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COHOMOLOGY AND EXTENSIONS OF REGULAR SEMIGROUPS

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Abstract

Let S be a regular semigroup and A a $D(S)$-module. We proved in a previous paper that the set $\mathrm{Ext}(S, A)$ of equivalence classes of extensions of A by S admits an abelian group structure and studied its functorial properties. The main aim of this paper is to describe $\mathrm{Ext}(S, A)$ as a second cohomology group of certain chain complex.

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Let S be a regular semigroup and A a $D(S)$-module. Denote by S the regular semigroup obtained from S by adjoining an identity element I, $I \notin S$, and by A^1 the $D(S^I)$-module obtained from A by taking $A^1_I = \varprojlim_{D(IS)} A$, where IS denotes the subsemigroup generated by the idempotents of S. In Loganathan (1982) we showed that the set $\mathrm{Ext}(S, A)$ of equivalence classes of extensions of A by S admits an abelian group structure and studied its functorial properties. One of the purposes of the present paper is to construct a chain complex $\mathbf{C}$ in the category of $D(S^I)$-modules and to show that the group $\mathrm{Ext}(S, A)$ is naturally isomorphic to the second cohomology group $H^2(\mathbf{C}, A^1)$. This generalizes the corresponding result for inverse semigroups due to Lausch (1975).

After Section 2 which gives necessary preliminaries, we construct in Section 3 the chain complex $\mathbf{C}$ and compare the lower dimensional cohomology groups of $\mathbf{C}$ and the category $D(S^I)$. It is shown that the second cohomology group $H^2(D(S^I), B)$ is isomorphic to a subgroup of $H^2(\mathbf{C}, B)$ and that the first cohomology group $H^1(D(S^I), B)$ is isomorphic to the group $H^1(\mathbf{C}, B)$, for any $D(S^I)$-module B. In Section 4 we prove that the group $\mathrm{Ext}(S, A)$ is isomorphic to

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the group $H^2(\mathbf{C}, A^1)$. The remainder of the paper is devoted to interpreting the groups $H^2(D(S^I), A^1)$ and $H^2(D(S^I), A^1)$ in terms of I-split extensions and automorphisms of extensions respectively.

2. Preliminaries

Let S be a regular semigroup, and $E(S)$ the set of idempotents of S. We denote the set of inverses of an element $x \in S$ by $V(x)$, that is,

$$V(x) = \{x' \in S : xx'x = x, x'xx' = x'\}.$$

If $x' \in V(x)$ then (x, x') is called a *regular pair* in S. For $e, f \in E(S)$, let $S(e, f)$ be the *sandwich* set of e and f, that is,

$$S(e, f) = \{h \in E(S) : he = h = fh \text{ and } ehf = ef\}.$$

LEMMA 2.1 (Nambooripad, 1979). *Let S be a regular semigroup and let $x, y \in S$. Suppose that $x' \in V(x)$, $y' \in V(y)$ and let $h \in S(x'x, yy')$. Then $y'hx' \in V(xy)$.*

A sequence $(e_0, e_1, \ldots, e_n)$ of idempotents of S is called an *$E(S)$-chain* if $e_i \mathcal{R} e_{i+1}$ or $e_i \mathcal{L} e_{i+1}$ for $i = 0, 1, \ldots, n-1$.

LEMMA 2.2 (Nambooripad, 1979). *Let S be a regular semigroup and IS the subsemigroup generated by the idempotents of S. Then*

(i) *given any x in IS there exists an $E(S)$-chain $(e_0, e_1, \ldots, e_n)$ such that $x = e_0 e_1 \cdots e_n$;*

(ii) *given any regular pair (x, x') in IS there exists an $E(S)$-chain $(f_0, f_1, \ldots, f_m)$ such that $(x, x') = (f_0 f_1 \cdots f_m, f_m \cdots f_1 f_0)$.*

We recall from Loganathan (1981) that if S is any regular semigroup then $C(S)$ is defined to be the category whose objects are the idempotents of S and whose morphisms from an object e to the object f are the triples (e, x, x') such that $x' \in V(x)$, $e \geqslant xx'$ and $x'x = f$. The category $D(S)$ is the quotient category of $C(S)$ by the congruence generated by the following relation. If (e, x, x'), $(e, y, y') : e \to f$ are morphisms from e to f then $(e, x, x') \sim (e, y, y')$ if and only if $x = y$ or $x' = y'$. We denote the image of (e, x, x') in $D(S)$ by $[e, x, x']$.

Finally we recall the definition of the cohomology of a small category. For more details we refer to Watts (1965) and to Loganathan (1981). Let Ab denote the category of abelian groups. Let $\mathcal{C}$ be any small category. A *$\mathcal{C}$-module* is a functor $A : \mathcal{C} \to \text{Ab}$. Let A, B be two $\mathcal{C}$-modules. A *$\mathcal{C}$-homomorphism* $\varphi : A \to B$ is a natural transformation from A to B. The group of all $\mathcal{C}$-homomorphisms from A

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to B is denoted by $\mathrm{Hom}_{\mathcal{C}}(A, B)$. The category of $\mathcal{C}$-modules and $\mathcal{C}$-homomorphisms is denoted by $\mathrm{Mod}(\mathcal{C})$. The inverse limit functor $\varprojlim_{\mathcal{C}}: \mathrm{Mod}(\mathcal{C}) \to \mathrm{Ab}$ is left exact. Therefore the right derived functors of $\varprojlim_{\mathcal{C}}$ can be defined. If A is a $\mathcal{C}$-module then the value of the nth right derived functor of $\varprojlim_{\mathcal{C}}$ on A, denoted by $H^n(\mathcal{C}, A)$, is called the nth *cohomology group* of $\mathcal{C}$ with coefficients in A.

Let $\Delta Z: \mathcal{C} \to \mathrm{Ab}$ be the constant $\mathcal{C}$-module at Z, the additive group of integers, that is $(\Delta Z)_e = Z$ for every object e of $\mathcal{C}$, and $(\Delta Z)u$ is the identity homomorphism for every morphism u of $\mathcal{C}$. Then $H^n(\mathcal{C}, A) = \mathrm{Ext}^n_{\mathcal{C}}(\Delta Z, A)$. Therefore the cohomology groups of $\mathcal{C}$ may be calculated using a projective resolution of the module ΔZ.

