Inside the Stars, IAU Colloquium 137
ASP Conference Series, Vol. 40, 1993
Werner W. Weiss and Annie Baglin (eds.)

PULSATION THEORY AND STELLAR STRUCTURE

J. CHRISTENSEN-DALSGAARD
Institut for Fysik og Astronomi, Aarhus Universitet,
DK 8000 Aarhus C, Denmark

ABSTRACT Observed periods of pulsating stars provide information about the properties of the stars. As examples I here consider double-mode pulsators, solar-like oscillations and δ Scuti stars. The ongoing expansion in the observational efforts in this area may be expected to lead to increasingly detailed tests of stellar evolution theory.

1 INTRODUCTION

The periods of the modes of oscillation of a star depend on the global properties of the star, such as mass and radius, and on its internal structure. Therefore, if one or more modes are observed to be excited one can use the observed periods to gain information about the star. It is evident that the amount of information increases with the number of modes that are observed. Extreme cases are the classical Cepheids which appear to pulsate in a single mode only, generally identified to be the fundamental radial mode; and the Sun where thousands of periods have been observed with great accuracy.

At the simplest level, the pulsation period Π can be estimated as being related to the free-fall time scale over a substantial part of the radius of the star:

$$\Pi \sim t_{\rm dyn} \equiv \left(\frac{GM}{R^3}\right)^{-1/2} , \qquad (1)$$

where G is the gravitational constant, and M and R are the stellar mass and radius. A similar relation results from considering the travel time of an acoustic wave over a stellar radius; the equivalence of the two approaches is a consequence of the virial theorem, which relates the gravitational and thermal properties of the star. For many modes, particularly those of radial oscillation, equation (1) gives a reasonable approximation to the dependence of the period on the stellar mass and radius. This already provides interesting information: $t_{\rm dyn}$ varies from being of order a year for red giant pulsators, over a few days for Cepheids and a few hours for typical main-sequence stars to a few seconds for white dwarfs. Hence even the simple detection of a single oscillation period for a red star can distinguish between its being a nearby main-sequence star or a distant giant.

Equation (1) clearly involves a factor which depends on the precise type of mode, and on the details of the internal structure of the star. To quantify this it is common, particularly in studies of classical pulsating stars, to express the period in terms of the *pulsation constant*

$$Q \equiv \Pi \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{R_\odot}\right)^{-3/2} , \tag{2}$$

where $M_\odot$ and $R_\odot$ are the solar mass and radius. Thus Q provides information about the more intricate properties of stellar internal structure. The behaviour depends strongly on the type of mode. For acoustic modes, including radial oscillations, where the dominant restoring force is pressure, Q varies relatively slowly with stellar type. However, for gravity modes, where the restoring force is dominated by buoyancy, Q is quite sensitive to fine details in the evolutionary state of the star. Examples of this are discussed in Section 4 below.

In computations of stellar models, the results depend on the assumed physical properties of matter in the star; these include the equation of state, the opacity and rate of nuclear reactions. In addition, the computations require specification of the initial composition of the star; an additional parameter in most calculations specifies the efficiency of convective heat transport near the stellar surface. However, as normally carried out the computations are based on additional assumptions, of uncertain validity. Thus it is common in such "standard" stellar evolution calculations to neglect effects of microscopic diffusion, overshoot from convective envelopes or cores, and mixing resulting from rotational instabilities or from weak motion induced by convective overshoot; furthermore, significant effects on the evolution of mass loss cannot be excluded. Although calculations have been carried out that include one or more of these effects, there are substantial uncertainties in how to treat them.

Such uncertainties must be kept in mind when interpreting the observed periods. Ideally, one would want to determine the basic parameters characterizing the star (*i.e.*, its mass, age and chemical composition); to learn about the properties of stellar matter; and to investigate possible departures from standard stellar evolution theory. Even taking into account other information of a more traditional nature this ideal is evidently entirely out of reach on the basis of data for a single star; this is the case even for the Sun, despite the richness of the oscillation data and the accurate measurements of the solar mass, radius and luminosity. However, with sufficiently detailed information about stars covering a large enough area of the Hertzsprung-Russell diagram, and including the information available for the Sun, there may be hope that we can make substantial progress towards a complete test of stellar evolution theory.

A discussion of the theory of stellar pulsation is evidently entirely outside the scope of the present paper. Reference may be made to monographs on the subject (*e.g.* Unno *et al.* 1989), or to reviews on solar or stellar oscillations (see, for example, Gough & Toomre 1991; Christensen-Dalsgaard & Berthomieu 1991; Chiosi 1990). Instead, I concentrate on a few examples of the relation between the structure of stars and their pulsation periods. In Section 2 I treat the case of double-mode pulsators in the Cepheid instability strip, which provide confirmation of recent opacity calculations. Section 3 is concerned with frequency separations amongst low-degree high-order acoustic modes, which have significant diagnostic potential for the structure of stellar cores. Finally, Section 4 discusses some properties of oscillations of δ Scuti stars, which promise a great deal of information about stellar evolution, so far barely exploited.

