of the Rayleigh-Ritz procedure for nonlinear two-point boundary value problems", *Numer. Math.* 24 (1975), 27-38.