Let $\mathcal{C}_0$ denote the discrete subcategory determined by the identity morphisms of $\mathcal{C}$. A $\mathcal{C}_0$-*set* is a functor from $\mathcal{C}_0$ to the category of sets, and a $\mathcal{C}_0$-*map* is a natural transformation between such functors. Note that a $\mathcal{C}$-module (resp. $\mathcal{C}$-homomorphism) may be regarded as a $\mathcal{C}_0$-set (resp. $\mathcal{C}_0$-map) in an obvious way.

Let X be a $\mathcal{C}_0$-set and F a $\mathcal{C}$-module. F is called a *free* $\mathcal{C}$-module on X if there exist a $\mathcal{C}_0$-map $i: X \to F$ such that to every $\mathcal{C}$-module A and to every $\mathcal{C}_0$-map $j: X \to A$ there is a unique $\mathcal{C}$-homomorphism $\varphi: F \to A$ such that $i\varphi = j$. Given a $\mathcal{C}_0$-set $X = \{X_e : e \in Ob\mathcal{C}\}$ a free $\mathcal{C}$-module F on X can be obtained by associating to each object e of $\mathcal{C}$ the free abelian group F_e generated by the symbols (x, u), where $u: h \to e$ runs through the morphisms of $\mathcal{C}$ with range e and $x \in X_h$, and to each morphism $v: e \to f$ the homomorphism $Fv: F_e \to F_f$, where Fv is given by $(x, u)(Fv) = (x, uv)$. The $\mathcal{C}_0$-map $i: X \to F$ is defined by $xi = (x, 1_e)$, where $x \in X_e$ and 1_e is the identity morphism of $\mathcal{C}$ at e. We usually identify X with its image in F under i.

3. Chain complexes over ΔZ

Let S be a regular semigroup. In this section we construct a chain complex **C** in the category of $D(S^I)$-modules. The cohomology of **C** will be used in Section 4 to describe the group $\mathrm{Ext}(S, A)$.

Throughout the remainder of this paper S will denote a regular semigroup with an inverse map $x \mapsto x^*: S \to S$; a map $x \mapsto x^*: S \to S$ is called *inverse* if (i) $x^* \in V(x)$ for each $x \in S$; (ii) $x^* \in H_e$ if $x \in H_e$. We extend $x \mapsto x^*: S \to S$ to S^I by defining $I^* = I$. If $x, y \in S^I$ then we denote the $D(S^I)$-morphisms

$$[y^*y, y^*y(xy)^*xy, (xy)^*xy]: y^*y \to (xy)^*xy$$

and

$$[x^*x, x^*xy, (xy)^*xh]: x^*x \to (xy)^*xy,$$

$h \in S(x^*x, yy^*)$, by $K_{x,y}$ and $J_{x,y}$ respectively.

LEMMA 3.1 (Loganathan 1982). *For* $x, y, z \in S^I$, *we have*
(i) $K_{y,z}K_{x,yz} = K_{xy,z}$;
(ii) $J_{x,y}J_{xy,z} = J_{x,yz}$;
(iii) $K_{x,y}J_{xy,z} = J_{y,z}K_{x,yz}$.

Let C_n, $n \geqslant 0$, be the free $D(S^I)$-module on the $D(S^I)_0$-set $S^n = \{S_e^n : e \in E(S^I)\}$, where for $n \geqslant 1$,

$$S_e^n = \{(x_1,\ldots,x_n) : x_i \in S,\ 1 \leqslant i \leqslant n,\ (x_1 \cdots x_n)^* x_1 \cdots x_n = e\}$$

and for $n = 0$, S_I^0 consists of a single element, denoted by $\langle\ \rangle$, and S_e^0 is empty if $e \neq I$. Note that S_I^n is an empty set for all $n \geqslant 1$. We define $D(S^I)$-homomorphisms $d_n : C_n \to C_{n-1}$ by

$$\begin{aligned}(x_1,\ldots,x_n)\,d_n = \big((x_2,\ldots,x_n),\ & K_{x_1,x_2\cdots x_n}\big)\\ &+ \sum_{i=1}^{n-1}(-1)^i(x_1,\ldots,x_ix_{i+1},\ldots,x_n)\\ &+ (-1)^n\big((x_1,\ldots,x_{n-1}), J_{x_1\cdots x_{n-1},x_n}\big), \qquad n > 1,\end{aligned}$$

and

$$(x)\,d_1 = (\langle\ \rangle, [I, x^*x, x^*x]) - (\langle\ \rangle, [I, x, x^*]).$$

A routine verification shows that $d_n d_{n-1} = 0$. Hence

(3.1) $$\mathbf{C} : \cdots \to C_n \to C_{n-1} \to \cdots \to C_0 \to 0$$

is a free chain complex in $\mathrm{Mod}(D(S^I))$. If B is a $D(S^I)$-module then the nth *cohomology group* of $\mathbf{C}$ with coefficients in B is the abelian group

$$H^n(\mathbf{C}, B) = H^n\big(\mathrm{Hom}_{D(S^I)}(\mathbf{C}, B)\big).$$

The following description of the second cohomology group of $\mathbf{C}$ is needed in Section 4. Suppose that A is a $D(S)$-module. Then we denote by A^1 the $D(S^I)$-module extended from A by taking $A_I^1 = \varprojlim_{D(IS)} A$ and defining, for every morphism $[I, x, x'] : I \to e$,

$$A^1([I, x, x']) : A_I^1 \to A_e^1\ (= A_e)$$

to be the composite

$$\varprojlim_{D(IS)} A \xrightarrow{p_{xx'}} A_{xx'} \xrightarrow{A([xx', x, x'])} A_e,$$

where $p_{xx'}$ is the projection from $\varprojlim_{D(IS)} A$ to $A_{xx'}$. If we regard $S \times S$ and S as $D(S)_0$-sets by taking for each $e \in E(S)$, $(S \times S)_e = S_e^2$ and $S_e = S_e^1$ respectively

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then, since S_I^n is an empty set for all $n \geqslant 1$, it follows that

$$\mathrm{Hom}_{D(S^I)}(C_2, A^1) = \mathrm{Hom}_{D(S)_0}(S \times S, A)$$

and

$$\mathrm{Hom}_{D(S^I)}(C_1, A^1) = \mathrm{Hom}_{D(S)_0}(S, A).$$

Hence, a 2-cocycle α can be considered as a $D(S)_0$-map $\alpha : S \times S \to A$ such that

$$(3.2) \quad (y, z)\alpha A(K_{x, yz}) - (xy, z)\alpha + (x, yz)\alpha - (x, y)\alpha A(J_{xy,z}) = 0,$$

for all $x, y, z \in S$; α is a coboundary if and only if there exists a $D(S)_0$-map $\beta : S \to A$ such that

$$(3.3) \qquad (x, y)\alpha = (y)\beta A(K_{x,y}) - (xy)\beta + (x)\beta A(J_{x,y}),$$

for all $x, y \in S$.