2 DOUBLE-MODE PULSATORS: THE EFFECT OF OPACITY

While measurement of a single period in principle gives information about the mass and radius of the star in the combination M/R^3, one might expect that measurement of two periods could be used to determine the mass and radius separately. (In both cases, an assumption is of course that the nature of the modes observed has been identified). There are in fact a number of stars in the Cepheid instability strip observed to oscillate in two modes, identified as the fundamental and first overtone of radial pulsation. Petersen (1973) showed that the periods could be used to determine the masses of these stars. The result was striking: the masses so obtained were smaller by approximately a factor three than the *evolution masses* inferred by comparing evolution calculations with the location of the stars in the HR diagram. This problem has led to a great deal of effort, although until recently without much definite result (for reviews of this and other Cepheid mass problems, see Cox 1980; Simon 1987).

It is common to illustrate the problem in a *Petersen* diagram, where the ratio Π_1/Π_0 between the periods of the first radial overtone and the fundamental is plotted against $\log \Pi_0$. The observed location of a star in such a diagram is given with great precision. The instability strip is so thin that it effectively defines a mean relation between the luminosity and effective temperature. From evolution calculations, assuming a specific evolutionary state for the stars considered, one can then obtain a unique relation between mass, luminosity and effective temperature. In this way one can calculate a sequence of envelope models along the instability strip, parametrized by the stellar mass; by computing Π_0 and Π_1 for these models, one obtains a curve in the Petersen diagram.

This procedure is illustrated in Fig. 1. Here I have used a simplified relation between mass, effective temperature and luminosity (Becker, Iben & Tuggle 1977) based on evolution calculations for Population I models. In all cases envelope models extending to a fractional radius of about 0.05 were used, and the periods were found by solving the equations of adiabatic oscillations. The solid curve was obtained from models computed with the Cox & Tabor (1976) opacities. A comparison between this curve and the observed points, plotted as crosses, illustrates the extent of the discrepancy; also, it is striking that its sign depends on the period and hence the mass of the star: for low-mass models, in the δ Scuti region, the observed ratio exceeds the computed value, whereas the observations are below the computations for Cepheids.

Simon (1982) pointed out that the situation for the double-mode Cepheids could be improved if one were to increase the opacity due to heavy elements by a factor 2 – 3 in the temperature region $T = 10^5 - 2 \times 10^6\,\mathrm{K}$. On this basis he made a plea for a careful recalculation of opacities. This suggestion was taken further by Andreasen & Petersen (1988), Andreasen (1988) and Petersen (1990); they showed that by increasing the opacities by a factor 2.5 in the range $5.2 < \log T < 5.9$, the period ratios could be brought into agreement with observations for both δ Scuti stars and Cepheids; the physical basis for the modification was at the time somewhat unclear, however.

The response of Π_1/Π_0 to an opacity modification can be investigated by means of the *opacity sensitivity function* $\psi_{0,1}$ (Petersen 1992). It is assumed that the modification to the logarithm of opacity κ is solely a function $\delta \log \kappa(T)$ of T.

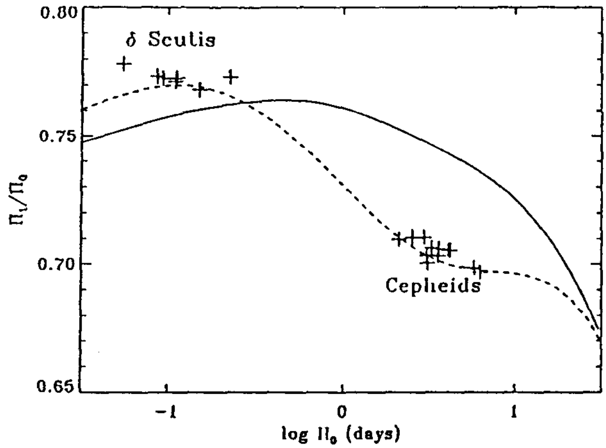

Fig. 1. Petersen diagram, plotting the ratio between the first overtone and fundamental radial period against the logarithm of the latter. The observed values are shown by crosses. The curves show the variation along the instability strip; the solid curve was based on models computed with the Cox & Tabor (1976) opacities, whereas the dashed curve used recent OPAL tables from Rogers & Iglesias (1992).

Assuming that the modification is sufficiently small, the change in the period ratio can be written as

$$\delta\left(\frac{\Pi_1}{\Pi_0}\right) = \int \psi_{0,1}(\log T)\delta \log \kappa(T) d \log T \, , \tag{3}$$

where the integral is over the model, and $\psi_{0,1}(\log T)$ gives the change to Π_1/Π_0 resulting from a delta-function change to $\log \kappa$ at T. Similar functions were used by Korzennik & Ulrich (1989) to interpret differences between observed and computed frequencies of solar oscillation in terms of errors in the opacity.

In practice $\psi_{0,1}$ is computed numerically, by making suitably localized small modifications to the opacity, at a number of points T, and recomputing the model and its periods. Opacity sensitivity functions for two models along the sequence illustrated in Fig. 1, in the δ Scuti and the Cepheid region, are shown in Fig. 2a. It is striking that $\psi_{0,1}$ has different sign for the two models around $\log T = 5.3$. This suggests that an opacity increase in this region would both increase the period ratio for the δ Scuti stars, and decrease it for the Cepheids, hence potentially bringing the models into agreement with the observations; this is evidently consistent with the results obtained by Andreasen & Petersen (1988).

