

## Energy Balance in Circumstellar Envelopes

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**Abstract.** We have successfully determined the kinetic temperature of the electron gas as a function of position in the circumstellar envelopes of Be stars. Our method yields a self-consistent solution of the equation for energy conservation, thus eliminating the necessity to assume arbitrarily a temperature for the gas. Our technique has been applied to Be stars of differing spectral classes, and we have also used several models for the distribution of the circumstellar material. The observed shape and relative line strength of the  $H\alpha$  line for several Be stars were matched successfully with these models. Recently we have begun to investigate the role of the diffuse radiation field in the Lyman continuum using the on-the-spot approximation. As a preliminary step to including metallic line cooling by the circumstellar gas, we have determined iron ionization fractions throughout the disks of both an early-type and a late-type Be star.

### 1. Introduction

A standard simplification used for Be star models has been to assume a temperature for the envelope and thus bypass the solution of the equation of energy conservation. We have developed a method for self-consistently enforcing the conservation of energy by adjusting the temperature throughout the envelope until the rates of energy gain and energy loss are balanced locally. The processes which are assumed to contribute to the rate of energy gain include photoionization, collisional de-excitation, and free-free absorption. Those included in the rate of energy loss include radiative recombination, collisional excitation, and free-free emission. The temperature is determined via an iterative process: first the atomic level populations are calculated, next the energy rates are determined based on these level populations, and finally the difference between the energy gain and loss rates are evaluated. This process is repeated until, at each position considered in the envelope, the rate of energy gain balances the rate of energy loss to within a prescribed value. Models have been produced for several notable Be stars, and the self-consistent temperature distributions have been used to match the relative strengths of the  $H\alpha$  emission lines. The effect of diffuse radiation produced within the envelope has been included via the on-the-spot approximation (Osterbrock 1974).

## 2. Envelope Temperature Distributions

Using the same parameters, with the exception of the density, as those in the investigation by Poekert & Marlborough (1978), hereafter PM, we determined the temperature distribution throughout the envelope of  $\gamma$  Cas by balancing the rates of energy gain and energy loss (Millar & Marlborough 1998; 1999b). The resultant temperature distribution is shown in Figure 1. In this model, the diffuse radiation field is included by the on-the-spot approximation. The diffuse field generally increases the degree of ionization in most locations in the envelope; consequently the density,  $\rho_0$ , which is assumed at one location in the equatorial plane for the equation of continuity, must be reduced by  $\sim \frac{1}{3}$  relative to the value assumed by PM in order to match the relative strength of the  $H\alpha$  emission line.

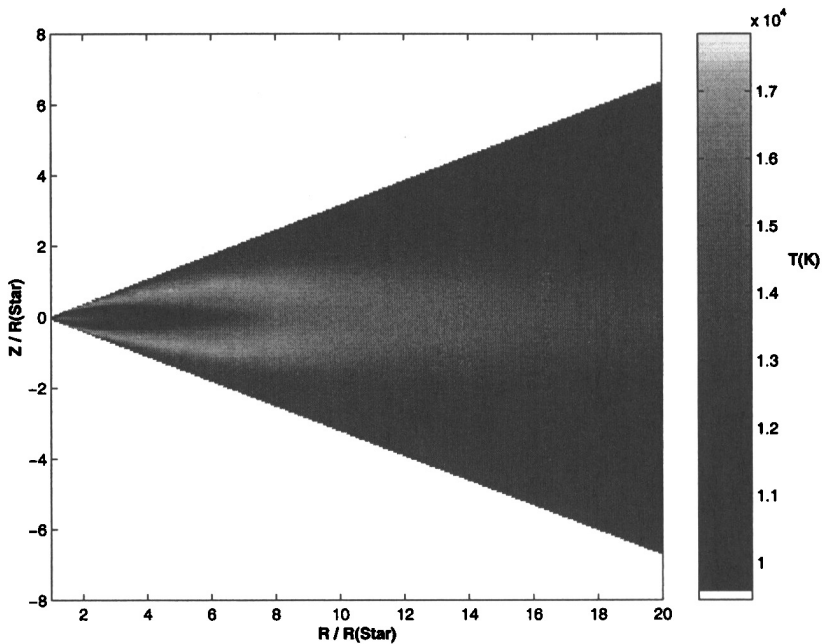


Figure 1. The predicted temperature for which the local energy gain balances energy loss for the circumstellar disk of  $\gamma$  Cas with the inclusion of the diffuse radiation field.