We would like to compare the lower dimensional cohomology groups of $\mathbf{C}$ and the small category $D(S^I)$. For this purpose we shall construct free resolutions of the $D(S^I)$-module ΔZ.

Let G_n, $n \geqslant 1$, be the free $D(S^I)$-module on the $D(S^I)_0$-set $Y^n = \{Y_e^n : e \in E(S^I)\}$, where Y_e^n consists of all composable sequences $\langle u_1, \ldots, u_n \rangle$ of morphisms of $D(S^I)$ with domain of $u_1 = I$, and range of $u_n = e$. Put $G_0 = C_0$. Define $D(S^I)$-homomorphisms $\varepsilon : G_0 \to \Delta Z$ by $(\langle\ \rangle)\varepsilon = 1$, the identity element of the group $(\Delta Z)_I = Z$, and $d_n : G_n \to G_{n-1}$ by

$$\begin{aligned}\langle u_1, \ldots, u_n \rangle d_n = \langle [I, e_1, e_1] u_2, u_3, \ldots, u_n \rangle \\ + \sum_{i=1}^{n-1} (-1)^i \langle u_1, \ldots, u_i u_{i+1}, \ldots, u_n \rangle \\ + (-1)^n (\langle u_1, \ldots, u_{n-1} \rangle, u_n),\end{aligned}$$

where $e_1 \in E(S^I)$ and domain of $u_2 = e_1$;

$$\langle [I, x, x'] \rangle d_1 = (\langle\ \rangle, [I, x'x, x'x]) - (\langle\ \rangle, [I, x, x']).$$

Define $D(S^I)_0$-homomorphisms $s_n : G_n \to G_{n+1}$ $(n \geqslant 0)$ and $\delta : \Delta Z \to G_0$ by

$$\begin{aligned}(\langle u_1, \ldots, u_n \rangle, v) s_n &= (-1)^{n+1} \langle u_1, \ldots, u_n, v \rangle; \\ (\langle\ \rangle, [I, x, x']) s_0 &= -\langle [I, x, x'] \rangle; \\ (1)\delta &= (\langle\ \rangle, [I, e, e]),\end{aligned}$$

where 1 is the identity element of the group $(\Delta Z)_e = Z$. It is easy to verify that

$$\delta\varepsilon = 1_{\Delta Z}, \quad s_0 d_1 + \varepsilon\delta = 1_{G_0}, \quad s_n d_{n+1} + d_n s_{n-1} = 1_{G_n} \qquad (n > 0).$$

Using these relations one can show as in Mac Lane (1963), page 115, that

$$\cdots \to G_n \xrightarrow{d_n} G_{n-1} \to \cdots \to G_0 \to 0$$

is a free resolution of ΔZ.

In G_n put $(\langle u_1, \ldots, u_n \rangle, v) = 0$, whenever one of the variables u_i = identity morphism or $u_1 = [I, f, f]$ for some $f \in E(S^I)$. Then we get another free resolution

$$\mathbf{F}: \cdots \to F_n \xrightarrow{d_n} F_{n-1} \to \cdots \to F_0 \to 0 \tag{3.4}$$

of ΔZ such that $F_0 = G_0 = C_0$, and F_n is the free $D(S^I)$-module on the $D(S^I)_0$-set $X^n = \{X_e^n : e \in E(S^I)\}$, where X_e^n is the subset of Y_e^n consisting of all $\langle u_1, \ldots, u_n \rangle$ such that $u_1 \neq [I, e, e]$ for any $e \in E(S^I)$ and such that none of the $u_1, \ldots, u_n$ are identity morphisms. Note that X_I^n is an empty set for all $n \geqslant 1$.

Now

$$\mathbf{C}: \to C_n \to C_{n-1} \to \cdots \to C_0 \to 0$$

is a free chain complex over ΔZ and

$$\mathbf{F}: \cdots \to F_n \to F_{n-1} \to \cdots \to F_0 \to 0$$

is a free resolution of ΔZ. Therefore the identity homomorphism of ΔZ can be lifted to a chain map $\varphi : \mathbf{C} \to \mathbf{F}$ and any two such chain maps are chain homotopic.

PROPOSITION 3.2. *Let $\varphi : \mathbf{C} \to \mathbf{F}$ be a chain map such that $\varphi_0 \varepsilon = \varepsilon$. Then for any $D(S^I)$-module B,*

(i) $$\varphi_1^* : H^1(D(S^I), B) \to H^1(\mathbf{C}, B)$$

is an isomorphism and

(ii) $$\varphi_2^* : H^2(D(S^I), B) \to H^2(C, B)$$

is a monomorphism.