As reviewed by Seaton in these proceedings, there have recently been two independent recalculations of stellar opacities, largely in response to Simon's plea; the results are in good agreement and both indicate substantial opacity increases in the temperature region relevant to the double-mode pulsators. Here I consider results obtained by the Livermore group in the OPAL project (Rogers & Iglesias 1992). Figure 2b shows the opacity change from the Cox & Tabor tables to the OPAL results, evaluated at the conditions in the two envelope

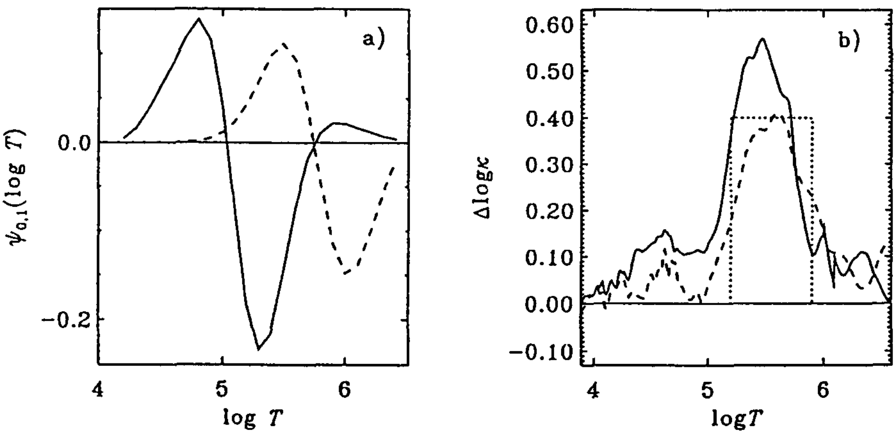

Fig. 2. (a) Opacity sensitivity functions (*cf.* eq. [3]); the solid curve is for a model with $\Pi_0 = 3.5$ days, corresponding to the Cepheid region in Fig. 1, whereas the dashed curve, for a model with $\Pi_0 = 0.09$ days, is representative for the δ Scutis. (b) The opacity increase in the OPAL tables, relative to Cox & Tabor (1976), at the conditions corresponding to the same two models. The dotted box illustrates the opacity increase proposed by Andreasen & Petersen (1988).

models considered in Fig. 2a. Evidently, there is a substantial increase in precisely the temperature range where the period ratio would be affected. The actual opacity change is compared with the change proposed by Andreasen & Petersen; although the details are different, the overall location and magnitude are evidently quite similar. This suggests that the revised opacities might in fact go a long way towards solving the double-mode problem. That this is indeed the case was found in recent calculations by Moskalik, Buchler & Marom (1992).

To illustrate the effect I have computed a second sequence of envelope models using the OPAL opacities. For consistency the relation between mass, luminosity and radius should have been based on evolution calculations with the new opacities; however, since such calculations are so far not available, I have used the same relation as for the previous calculations. The results are illustrated by the dashed line in Fig. 1; there is now generally good agreement between theory and observations.

These results provide a striking example of the detection of errors in the treatment of the basic physical properties of stellar matter through observations of pulsating stars, the errors subsequently being confirmed by a more careful analysis of the physics. It is interesting that this was possible based on observations of only two periods, but for a substantial and fairly uniform sample of stars. Evidence for the need for an opacity increase has also been obtained from analysis of the observed solar oscillation frequencies (*e.g.* Christensen-Dalsgaard *et al.* 1985; Korzennik & Ulrich 1989). In the solar case, however, the structure of the model is insensitive to the opacity at temperatures lower than 2×10^6 K where energy transport occurs through convection. Hence the solar data provide no information about the temperature region corresponding to the massive increase in opacity found in the recent calculations; this illustrates the complementary nature of stellar and solar observations, despite the richness of the latter. Dziembowski, Pamyatnykh & Sienkiewicz (1992) did indeed find that the modest opacity increases at higher temperature found in the new calculations resulted in a substantial improvement in the agreement between solar observations and models; however, some doubt has been cast on this conclusion by the recent revision of the determination of the solar photospheric iron abundance.

3 SOLAR-LIKE OSCILLATIONS IN OTHER STARS

The modes observed in the Sun are concentrated in the period range between 15 and 3 minutes; at low degree, which is the only case relevant to distant stars, such periods correspond to acoustic oscillations of high radial order. Detection of solar-like oscillations in other stars is hindered by their expected very small amplitudes; in the Sun the maximum amplitudes in velocity and relative intensity are $15\,\mathrm{cm\,sec^{-1}}$ and about 3×10^{-6}, respectively. However, there has been some evidence for oscillations resembling those of the Sun in Procyon (Brown *et al.* 1991) and α Centauri A (Pottasch, Butcher & Hoesel 1992); Hence it is of considerable interest to investigate the information content of such modes.

As the modes are of high radial order, they can be treated with reasonable accuracy by means of asymptotic theory. This shows that the cyclic frequency $\nu_{n,l}$ of a mode of degree l and radial order n is approximately given by

$$\nu_{nl} \sim \Delta\nu_0[n + \tfrac{1}{2}l + \tfrac{1}{4} + \alpha(\nu)] + \epsilon_{nl} \tag{4}$$

(*e.g.* Tassoul 1980). Here

$$\Delta\nu_0 = \left(2\int \frac{dr}{c}\right)^{-1}, \tag{5}$$

c being the adiabatic sound speed and r the distance to the centre of the star, is the sound travel time across a stellar diameter; α is related to the properties of the layer near the stellar surface, and ϵ_{nl} is a small correction term.