A similar investigation was conducted for the late Be shell star 1 Del (Millar & Marlborough 1999c). The parameters of Marlborough & Cowley (1974) were used, with the exception of  $\rho_0$  which was adjusted until the relative line strength of  $H\alpha$  matched observations. The resultant temperature distribution for this star has a cooler neutral region in and surrounding the equatorial plane, with hotter regions further from the equatorial plane. The late-type central star produces relatively fewer ionizing photons, and, as a result, the portions of the envelope in and near the equatorial plane are cool and largely neutral.

### 3. Integrated Flux Calculations

We have also calculated the total integrated flux emitted in  $H\alpha$ ,  $P\alpha$ , and  $Br\alpha$  from the envelopes of a range of Be stars using models which have temperatures determined self-consistently. In principle, for Be stars, there are enough ionizing photons from the central star to account for the observed lines; the question is really one of energy conversion efficiency into line emission. Apparao & Tarafdar (1987) claim that this conversion efficiency is not high enough for the later Be stars, and thus an additional source of ionizing photons is required. Our flux calculations for  $H\alpha$  are displayed in Table 1. Column 3 corresponds to case B recombination, that is, the flux is simply proportional to the emission measure of the disk, and all  $H\alpha$  photons generated by recombination escape (similar to the assumptions made by Apparao & Tarafdar). We have found, however, that this is not a reliable way to determine these fluxes. One must include optical depth effects, a realistic geometry, and collisional excitation. Column 4 is an (ad-hoc) improvement to column 3 by weighting the emission from each volume element of the envelope by an escape probability,  $P_{ji}^{esc}$ , estimated from the optical depth from the point of interest to the nearest edge of the envelope along a path perpendicular to the equatorial plane. Column 5 displays our best estimate using the standard escape probability approximation for the flux,

$$F(ji) = h\nu_{ji} A_{ji} \int_{\text{Disk}} N_j P_{ji}^{esc} dV. \quad (1)$$

See Millar, Sigut, & Marlborough (1999) for the  $P\alpha$  and  $Br\alpha$  flux calculations. The results from Column 5 agree adequately with the observations of Ashok et al. (1984) and demonstrate that the ionizing radiation field of the central star can be converted into hydrogen line emission with the required efficiency. We find that no additional source of ionizing radiation is required (Millar et al. 1999).

Table 1. Calculated hydrogen line flux for  $H\alpha$ .

Star	Diffuse Radiation	Case B [erg s <sup>-1</sup> ]	Ad-Hoc $P_{ji}^{esc}$ [erg s <sup>-1</sup> ]	$P_{ji}^{esc}$ [erg s <sup>-1</sup> ]
$\gamma$ Cas	yes	8.3x10 <sup>35</sup>	2.1x10 <sup>32</sup>	5.9x10 <sup>33</sup>
	no	2.3x10 <sup>36</sup>	6.3x10 <sup>31</sup>	6.9x10 <sup>33</sup>
1 Del	yes	2.4x10 <sup>34</sup>	7.5x10 <sup>28</sup>	1.3x10 <sup>32</sup>
	no	3.1x10 <sup>33</sup>	4.9x10 <sup>29</sup>	2.3x10 <sup>32</sup>

$\gamma$  Cas is B0IVe star with a luminosity of  $1.3 \times 10^{38}$  erg s<sup>-1</sup>.