PROOF. We choose φ so that $\varphi_n : C_n \to F_n$, $n = 0, 1, 2$ are given by

$$\varphi_0 = \text{identity homomorphism};$$
$$(x)\varphi_1 = \langle J_{I,x} \rangle;$$
$$(x, y)\varphi_2 = \langle J_{I,x}, J_{x,y} \rangle + \langle J_{I,y}, K_{x,y} \rangle.$$

(i) Suppose $\beta : F_1 \to B$ is a cocycle such that $\varphi_1 \beta$ is a coboundary. Then there exists a unique $b \in B_I$ such that

$$(x)\varphi_1\beta = (J_{I,x})\beta = bB(J_{I,x*x}) - bB(J_{I,x}),$$

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for all $x \in S$. Since

$$\langle [I, x, x^*], [x^*x, x^*x, x'x] \rangle (\beta) d_2^* = 0,$$

it follows that

$$\langle [I, x, x'] \rangle \beta = bB([I, x'x, x'x]) - bB([I, x, x']),$$

for all $\langle [I, x, x'] \rangle \in X^1$. Hence β is a coboundary. Thus φ_1^* is a monomorphism. Now suppose that $\beta : C_1 \to B$ is a cocycle. Then $(x)\beta = 0$ for all $x \in IS$. If we define $\beta' : F_1 \to B$ by

$$\langle [I, x, x'] \rangle \beta' = (x)\beta B([x^*x, x^*x, x'x]), \qquad \langle [I, x, x'] \rangle \in X^1,$$

then β' is a cocycle and $\varphi_2 \beta' = \beta$. Hence φ_2^* is an epimorphism.

(ii) Suppose that $[\alpha] \in \ker \varphi_2^*$, and let $\alpha' = \varphi_2 \alpha$. Then there exists a $D(S^I)$-homomorphism $\beta' : C_1 \to B$ such that

$$(x, y)\alpha' = (x)\beta' B(J_{x,y}) + (y)\beta' B(K_{x,y}) - (xy)\beta', \tag{3.5}$$

for all $(x, y) \in S^2$. If $x, y \in IS$ then $(x, y)\alpha' = 0$ because

$$\begin{aligned}(x, y)\alpha' &= (x, y)\varphi_2 \alpha \\ &= \langle J_{I,x}, J_{x,y} \rangle \alpha + \langle J_{I,y}, K_{x,y} \rangle \alpha \\ &= \langle J_{I,x^*x}, J_{x,y} \rangle \alpha + \langle J_{I,y^*y}, K_{x,y} \rangle \alpha \\ &= 0.\end{aligned}$$

Since every element of IS can be expressed as a product of idempotents of S, using (3.5) one can prove by an induction argument that $(x)\beta' = 0$ for all $x \in IS$. This implies that $\beta : F_1 \to B$ given by

$$\langle [I, x, x'] \rangle \beta = (x)\beta' B(K_{x'x, x^*x}) - \langle J_{I,x}, K_{x'x, x^*x} \rangle \alpha \tag{3.6}$$

is well defined and it is a $D(S^I)$-homomorphism from F_1 to B. We claim that $\alpha = (\beta) d_2^*$. To prove this let

$$\langle [I, x, x'], [e, y, y'] \rangle \in X_f^2, \qquad f \in E(S).$$

Consider

$$\begin{aligned}\langle [I, x, x'], [e, y, y'] \rangle (\beta) d_2^* &= \langle [I, y, y'] \rangle \beta - \langle [I, xy, y'x'] \rangle \beta \\ &\quad + \langle [I, x, x'] \rangle \beta B([e, y, y']) \\ &= (x, y)\alpha' B(K_{f,(xy)^*xy}) - \langle J_{I,y}, K_{f,y^*y} \rangle \alpha \\ &\quad + \langle J_{I,xy}, K_{f,(xy)^*xy} \rangle \alpha \\ &\quad - \langle J_{I,x}, K_{e,x^*x} \rangle \alpha B([e, y, y']) \\ &\qquad \text{(using (3.5) and (3.6))}.\end{aligned} \tag{3.7}$$

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Now

$$
\begin{aligned}
(x, y)\alpha' B\left(K_{f,(xy)^*xy}\right) &= \left\langle J_{I,x}, J_{x,y}\right\rangle \alpha B\left(K_{f,(xy)^*xy}\right) \\
&\quad + \left\langle J_{I,y}, K_{x,y}\right\rangle \alpha B\left(K_{f,(xy)^*xy}\right) \\
&= \left\langle K_{x,I} J_{x,y}, K_{f,(xy)^*xy}\right\rangle \alpha - \left\langle J_{I,x} J_{x,y}, K_{f,(xy)^*xy}\right\rangle \alpha \\
&\quad + \left\langle J_{I,x}, J_{x,y} K_{f,(xy)^*xy}\right\rangle \alpha \\
&\quad - \left\langle J_{I,y} K_{x,y}, K_{f,(xy)^*xy}\right\rangle \alpha \qquad (3.8) \\
&\quad + \left\langle J_{I,y}, K_{x,y} K_{f,(xy)^*xy}\right\rangle \alpha \\
&= -\left\langle J_{I,xy}, K_{f,(xy)^*xy}\right\rangle \alpha + \left\langle J_{I,x}, J_{x,y} K_{f,(xy)^*xy}\right\rangle \alpha \\
&\quad + \left\langle J_{I,y}, K_{f,y^*y}\right\rangle \alpha,
\end{aligned}
$$

since $K_{x,I} J_{x,y} = J_{I,y} K_{x,y}$ by Lemma 3.1; and, since $\langle J_{I,x}, K_{e,x^*x}, [e, y, y']\rangle(\alpha) d_3^* = 0$,

$$
\begin{aligned}
\left\langle J_{I,x}, K_{e,x^*x}\right\rangle \alpha B([e, y, y']) &= -\left\langle J_{I,x} K_{e,x^*x}, [e, y, y']\right\rangle \alpha \\
&\quad + \left\langle J_{I,x}, K_{e,x^*x}[e, y, y']\right\rangle \alpha \\
&= -\langle [I, x, x'], [e, y, y']\rangle \alpha \qquad (3.9) \\
&\quad + \left\langle J_{I,x}, [x^*x, x^*xy, y'e]\right\rangle \alpha.
\end{aligned}
$$

Substituting (3.8) and (3.9) in (3.7) we get

$$
\begin{aligned}
\langle [I, x, x'], [e, y, y']\rangle(\beta) d_2^* &= \left\langle J_{I,x}, J_{x,y} K_{f,(xy)^*xy}\right\rangle \alpha \\
&\quad + \langle [I, x, x'], [e, y, y']\rangle \alpha \\
&\quad - \left\langle J_{I,x}, [x^*x, x^*xy, y'e]\right\rangle \alpha \\
&= \langle [I, x, x'], [e, y, y']\rangle \alpha,
\end{aligned}
$$

since $J_{x,y} K_{f,(xy)^*xy} = [x^*x, x^*xy, y'y(xy)^*xh] = [x^*x, x^*xy, y'e]$. Thus $\alpha = (\beta) d_2^*$. Hence φ_2^* is a monomorphism.