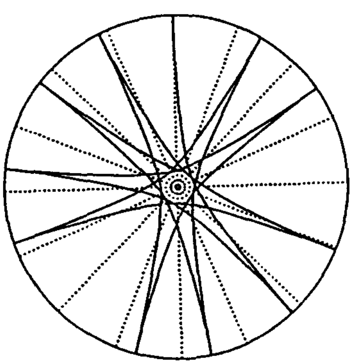

Fig. 3. The propagation of acoustic rays corresponding to a mode of degree 0 (dotted lines) and degree 2 (solid lines), in a solar model.

https://doi.org/10.1017/S0252921100018327 Published online by Cambridge University Press

To leading order, and neglecting the frequency-dependence of α, equation (4) predicts a uniformly spaced frequency spectrum, consisting of superposed peaks of even and odd degree, with a basic separation of $\Delta\nu_0/2$. This separation scales roughly as $(M/R^3)^{1/2}$ and hence provides information of a similar nature to the fundamental radial pulsation period. The term in ϵ_{nl} causes a departure from this simple pattern, conveniently measured by the small separation

$$\delta\nu_{nl} = \nu_{nl} - \nu_{n-1\,l+2} \,. \tag{6}$$

According to asymptotic theory $\delta\nu_{nl}$ is largely determined by the sound-speed gradient in the stellar core. That this separation is predominantly determined by conditions in the core can be understood quite simply by considering the oscillations in terms of acoustic rays (*cf.* Fig. 3). Radial oscillations, with $l = 0$, correspond to rays that propagate radially and hence pass through the centre of the star. Consider now rays corresponding to a mode of degree 2 with almost the same frequency: near the surface they propagate almost vertically, and hence behave in a manner very similar to the rays for the radial mode; but due to their slight inclination from the radial direction they are refracted by the increase in sound speed with increasing depth, and hence do not reach the centre of the star. Thus the behaviour of the rays differ only in the core; and it is plausible that the frequency difference between the two modes provides information about the core. Although the actual behaviour of the modes is more complex, a detailed analysis largely confirms this sensitivity (see Gabriel 1989; Gough & Novotny 1990; Christensen-Dalsgaard 1992a).

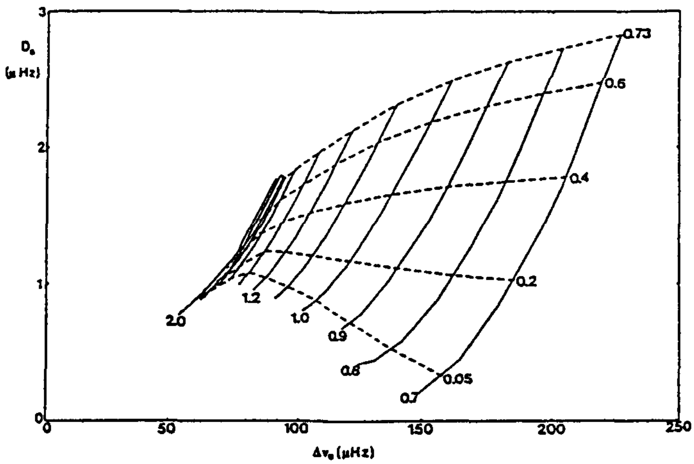

Fig. 4. Evolution tracks (solid lines) and curves of constant central hydrogen abundance X_c (dashed lines), in a $(\Delta\nu_0, D_0)$ diagram. Here $\Delta\nu_0$ is the separation between modes of the same degree, whereas D_0 is a measure of $\delta\nu_{nl}$. The stellar masses, in solar units, and the values of X_c are indicated. (From Christensen-Dalsgaard 1988).

The sound speed in stellar interiors is approximately given by the the relation valid for an ideal gas:

$$c^2 = \frac{\gamma k_B T}{\mu m_u}, \tag{7}$$

where $\gamma \simeq 5/3$, k_B is Boltzmann's constant, m_u is the atomic mass unit and μ is the mean molecular weight. Hence $\delta\nu_{nl}$ measures the combination T/μ. In the solar case, because of their dependence on temperature the separations are useful constraints on models designed to reduce the predicted neutrino flux, effectively eliminating several such models (see, for example Christensen-Dalsgaard 1992b). The dependence on mean molecular weight makes $\delta\nu_{nl}$ a measure of stellar evolution (Ulrich 1986; Christensen-Dalsgaard 1988). This is illustrated in Fig. 4, where evolutionary tracks are plotted in a $(\Delta\nu_0, D_0)$ diagram, D_0 being an average of $(4l+6)^{-1}\delta\nu_{nl}$ which according to asymptotic theory is independent of l. The dependence of $\Delta\nu_0$ on M/R^3 serves to separate the models according to mass, at least for $M \lesssim 1.3 M_\odot$. Given such a diagram, one might hope to use measured values of $\Delta\nu_0$ and D_0 to determine the mass and evolutionary state of main-sequence stars. As stressed by Gough (1987), however, such a determination is affected by uncertainties in other stellar parameters, and in the physics of stellar interiors.

TABLE I

Properties of models without overshoot, and with overshoot over a distance of 0.2 pressure scale heights. The models have the same mass and initial composition.

