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Some discussion of the nonlocal treatment of the dissipation in the Reynolds stress models

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Abstract. We investigate the weakness of the present turbulence model with the nonlocal treatment of dissipation rate. A revised version is well tested for the solar convection. The suggestion of constant mixing length parameter of MLT could not hold any more if we refer to the nonlocal description of the dissipation rate, especially in the region of overshooting zone.

Keywords. Convection, hydrodynamics, sun:interior

1. Introduction

Discarding of the traditional local mixing length theory(MLT) (Böhm-Vitense (1958)), Xiong developed a nonlocal turbulence model by using the Reynolds stress methods for studying the stellar convection (Xiong (1979)). However, Canuto argued that Xiong's model is not a fully nonlocal turbulence model since it still uses a local description of dissipation rate, hence he developed a fully nonlocal turbulence model by describing the dissipation rate in a nonlocal way (Canuto & Dubovikov (1998)). By using this fully nonlocal model, Kupka and his coworkers have calculated the cases of A-star and DA/DB white dwarfs with thin convection zones (Kupka & Montgomery (2002), Montgomery & Kupka (2004)). Meanwhile the more attractive case of the sun with a deep convection zone is still unclear. The purpose of this paper is to try to explain where the difficulty comes from and what's the difference if we introduce the nonlocal description of the dissipation rate.

2. Results and discussion

The fully nonlocal turbulence model with the nonlocal treatment of dissipation rate is given by (Canuto & Dubovikov (1998))

$$\frac{\partial K}{\partial t} + D_f = g\alpha J - \epsilon \tag{2.1}$$

$$\frac{\partial}{\partial t}\frac{1}{2}\theta^2 + D_f = \beta J - \tau_\theta^{-1}\overline{\theta^2} + \frac{1}{2}\frac{\partial}{\partial r}(\chi\frac{\partial}{\partial r}\overline{\theta^2}) \tag{2.2}$$

$$\frac{\partial}{\partial t}J + D_f = \beta\overline{\omega^2} + (1-\gamma_1)g\alpha\overline{\theta^2} - \tau_{p\theta}^{-1}J + \frac{1}{2}\frac{\partial}{\partial r}(\chi\frac{\partial}{\partial r}J) \tag{2.3}$$

$$\frac{\partial}{\partial t}\frac{1}{2}\overline{\omega^2} + D_f = -\tau_{pv}^{-1}(\overline{\omega^2} - \frac{2}{3}K) + \frac{1}{3}(1+2\beta_5)g\alpha J - \frac{1}{3}\epsilon \tag{2.4}$$

$$\frac{\partial \epsilon}{\partial t} + D_f = 2c_1\tau^{-1}g\alpha J - 2c_2\tau^{-1}\epsilon + c_3\epsilon N \tag{2.5}$$

$$N = (g\alpha|\beta|)^{\frac{1}{2}} \tag{2.6}$$

$$\tau = \frac{2}{\epsilon}K \tag{2.7}$$

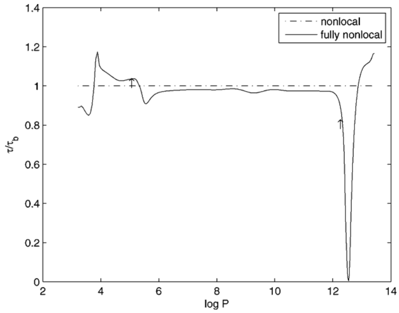

Figure 1. The ratio of the dissipative timescale over the buoyant timescale of the fully non-local (solid curve) and nonlocal model (dash-dotted curve) in the solar envelope. The arrows indicate the boundaries of the convective instable zone where $\nabla = \nabla_{ad}$.The ratio in the lower overshooting zone is very small due to the nonlocal treatment of the dissipation rate.

When K^2/ϵ is very small compared with $1/\tau$, the diffusion term in the equations can be neglected. Then suppose that $(K, \overline{\theta^2}, J, \overline{w^2}, \epsilon)$ is the solution of the system. It is not difficult to check that $\lambda(K, \overline{\theta^2}, J, \overline{w^2}, \epsilon)$ is also the solution by substituting it into the equations if we notice that the timescales remain constant at the same level and the system can be viewed as a linear system, here λ is a constant. This tells us that the solution is not unique and that's the reason why steady solution is hard to be obtained by solving these equations. To overcome this weakness, a new formula of Brunt-Väisälä frequency is introduced for replacing the former one. It can be defined by

$$N = \frac{c_\epsilon}{\alpha_m H_p} K^{1/2}. \tag{2.8}$$

$\tau_b = 2/N$ could be viewed as the buoyant characteristic timescale describing the travelling time of the bulb to go through the mixing length. We know that τ is the dissipative characteristic timescale for the bulb to consume all of its kinetic energy. If $\tau/\tau_b > 1$, the bulb would reserve some energy when finishing travelling the mixing length. If $\tau/\tau_b < 1$, the bulb would use up all of its kinetic energy within the mixing length. If $\tau/\tau_b = 1$, which is the local description of dissipation rate, the kinetic energy of the bulb is exactly used up after the bulb goes through the mixing length. This may provide us information to explain why the mixing length parameter α_m is supposed to have different values at different levels when using the local mixing length model. This disadvantage will disappear when we refer back to the nonlocal description. If $\tau/\tau_b > 1$, an adjustment of increasing the mixing length parameter would balance the timescales, hence it is corresponding to a larger mixing length parameter of MLT. While $\tau/\tau_b < 1$ is corresponding to a smaller mixing length parameter of MLT.

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