If S is an inverse semigroup then the chain complex $\mathbf{C}$ is exact and hence a free resolution of ΔZ. In this case $\boldsymbol{\varphi}$ becomes a chain equivalence inducing isomorphism on the cohomology groups. In the general case, $\boldsymbol{\varphi}^*$ need not be an isomorphism. The reader is advised to compare Proposition 3.1 with Theorem 7.5 and the subsequent Remark in Lausch (1975).

4. Description of $\operatorname{Ext}(S, A)$

Let $\pi : T \to S$ be an idempotent separating homomorphism from a regular semigroup T onto S. Then, for each $e \in E(S)$,

$$
(\operatorname{Ker} \pi)_e = \{t \in T : t\pi = e\}
$$

is a subgroup of T and the following two properties hold:

(4.1) $af = fa$, for all $a \in (\operatorname{Ker}\pi)_e$ and all $f \in E(T)$ such that $e \geqslant f\pi$;

(4.2) $x'(\operatorname{Ker}\pi)_e x \subseteq (\operatorname{Ker}\pi)_{(x'x)\pi}$, for all regular pairs (x, x') in T such that $e \geqslant (xx')\pi$.

Suppose now that the groups $(\operatorname{Ker}\pi)_e$, $e \in E(S)$, are abelian. Thus, using (4.1) and (4.2), it is easy to see that π defines a $D(S)$-module, denoted by $\operatorname{Ker}\pi$, which associates to each object e the abelian group $(\operatorname{Ker}\pi)_e$ and to each morphism $[e, x, x'] : e \to f$ the homomorphism

$$(\operatorname{Ker}\pi)[e, x, x'] : (\operatorname{Ker}\pi)_e \to (\operatorname{Ker}\pi)_f,$$

given by $a((\operatorname{Ker}\pi)[e, x, x']) = y'ay$, where (y, y') is a regular pair in T satisfying $(y\pi, y'\pi) = (x, x')$.

Let A be a $D(S)$-module. We recall from Loganathan (1982) that an *extension* of A by S is a triple $E = (T, \pi, i)$ consisting of a regular semigroup T, an idempotent separating homomorphism π from T onto S such that the groups $(\operatorname{Ker}\pi)_e$, $e \in E(S)$, are abelian, and an isomorphism $i : A \to \operatorname{Ker}\pi$ of $D(S)$-modules. Two extensions $E_1 = (T_1, \pi_1, i_1)$ and $E_2 = (T_2, \pi_2, i_2)$ are said to be *equivalent* if there exists a homomorphism (in fact an isomorphism) $\theta : T_1 \to T_2$ such that $\theta\pi_2 = \pi_1$ and $ai_1\theta = ai_2$, for all $a \in A$. Let $\operatorname{Ext}(S, A)$ denote the set of all equivalence classes of extensions of A by S. We have shown in Loganathan (1982) that $\operatorname{Ext}(S, A)$ admits an abelian group structure. We now show that the abelian group $\operatorname{Ext}(S, A)$ is naturally isomorphic to the group $H^2(\mathbf{C}, A^1)$.

LEMMA 4.1. *Let $\pi : T \to S$ be an idempotent separating homomorphism from a regular semigroup T onto S. Suppose that $t\pi = u\pi = x$, $t, u \in T$. Then, for each $e \in E(S) \cap L_x$, there exists a unique element a in T such that $u = ta$ and $a\pi = e$.*

PROOF. Let x' be an inverse of x such that $x'x = e$. Choose $t' \in V(t)$ and $u' \in V(u)$ such that $t'\pi = x' = u'\pi$. Then, since π is idempotent-separating, $tt' = uu'$, and $t't = u'u$. If we take $a = t'u$ then $u = uu'u = tt'u = ta$, and $a\pi = x'x = e$. The element a is unique, for if b is another element of T satisfying $u = tb$ and $b\pi = e$ then $b = t'u = a$.

Let now $E = (T, \pi, i)$ be an extension of A by S. Fix an inverse map $t \mapsto t^* : T \to T$ such that $(t^*)\pi = (t\pi)^*$ for all $t \in T$. Choose a section $j : S \to T$; that is, j is a map from S to T such that $xj\pi = x$, for all $x \in S$. Since $((xj)(yj))\pi = xy = (xy)j\pi$, it follows from Lemma 4.1 that there exists a $D(S)_0$-map $\alpha : S \times S \to A$ such that

$$(x)j(y)j = (xy)j((x, y)\alpha)i, \quad \text{for all } x, y \in S.$$

We shall prove that α is a 2-cocycle. First we prove a lemma.

LEMMA 4.2. *Let* $t, u \in T$, *and let* $a \in A_{(t\pi)^*t\pi}$, $b \in A_{(u\pi)^*u\pi}$. *Then*

$$t(a)iu(b)i = tu(aA(J_{t\pi,u\pi}) + bA(K_{t\pi,u\pi}))i.$$

PROOF. Let $h \in S(t^*t, uu^*)$. Then $t^*thuu^* = t^*tuu^*$, $t^*t \geq t^*th$ and $uu^* \geq huu^*$. Since $a \in A_{(t\pi)^*(t\pi)}$, $(a)it^*t = t^*t(a)i = (a)i$. Now

$$\begin{aligned} t(a)iu(b)i &= t(a)it^*thuu^*u(b)i \\ &= tt^*th(a)iu(b)i \quad \text{(by (4.1), since } t^*t \geq t^*th) \\ &= tuu^*h(a)iu(b)i. \end{aligned}$$

Since $(tuu^*h(a)iu)\pi = (tu)\pi$ and since π is idempotent separating, it follows that

$$tuu^*h(a)iu = tuu^*h(a)iuk, \quad \text{where } k = (tu)^*tu.$$

Hence

$$\begin{aligned} t(a)iu(b)i &= tuu^*h(a)iuk(b)i \\ &= tu(ku^*h(a)it^*tu)(k(b)iuu^*k) \quad \text{(by (4.1), since } u^*u \geq u^*uk) \\ &= tu(aA(J_{t\pi,u\pi}) + bA(K_{t\pi,u\pi}))i. \end{aligned}$$

Hence the result.