	No overshoot	Overshoot
Age	3.6 Gyr	3.8 Gyr
$L/L_\odot$	3.71	3.79
$T_{\rm eff}$	6340 K	6350 K

An important issue is the extent to which pulsation observations can be used to detect non-standard features in stars that are superficially similar. As an example, I consider the effect of overshoot from a convective core. This affects the hydrogen profile and hence the core structure, therefore potentially leading to detectable effects in the small frequency separations. I have selected two models from the evolution calculations of Schaller *et al.* (1992). Both have a mass of $1.25 M_\odot$ and an initial composition characterized by a hydrogen and heavy element abundance of 0.68 and 0.02. One model was computed without convective overshoot, the second included overshoot from the core over 0.2 pressure scale heights. Some properties of the models are given in Table I; clearly the stars would be indistinguishable on the basis of classical surface observations.

The profile of hydrogen abundance X, against mass fraction m/M, is shown in Fig. 5a. In the overshoot model, the region of hydrogen depletion extends substantially further than in the normal model, and as a result the central hydrogen abundance is larger. This is also reflected in the total main-sequence lifetimes of the models which are 3.95 Gyr for the normal, and 4.91 Gyr for the

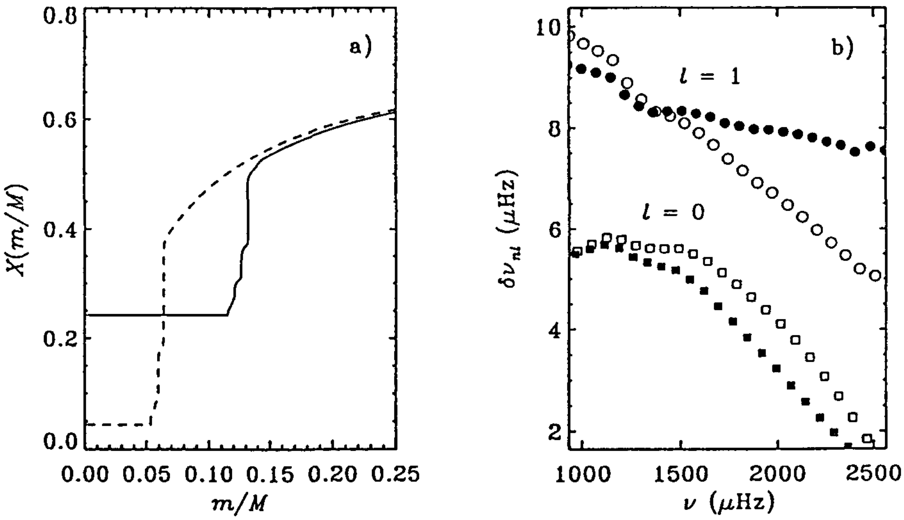

Fig. 5. (a) The hydrogen abundance in the inner portions of a model without overshoot (dashed line) and with overshoot (solid line), against mass fraction. (b) Computed frequency separations $\delta\nu_{nl}$ for $l = 0$ and 1. Open and filled symbols are for the models without and with overshoot, respectively.

overshoot, model. Hence overshoot would have a very substantial effect on age determinations based on the location of clusters in the HR diagram.

Figure 5b shows frequency separations computed for the two models. It is evident that there are quite substantial differences, both in absolute value and in the dependence on frequency. A careful analysis is required to understand how this behaviour is related to the difference in structure between the models; however, it seems plausible that one would be able to distinguish between the models on this basis.

It should perhaps be emphasized again that in reality the analysis would require that other uncertainties in the model parameters be taken into account. Hence it is perhaps unlikely that the presence of overshoot could be unambiguously detected from observations of a single star. Nevertheless, by combining data from several stars, one may hope to be able to probe the extent of overshoot and hence estimate its likely effect on the overall properties of the stars. This would be of obvious importance to our understanding of stellar evolution.

4 SOME REMARKS ABOUT δ SCUTI STARS

There is mounting evidence that many δ Scuti stars have far more complex spectra of oscillation, involving both radial and nonradial modes, than the two radial modes considered in Section 2. This realization is largely the result of extended observing campaigns, involving coordinated observations from several sites (*e.g.* Belmonte *et al.* 1991; Michel & Baglin 1991; Michel *et al.* 1992). Although the interpretation of the observed spectra is difficult (see, for example, Mangeney *et al.* 1991) the observations in principle provide a substantial amount of information about these stars, which are in many cases in the very interesting evolutionary phase near the end of central hydrogen burning.

