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A NOTE ON EXTENDED AMBIGUOUS POINTS

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Let f be an arbitrary function from the open unit disk D of the complex plane into the Riemann sphere S. If p is any point on the unit circle C, $C(f, p)$ is the set of all points w such that there exists in D a sequence of points $\{z_j\}$ such that $z_j \to p$ and $f(z_j) \to w$. $C_\Delta(f, p)$ is defined in the same way, but the sequence $\{z_j\}$ is restricted to $\Delta \subset D$. If α and β are two arcs in D terminating at p and $C_\alpha(f, p) \cap C_\beta(f, p) = \Phi$, p is called an ambiguous point for f.

Mathews (3, p. 138) defined an extended ambiguous point for f in the following way: Let α be an arc in $\bar{D} - \{p\}$ terminating at p. $EC_\alpha(f, p) = \cap \overline{\cup C(f, q)}$ where the intersection is taken over all neighborhoods N of p and the union over all q on $\alpha \cap N$, $q \neq p$, and $C(f, q) = \{f(q)\}$ if $q \in D$.

Bagemihl (1) proved that an arbitrary function from D into S can have at most countably many ambiguous points. Mathews (3, p. 138, proof of Theorem 1) has proved that a countinuous function from D into S can have at most countably many extended ambiguous points. This proof does not hold if f is not continuous (the statement that $C_{BL}(g, 1)$ is contained in either $EC_\alpha(f, p)$ or $EC_\beta(f, p)$ only holds if f is continuous). The theorem, however, does hold.

THEOREM (*Mathews*): *If f is an arbitrary function from D into S and if a point p on C is an extended ambiguous point for f, then p is an ambiguous point for f.*

Proof. Let α be any arc in $\bar{D} - \{p\}$ such that α tends to p. As Mathews' indicates, it is sufficient to find an arc $\alpha' \subset D$ and tending to p such that $C_{\alpha'}(f, p) \subset EC_\alpha(f, p)$. Use the points $q \in \alpha \cap C$ and the method of Gross (2, p. 249) to construct a "wedge" Z_1 in D such that for every sequence of points $\{z_k\}$ in Z_1 tending to p, $\{f(z_k)\}$ has limit points only in $\cap \overline{\cup C(f, q)}$

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where $q \in \alpha \cap C$, $q \neq p$, and the intersection is taken over all neighborhoods of p.

In general the set Z_1 will not be connected since $\alpha \cap C$ will not always contain a terminal part of α. However, $\alpha \cap D$ may be used to join the components of Z_1 so that $(\alpha \cap D) \cup Z_1$ will contain an arc α' tending to p. It is clear that $C_{\alpha'}(f, p) \subset EC_{\alpha}(f, p)$, and the theorem follows.

References

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