Let α be as above. Suppose $x, y, z \in S$. Put $e = (xyz)j^*(xyz)j$ and $f = (yz)j^*(yz)j$. Then

$$\begin{aligned} &(xj)((yj)(zj)) = (xj)(yz)j(y, z)\alpha i \\ &= (xyz)j(x, yz)\alpha i(y, z)\alpha i \\ &= (xyz)j(x, yz)\alpha iefe(y, z)\alpha i \quad \text{(since } ef = e \text{ and } (x, yz)\alpha ie = (x, yz)\alpha) \\ &= (xyz)j(x, yz)\alpha ie(y, z)\alpha ife \quad \text{(by (4.1), since } f \geq fe) \\ &= (xyz)j((x, yz)\alpha + (y, z)\alpha A(K_{x,yz}))i; \end{aligned}$$

where as

$$\begin{aligned} ((xj)(yj))(zj) &= (xy)j(x, y)\alpha i(zj) \\ &= (xy)j(zj)((x, y)\alpha A(J_{xy,z}))i \quad \text{(using Lemma 4.2)} \\ &= (xyz)j((xy, z)\alpha + (x, y)\alpha A(J_{xy,z}))i. \end{aligned}$$

Since $(xj)((yj)(zj)) = ((xj)(yj))(zj)$, Lemma 4.1 implies that

$$(x, yz)\alpha + (y, z)\alpha A(K_{x,yz}) = (xy, z)\alpha + (x, y)\alpha A(J_{xy,z}).$$

That is,

$$(y, z)\alpha A(K_{x,yz}) - (xy, z)\alpha + (x, yz)\alpha - (x, y)\alpha A(J_{xy,z}) = 0.$$

Hence α is a 2-cocycle by (3.2).

Suppose $E' = (T', \pi', i')$ is another extension of A by S which is equivalent to $E = (T, \pi, i)$ and $\theta : T \to T'$ is an isomorphism such that $\theta\pi' = \pi$ and $ai\theta = ai'$, for all $a \in A$. Let $\alpha' = S \times S \to A$ be the cocycle induced by a section $j' : S \to T'$. Since $j\theta\pi' = j'\pi'$, it follows from Lemma 4.1 that there exists a $D(S)_0$-map $\beta : S \to A$ such that $xj\theta = (xj')(x\beta i')$, for all $x \in S$. It is easily seen that $\alpha - \alpha' = (\beta)d_2^*$. Consequently, the cohomology class of α does not depend on the extension E but only on the equivalence class $[E]$. Hence we have a well defined mapping

$$[E] \mapsto ([E])\Sigma : \mathrm{Ext}(S, A) \to H^2(\mathbf{C}, A^1).$$

PROPOSITION 4.3. *Σ is a homomorphism of abelian groups.*

PROOF. Consider two extensions $E_1 = (T_1, \pi_1, i_1)$, $E_2 = (T_2, \pi_2, i_2)$ with sections $j_1 : S \to T_1$, $j_2 : S \to T_2$ and corresponding 2-cocycles $\alpha_1 : S \times S \to A$, $\alpha_2 : S \times S \to A$. Let $E_1 + E_2 = \underline{(T_1 + T_2}, \pi, i)$ be the sum of E_1 and E_2. If we define $j : S \to T_1 + T_2$ by $xj = \overline{(xj_1, xj_2)}$ then j is a section and the 2-cocycle induced by j is $\alpha_1 + \alpha_2$. Therefore

$$([E_1] + [E_2])\Sigma = ([E_1 + E_2])\Sigma = [\alpha_1 + \alpha_2] = [\alpha_1] + [\alpha_2]$$
$$= ([E_1])\Sigma + ([E_2])\Sigma.$$

Hence Σ is a homomorphism.

THEOREM 4.4. *$\Sigma : \mathrm{Ext}(S, A) \to H^2(\mathbf{C}, A^1)$ is an isomorphism of abelian groups.*

PROOF. To show that Σ is a monomorphism, assume that $E = (T, \pi, i)$ is an extension of A by S such that $([E])\Sigma = 0$. Then there exists a section $j : S \to T$ such that the 2-cocycle α induced by j is of the form $\alpha = (\beta)d_2^*$ for some $D(S)_0$-map $\beta : S \to A$. Now define $\mu : S \to T$ by $(x)\mu = (x)j(-(x\beta))i$. Then, for $x, y \in S$,

$$\begin{aligned}(x)\mu(y)\mu &= (x)j(-(x\beta))i(y)j(-(y\beta))i\\ &= (xy)j\big[(x, y)\alpha - (x)\beta A(J_{x,y}) - (y)\beta A(K_{x,y})\big]i \\ &\qquad\qquad\qquad\qquad\qquad\qquad\text{(by Lemma 4.2)}\\ &= (xy)j(-(xy)\beta)i \quad (\text{since } \alpha = (\beta)d_2^*)\\ &= (xy)\mu.\end{aligned}$$

Thus μ is a homomorphism. Further, $\mu\pi = 1_S$. Hence $E = (T, \pi, i)$ is a split extension of A by S and so, by Theorem 3.3 of Loganathan (1982), $[E]$ is the zero element of Ext(S, A).

To show that Σ is an epimorphism, let $\alpha : S \times S \to A$ be a 2-cocycle. Set

$$T_\alpha = \{(x, a) : x \in S, a \in A_{x*x}\}$$

and define a multiplication on T_α by

$$(x, a)(y, b) = \big(xy, (x, y)\alpha + aA(J_{x,y}) + bA(K_{x,y})\big).$$

Using Lemma 3.1 and (3.2), it is easily seen that the above multiplication is associative. The set $E(T_\alpha)$ of idempotents of T_α is

$$E(T_\alpha) = \{(e, -(e, e)\alpha) : e \in E(S)\}.$$

If $(x, a) \in T_\alpha$ then, for each $y \in V(x)$,

$$\big(y, (-(yx, yx)\alpha - (y, x)\alpha)A(J_{yx,y}) - aA(J_{x,y}K_{y*y,xy})\big)$$

is an inverse of (x, a). Hence T_α is a regular semigroup. Define $\pi : T_\alpha \to S$ by $(x, a)\pi = x$. Then π is an idempotent separating homomorphism from T onto S such that

$$(\operatorname{Ker}\pi)_e = \{(e, a) : a \in A_e\}, \qquad e \in E(S).$$

Define $i : A \to \operatorname{Ker}\pi$ by

$$(a)i = (e, -(e, e)\alpha + a), \qquad a \in A_e.$$

Then $E_\alpha = (T_\alpha, \pi, i)$ is an extension of A by S. If we define a section $j : S \to T_\alpha$ by $(x)j = (x, O_{x*x})$, $x \in S$, then the induced 2-cocycle is α so that $([E_\alpha])\Sigma$ is the cohomology class determined by α. Thus Σ is an epimorphism and hence an isomorphism.