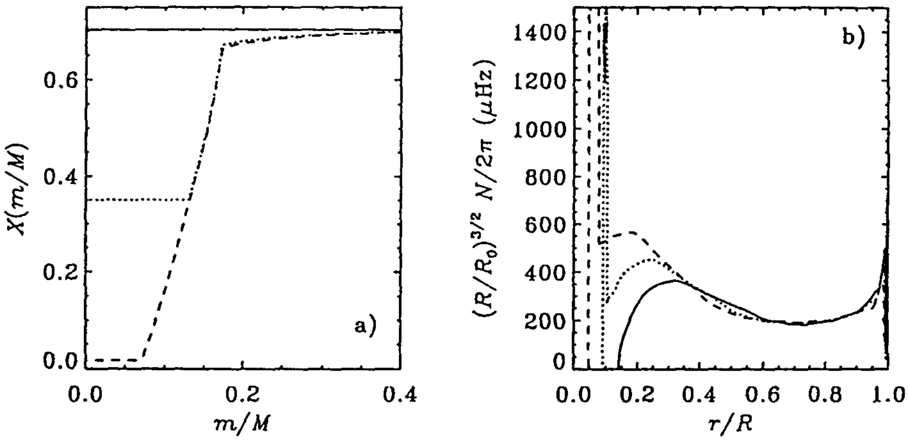

Fig. 6. (a) Hydrogen content X against mass fraction m/M for three models in a $2.2M_\odot$ evolution sequence. The solid line is for age 0, the dotted line for age 0.470 Gyr, and the dashed line for age 0.706 Gyr. (b) Scaled buoyancy frequency, expressed in cyclic frequency, for the same three models, against fractional radius r/R; for the model of age 0.706 Gyr, the maximum value of $N/2\pi$ is 2.4 mHz. In the scaling factor, R and R_0 are the radii of the current and the zero-age main sequence models, respectively.

The modes discussed in Sections 2 and 3 were either purely (in the case of the radial modes) or predominantly (for the high-order p modes) of an acoustic nature. In the case of the δ Scuti stars, however, one has to take into account also the presence of the g mode spectrum. While the p modes predominantly depend on the distribution of sound speed in the star, the behaviour of the g modes is controlled by the buoyancy frequency N, given by

$$N^2 = g\left(\frac{1}{\gamma}\frac{d\ln p}{dr} - \frac{d\ln\rho}{dr}\right) , \tag{8}$$

where g is the local gravitational acceleration, p is pressure and ρ is density. The increasing importance of the g modes, in the period range corresponding to the observations, arises from the build-up of a steep gradient in the hydrogen abundance outside the convective core in these stars. This increases the density gradient, in absolute value, in this region and hence increases N. Consequently the g mode frequencies are increased (see, for example, Dziembowski & Pamyatnykh 1991).

The changes in the hydrogen profile and the buoyancy frequency with stellar evolution are illustrated in Fig. 6, on the basis of an evolution sequence for a $2.2M_\odot$ star. The convective core is fully mixed and here, therefore, the composition is uniform. However, in stars of this mass the convective core shrinks during most of evolution, leaving behind a steep gradient in X, as shown in Fig. 6a.

This causes the sharp peak in the buoyancy frequency seen in Fig. 6b; in the convective core N is essentially zero. Note that in the outer parts of the model N follows a scaling corresponding to equation (1), *i.e.*, decreases as $R^{-3/2}$; to compensate for this I show N scaled by $(R/R_0)^{3/2}$, R_0 being the radius of the star on the zero-age main sequence.

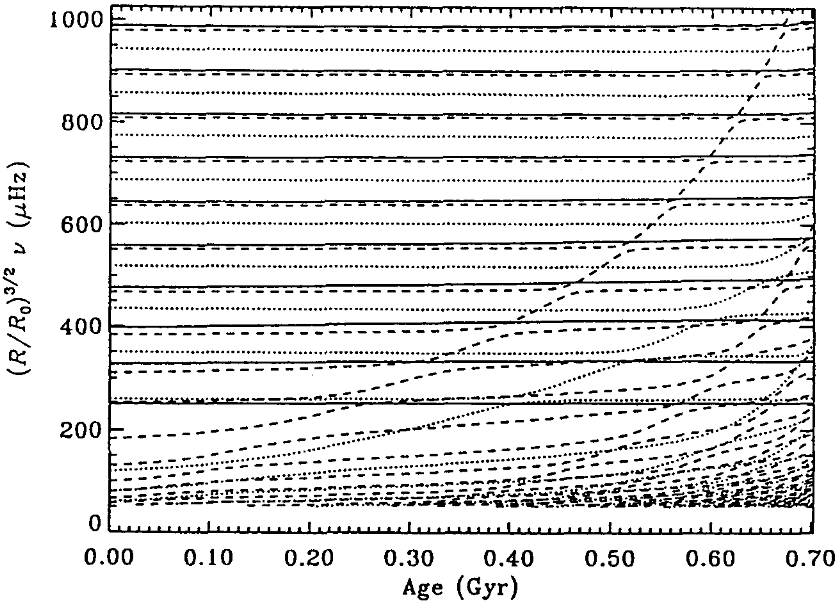

Fig. 7. Scaled oscillation frequencies, as functions of age, in a $2.2M_\odot$ evolution sequence. Modes of the same radial order have been connected. The solid lines are for radial modes, of degree $l = 0$, the dotted lines are for $l = 1$ and the dashed lines for $l = 2$.

The behaviour of the oscillation frequencies, as functions of stellar age, are shown in Fig. 7; here again I have applied the scaling corresponding to equation (1). As a result, the frequencies of largely acoustic modes, including the radial modes, change very little with age. It should be noticed also that except at low order, these clearly exhibit the asymptotic pattern predicted by equation (4); indeed, there seems to be a pairing of the radial and $l = 2$ modes even at the lowest orders. Such a pattern of closely-spaced peaks has in fact been observed in some cases (*e.g.* Michel *et al.* 1992).

The most striking feature of the computed frequencies, however, is the interaction for $l = 1$ and 2 between the p modes and the g modes (since the g modes depend on buoyancy, they clearly cannot exist for spherically symmetric modes). As was first found by Osaki (1975), this interaction takes place through a sequence of *avoided crossings*, where the frequencies approach closely without actually crossing. At the avoided crossing the two modes exchange nature. This type of behaviour is common to a wide variety of eigenvalue problems; a very clear discussion of the phenomenon, in the context of atomic physics, was given by von Neuman & Wigner (1929). It is evident that the presence of the g-like modes in the p-mode spectrum, particularly at late evolutionary stages,

https://doi.org/10.1017/S0252921100018327 Published online by Cambridge University Press

complicates the analysis of observed frequencies. Dziembowski & Królikowska (1990) pointed out that mode selection might be affected by the larger energy of the modes that behave like g modes, thereby restricting the choice of modes in the identification. However, such arguments depend on the mechanisms responsible for exciting the modes and limiting their amplitudes, which are so far incompletely understood. It should also be noted that *if* g-mode like pulsations could in fact be identified, their frequencies would give strong constraints on conditions in the region just outside the stellar core. In fact, Dziembowski & Pamyatnykh (1991) pointed out that measurement of g-mode frequencies might provide a measure of the extent of convective overshoot from the core.

5 CONCLUDING REMARKS

I hope that the examples considered here have given some impression of the richness of information that is actually and potentially available in observed periods of pulsating stars. It should be noted that other types of stars offer equally exciting prospects. The rapidly oscillating Ap stars (Kurtz 1990; Shibahashi 1991), although pulsating in modes of the same nature as seen in the Sun, have distinct features; particularly interesting is the apparent association between the pulsations and the large-scale magnetic field of these stars. A very active field of research is the study of pulsating white dwarfs (*e.g.* Kawaler 1990; Winget 1991), where the modes detected have been identified as g modes; the observations have led to accurate mass determinations for some stars, as well as to the detection of evolutionary period changes at several points along the white-dwarf cooling sequence. There is also likely to be increasing activity in the study of B-star variability, including the β Cephei stars; a particularly striking feature is the fact that the instability of these stars can apparently be understood in terms of the classical opacity mechanism, when the recent OPAL opacities are used (Moskalik & Dziembowski 1992).

Detailed information about stellar interiors require the study of stars pulsating in many modes simultaneously. This immediately puts stringent constraints on the observations: the time series must be sufficiently long to ensure adequate frequency resolution; furthermore, the daily gaps in the data from a single site introduce side-band structure in the observed spectra which greatly complicates the mode identification. An additional difficulty, particularly in the case of solar-like oscillations, is the expected very small amplitudes which require that the observational techniques be pushed to the limit of sensitivity.

To avoid the daily gaps, data from several sites must be combined. A very ambitious and successful project to observe white dwarfs is the so-called Whole Earth Telescope (Winget 1991), which involves twelve sites distributed around the Earth. Coordinated observations have also yielded impressive results in the STEPHI project on δ Scuti stars (Michel & Baglin 1991). It is worth pointing out that if CCD photometry is used observations of δ Scuti stars can in fact be carried out with relatively modest means: Jones *et al.* (1992) obtained very precise measurements using a relatively small telescope in an urban setting. This points to the possibility of using the large number of existing small university telescopes for coordinated observations of selected stars; such a project is very well suited as part of student training.

The most promising prospects for the study of solar-like oscillations are probably observation from space. This eliminates the noise from the Earth's atmosphere and hence allows simple photometric measurements with the required accuracy; also, with a proper choice of orbit gaps in the data can be avoided. The EVRIS project (Baglin 1991) will observe a limited number of stars from the Russian Mars probe MARS 94. Much more extensive data will be available from the PRISMA satellite (Lemaire *et al.* 1991) which is currently undergoing Phase A studies within ESA. If approved, this project will provide detailed information about stars covering a wide range of parameters.

ACKNOWLEDGEMENTS

I am grateful to N. H. Andersen for assistance with the computations required for this paper. J. Otzen Petersen is thanked for extensive discussions about double-mode variables and for commenting on an early version of the manuscript. I thank W. Däppen and G. Houdek for help with implementing the OPAL opacities, and G. Meynet for providing me with models without and with convective overshoot. The High Altitude Observatory is thanked for hospitality during the time where the paper was written. The work reported here was supported by the Danish Natural Science Research Council.