By Proposition 3.2, $H^2(D(S^I), A^1)$ can be identified with its isomorphic image in $H^2(\mathbf{C}, A^1)$. We next characterize the subgroup of Ext(S, A) which corresponds to $H^2(D(S^I), A^1)$ under the isomorphism Σ.

An extension $E = (T, \pi, i)$ of A by S is called *I-split* if $\pi \mid IT : IT \to IS$ is an isomorphism of regular semigroups. If $E = (T, \pi, i)$ is an I-split extension of A by S then any extension which is equivalent to E is itself I-split. Further, the subset $E(S, A)$ of Ext(S, A) consisting of all equivalence classes of I-split extensions of A by S is closed under taking sums and inverses. Hence $E(S, A)$ is a subgroup of Ext(S, A).

THEOREM 4.5 (Loganathan, 1978). *$\Sigma \mid E(S, A)$ is an isomorphism of abelian groups from $E(S, A)$ onto $H^2(D(S^I), A^1)$.*

We first prove the following lemma.

LEMMA 4.6. *Let $\pi : T \to S$ be a homomorphism from a regular semigroup T onto S such that $\pi \mid IT : IT \to IS$ is an isomorphism. Let $(\pi, \pi): RP(T) \to RP(S)$ be the induced map, where $RP(T)$ and $RP(S)$ denote the set of all regular pairs in T and S respectively. Then there exists a section (j_1, j_2) of (π, π) satisfying the following conditions.*

(i) *If $e \in E(S)$, then $((e, e)j_1, (e, e)j_2) = (\bar{e}, \bar{e})$, where $\bar{e}$ is the unique idempotent of T such that $\bar{e}\pi = e$.*

(ii) *If (y, y'), (x, x') are regular pairs in T such that $(y, y') = (e_n \cdots e_0 x, x'e_0 \cdots e_n)$ for some $E(S)$-chain $(e_0, \ldots, e_n)$, with $e_0 = xx'$ and $e_n = yy'$, then*

$$((y, y')j_1, (y, y')j_2) = (\bar{e}_n \cdots \bar{e}_0 (x, x')j_1, (x, x')j_2 \bar{e}_0 \cdots \bar{e}_n).$$

PROOF. Consider the equivalence relation ρ on $RP(S)$ defined by $(y, y')\rho(x, x')$ if and only if $(y, y') = (e_n \cdots e_0 x, x'e_0 \cdots e_n)$ for some $E(S)$-chain $(e_0, \ldots, e_n)$ satisfying $e_0 = xx'$ and $e_n = yy'$. (Note that $(y, y')\rho(x, x')$ if and only if $[I, y, y'] = [I, x, x']$ in $D(S^I)$.) Let U be a transversal of ρ such that $(e, e) \in U$ for all $e \in E(S)$. Define $(j_1, j_2): U \to RP(T)$ such that $(e, e)j_1 = (e, e)j_2 = \bar{e}$ for all $e \in E(S)$, and $((x, x')j_1\pi, (x, x')j_2\pi) = (x, x')$ for all $(x, x') \in U$. We extend (j_1, j_2) to $RP(S)$ as follows. Suppose that $(y, y') \in RP(S)$. Then there exists a unique $(x, x') \in U$ and an $E(S)$-chain $(e_0, \ldots, e_n)$, with $e_0 = xx'$ and $e_n = yy'$, such that $(y, y') = (e_n \cdots e_0 x, x'e_0 \cdots e_n)$. We define

$$((y, y')j_1, (y, y')j_2) = (\bar{e}_n \cdots \bar{e}_0 (x, x')j_1, (x, x')j_2 \bar{e}_0 \cdots \bar{e}_n).$$

Since $\pi \mid IT$ is an isomorphism, the above map is well defined. It is quite obvious from the definition of (j_1, j_2) that it satisfies (i) and (ii).

PROOF OF THEOREM 4.5. Suppose $E = (T, \pi, i)$ is an I-split extension of A by S. We must show that $([E])\Sigma \in H^2(D(S^I), A^1)$. To prove this, take any section (j_1, j_2) of $(\pi, \pi): RP(T) \to RP(S)$ satisfying (i) and (ii) of Lemma 4.6. Let $j : S \to T$ be the section of π defined by $(x)j = (x, x^*)j_1$, $x \in S$, and let α' be the corresponding 2-cocycle so that $[\alpha'] = ([E])\Sigma$. Define $\alpha : F_2 \to A^1$ implicitly by

$$\langle [I, x, x'], [x'x, y, y'] \rangle \alpha i = (xy, y'x')j_2 (x, x')j_1 (y, y')j_1,$$

$\langle [I, x, x'], [x'x, y, y'] \rangle \in X^2$. Then using Lemma 4.6 and the fact that $\bar{e}$ is the identity element of the group $(\operatorname{Ker} \pi)_e$ it follows that α is well defined and $(\alpha)d_3^* = 0$. We claim that $(\alpha)\varphi_2^* = \alpha'$, implying that $([\alpha])\varphi_2^* = [\alpha'] = ([E])\Sigma$. To prove this take any $x, y \in S$. Put $e = x^*x$, $f = yy^*$, and $k = (xy)^*xy$. Then

$$\begin{aligned}(x, y)(\alpha)\varphi_2^* i &= \left(\langle J_{I,x}, J_{x,y} \rangle \alpha + \langle J_{I,y}, K_{x,y} \rangle \alpha \right) i \\ &= (xy, ky^*hx^*)j_2 (x, x^*)j_1 (ey, ky^*h)j_1 \\ &\quad \times (yk, ky^*)j_2 (y, y^*)j_1 (fk, k)j_1,\end{aligned}$$

where $h \in S(e, f)$. Now by Lemma 4.6,

$$(xy, ky^*hx^*)j_2 = (xy, (xy)^*)j_2(x, x^*)j_1\bar{h}(x, x^*)j_2$$

and

$$(ey, ky^*h)j_1 = \overline{eh}\,\bar{h}\,\overline{hf}(yk, ky^*)j_1 = \bar{e}\bar{f}(yk, ky^*)j_1.$$

It follows that

$$(x, y)(\alpha)\varphi_2^* i = (xy, (xy)^*)j_2(x, x^*)j_1(y, y^*)j_1 = (x, y)\alpha' i.$$

Hence $(\alpha)\varphi_2^* = \alpha'$.