REFERENCES

Andreasen, G. K., 1988. *Astr. Ap.*, **201**, 72.
Andreasen, G. K. & Petersen, J. O., 1988. *Astr. Ap.*, **192**, L4.
Baglin, A., 1991. *Solar Phys.*, **133**, 155.
Becker, S. A., Iben, I. & Tuggle, R. S., 1977. *Ap. J.*, **218**, 633.
Belmonte, J. A., Chevreton, M., Mangeney, A., Praderie, F., Saint-Pé, O., Puget, P., Alvarez, M. & Roca Cortés, T., 1991. *Astr. Ap.*, **246**, 71.
Brown, T. M., Gilliland, R. L., Noyes, R. W. & Ramsey, L. W., 1991. *Ap. J.*, **368**, 599.
Chiosi, C., 1990. In *Confrontation between stellar pulsation and evolution*, *A.S.P. Conf. Ser.*, **11**, p. 158, eds Cacciari, C. & Clementini, G.
Christensen-Dalsgaard, J., 1988. *Proc. IAU Symposium No 123, Advances in helio- and asteroseismology*, p. 295, eds Christensen-Dalsgaard, J. & Frandsen, S., Reidel, Dordrecht.
Christensen-Dalsgaard, J., 1992a. *Geophys. Astrophys. Fluid Dyn.*, **62**, 123.
Christensen-Dalsgaard, J., 1992b. In *The Sun: A Laboratory for Astrophysics* eds Schmelz, J. T. & Brown, J. C., Kluwer Academic Press, Dordrecht, in the press.
Christensen-Dalsgaard, J. & Berthomieu, G., 1991. In *Solar interior and atmosphere*, p. 401, eds Cox, A. N., Livingston, W. C. & Matthews, M., University of Arizona Press.
Christensen-Dalsgaard, J., Duvall, T. L., Gough, D. O., Harvey, J. W. & Rhodes, E. J., 1985. *Nature*, **315**, 378.
Cox, A. N., 1980. *Ann. Rev. Astron. Astrophys.*, **18**, 15.
Cox, A. N. & Tabor, J. E., 1976. *Ap. J. Suppl.*, **31**, 271.
Dziembowski, W. A. & Królikowska, M., 1990. *Acta Astron.*, 40, 19.

Dziembowski, W. A. & Pamyatnykh, A. A., 1991. *Astr. Ap.*, **248**, L11.
Dziembowski, W. A., Pamyatnykh, A. A. & Sienkiewicz, R., 1992. *Acta Astron.*, **42**, 5.
Gabriel, M., 1989. *Astr. Ap.*, **226**, 278.
Gough, D. O., 1987. *Nature*, **326**, 257.
Gough, D. O. & Novotny, E., 1990. *Solar Phys.*, **128**, 143.
Gough, D. O. & Toomre, J., 1991. *Ann. Rev. Astron. Astrophys.*, **29**, 627.
Jones, A., Kjeldsen, H. Frandsen, S., Christensen-Dalsgaard, J., Hjorth, J., Sodemann, M., Thomsen, B. & Viskum, M., 1992. To be submitted to *Astr. Ap.*
Kawaler, S. D., 1990. In *Confrontation between stellar pulsation and evolution*, *A.S.P. Conf. Ser.*, **11**, p. 494, eds Cacciari, C. & Clementini, G.
Korzennik, S. G. & Ulrich, R. K., 1989. *Ap. J.*, **339**, 1144.
Kurtz, D. W., 1990. *Ann. Rev. Astron. Astrophys.*, **28**, 607.
Lemaire, P., Appourchaux, T., Catala, C., Catalano, S., Frandsen, S., Jones, A. & Weiss, W., 1991. *Adv. Space. Res.*, **Vol. 11, No. 4**, 141.
Mangeney, A., Däppen, W., Praderie, F. & Belmonte, J. A., 1991. *Astr. Ap.*, **244**, 351.
Michel, E. & Baglin, A., 1991. *Adv. Space Res.*, **vol. 11, No. 4**, 167.
Michel, E., Belmonte, J. A., Alvarez, M., Jiang, S. Y., Chevreton, M., Auvergne, M., Goupil, M. J., Baglin, A., Mangeney, A., Roca Cortés, T., Liu, Y. Y., Fu, J. N., Dolez, N., 1992. *Astr. Ap.*, **255**, 139.
Moskalik, P. & Dziembowski, W. A., 1992. *Astr. Ap.*, **256**, L5.
Moskalik, P., Buchler, J. R. & Marom, A., 1992. *Ap. J.*, **385**, 685.
Osaki, Y., 1975. *Publ. Astron. Soc. Japan*, **27**, 237.
Petersen, J. O., 1973. *Astr. Ap.*, **27**, 89.
Petersen, J. O., 1990. *Astr. Ap.*, **238**, 160.
Petersen, J. O., 1992. *Astr. Ap.*, in the press.
Pottasch, E. M., Butcher, H. R. & van Hoesel, F. H. J., 1992. *Astr. Ap.*, in the press.
Rogers, F. J. & Iglesias, C. A., 1992. *Ap. J. Suppl.*, **79**, 507.
Schaller, G., Schaerer, D., Meynet, G. & Maeder, A., 1992. *Astr. Ap. Suppl.*, in the press.
Shibahashi, H., 1991. In *Challenges to theories of the structure of moderate-mass stars*, *Lecture Notes in Physics*, vol. **388**, p. 393, eds Gough, D. O. & Toomre, J., Springer, Heidelberg.
Simon, N. R., 1982. *Ap. J.*, **260**, L87.
Simon, N. R., 1987. In *Stellar pulsation*, *Lecture Notes in Physics*, vol. **274**, p. 148, eds Cox, A. N., Sparks, W. M. & Starrfield, S. G., Springer, Berlin.
Tassoul, M., 1980. *Ap. J. Suppl.*, **43**, 469.
Ulrich, R. K., 1986. *Ap. J.*, **306**, L37 - L40.
Unno, W., Osaki, Y., Ando, H., Saio, H. & Shibahashi, H., 1989. *Nonradial Oscillations of Stars, 2nd Edition* (University of Tokyo Press).
von Neuman, J. & Wigner, E., 1929. *Phys. Z.*, **30**, 465.
Winget, D. E., 1991. In *White dwarfs*, p. 129, eds Vauclair, G. & Sion, E., Kluwer, Dordrecht, The Netherlands.