Next suppose $[\alpha] \in H^2(D(S^I), A^1) \subseteq H^2(\mathrm{C}, A^1)$ and let α be a representative of $[\alpha]$. Then $(x, y)\alpha = 0$ for all $x, y \in IS$. It follows that the associated extension $E_\alpha = (T_\alpha, \pi, i)$ is I-split and $([E_\alpha])\Sigma = [\alpha]$.

Remark. If S is an inverse semigroup then every extension of A by S is I-split. Hence $E(S, A) = \mathrm{Ext}(S, A)$ and $\Sigma \mid E(S, A) = \Sigma$. In this case, Theorem 4.5 is equivalent to Theorem 7.4 of Lausch (1975).

5. The group $H^1(D(S^I), A^1)$

In this section we interpret the group $H^1(D(S^I), A^1)$ in terms of automorphisms of extensions. We begin by describing the group $H^1(D(S^I), A^1)$. Since $\mathrm{Hom}_{D(S^I)}(C_1, A^1) = \mathrm{Hom}_{D(S)_0}(S, A)$, a 1-cocycle β can be considered as a $D(S)_0$-map $\beta : S \to A$ such that

$$(5.1) \qquad (y)\beta A(K_{x,y}) - (xy)\beta + (x)\beta A(J_{x,y}) = 0,$$

for all $x, y \in S$. Since $\mathrm{Hom}_{D(S^I)}(C_0, A^1) = \varprojlim_{D(IS)} A$, a 1-cocycle $\beta : S \to A$ is a coboundary if and only if there exists a σ in $\varprojlim_{D(IS)} A$ such that

$$(5.2) \qquad (x)\beta = (x^*x)\sigma - (xx^*)\sigma A(J_{xx^*,x}),$$

for all $x \in S$. Hence $H^1(D(S^I), A')$ is the group of all $D(S)_0$-maps satisfying (5.1) modulo the subgroup of all $D(S)_0$-maps satisfying (5.2).

Let $E = (T, \pi, i)$ be an extension of A by S. Let Aut E denote the group of all automorphisms θ of T satisfying $\theta\pi = \pi$, and $ai\theta = ai$ for all $a \in A$. For each $\sigma \in \varprojlim_{D(IS)} A$ define $\theta_\sigma : T \to T$ by

$$(5.3) \qquad (t)\theta_\sigma = t(t\pi)\beta i,$$

where $\beta: S \to A$ is the 1-cocycle defined by (5.2), θ_σ is a homomorphism because

$$\begin{aligned}(t)\theta_\sigma(u)\theta_\sigma &= t(x)\beta iu(y)\beta i\\ &= tu\big((x)\beta A(J_{x,y}) + (y)\beta A(K_{x,y})\big)i \quad \text{(by Lemma 4.2)}\\ &= tu(xy)\beta i \quad \text{(since } \beta \text{ is a cocycle)}\\ &= tu(tu)\pi\beta i\\ &= (tu)\theta_\sigma,\end{aligned}$$

where $x = t\pi$ and $y = u\pi$. Clearly $\theta_\sigma\pi = \pi$, and $(a)i\theta_\sigma = (a)i$ for all $a \in A$. Hence θ_σ is an automorphism of T and $\theta_\sigma \in \operatorname{Aut} E$. Let

$$V = \left\{\theta_\sigma : \sigma \in \varprojlim_{D(IS)} A\right\}.$$

We now will define a map $\eta: \operatorname{Aut} E \to Z_1(D(S^I), A^1)$, where $Z_1(D(S^I), A^1)$ denote the group of all 1-cocycles. Let $\theta \in \operatorname{Aut} E$. Since $\theta\pi = \pi$, by Lemma 4.1 we can associate with each t in T a unique element a_t in $A_{(t\pi)^*t\pi}$ such that $t\theta = t(a_t)i$. If $t\pi = u\pi$ then $a_t = a_u$. Hence we obtain a $D(S)_0$-map $\beta: S \to A$ such that $(t\pi)\beta = a_t$, $t \in T$. If $t, u \in T$ then it follows from Lemmas 4.1 and 4.2 that

$$a_{tu} = a_t A(J_{t\pi,u\pi}) + a_u A(K_{t\pi,u\pi}).$$

Hence β is a cocycle. We associate with θ the element $(\theta)\eta = \beta$. Clearly η is a homomorphism of groups.

THEOREM 4.6. *Let $E = (T, \pi, i)$ be an extension of A by S. Then the homomorphism*

$$\eta: \operatorname{Aut} E \to Z_1\big(D(S^I), A^1\big)$$

induces an isomorphism

$$(\operatorname{Aut} E) \mid V = H^1\big(D(S^I), A^1\big).$$

PROOF. If $\beta: S \to A$ is a 1-cocycle then the map $\theta: T \to T$, defined by $(t)\theta = t(t\pi)\beta i$, is a homomorphism by Lemma 4.2. Further $\pi\theta = \pi$ and $(a)i\theta = (a)i$ for all $a \in A$. Hence $\theta \in \operatorname{Aut} E$. Obviously $(\theta)\eta = \beta$.

It is easy to see that $\theta \in V$ if and only if $(\theta)\eta$ is a coboundary. Hence $H^1(D(S^I), A^1)$ is isomorphic to the quotient group $(\operatorname{Aut} E) \mid V$.

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