1 Del is a B8-9e shell star with a luminosity of  $9.5 \times 10^{35}$  erg s<sup>-1</sup>.

#### 4. Iron Ionization Zones

Our self-consistent temperature distributions can be used to delineate the envelope ionization zones accurately. These results can also be used to determine where in the envelope various lines are formed. Determining the ionization zones of metals is the first step to include metallic line cooling in the energy balance calculation. We currently are considering iron with 4 stages of ionization, balancing rates of photoionization and recombination, and including the effects of charge exchange. For example, the model for 1 Del discussed in Section 2 has an envelope which is predominantly FeII with the exception of the hotter regions further from the equatorial plane, which are FeIII.

#### 5. Summary

Given a density and velocity distribution, both a self-consistent ionization-excitation equilibrium and electron temperature can be obtained. This will improve a model's use as a diagnostic for the physical conditions in circumstellar winds. Self-consistent temperatures are critical to determining the correct thermal structure and ionization balance within the wind. We are extending our work with metals to determine whether or not line cooling plays a significant role.

The results for the self-consistent temperatures for  $\gamma$  Cas and 1 Del are:

- $\gamma$  Cas: density weighted average temperature of 10700 K. This result agrees with the temperature of  $9500 \pm 1000$  K obtained by Honny et al. (1999).
- 1 Del: density weighted average temperature of 5900 K.

Both of these results are *much lower* than the stellar effective temperatures of 25000 K and 14000 K, respectively, for  $\gamma$  Cas and 1 Del. The H $\alpha$  fluxes from our Be star models agree well with observations and an additional source of ionizing photons is not necessary.

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## Discussion

**R. Hirata:** In some active shell stars with late B central stars, like 28 Tau, 88 Her and 4 Her, the broad Ca II K absorption feature is observed in its initial shell development phase. Can such a feature be realized in your temperature distribution?

**C. Millar:** Our models are pure hydrogen. Recently we have begun to add metals, namely iron, to investigate line cooling. At the present time, I do not know whether or not such a feature could be reproduced by our models.

**J. Porter:** In your simulation of  $\gamma$  Cas, you have regions where high temperature gas is in the plane with lower temperature gas lying above it (and vice-versa). Are regions in the disc convectively unstable? (I'm thinking more about disc structure implications than energy balance here.)

**C. Millar:** In our models, the density distribution perpendicular to the equatorial plane is exponential. Although there are regions where cool gas lies above hot gas, the cool gas will be less dense due to the exponential decrease in density. Hence, I do not expect regions in the disk that are convectively unstable; however, perhaps a more complete investigation may be valuable.

**N. Ashok:** Will the model developed by you be able to explain the H $\alpha$  line flux in Ae type stars?

**C. Millar:** We were able to successfully explain the line flux for the star 1 Del which is a late B or early A shell star. However, we have not yet extended our investigations to cooler objects.

**J. Kubat:** How do you determine the radiation field? Do you solve the transfer equation?

**C. Millar:** The local radiation field is determined by taking into account geometrical dilution and optical depth effects for the direct radiation field coming from the central star. However, along the path from the central star to the point of interest, emission from the envelope is not included. The diffuse radiation field is approximated by the on-the-spot approximation which removes the necessity of solving the transfer equation about  $4\pi$  steradians.

**M. Smith:** Your figure showing the spatial temperature distribution indicates a rise in temperature as one moves away from the star in the equatorial plane. Can you tell me why, as it doesn't look intuitively obvious?

**C. Millar:** The model for  $\gamma$  Cas with the inclusion of the diffuse radiation field has a temperature distribution which decreases near the stellar surface along

the equatorial plane of the envelope. Generally, inclusion of the diffuse field increases the degree of ionization and heating. However, in the equatorial plane between the stellar surface and approximately 2 stellar radii, the densities are large and the column densities in neutral hydrogen are also large; consequently the number of recombinations to the ground state are correspondingly low. As a result, the temperature remains low at approximately the same values as the original Poeckert-Marlborough model without the diffuse field. With increasing distance from both the central star and the equatorial plane, the optical depths for some lines of sight become smaller, resulting in more ionizations and heating. Therefore the temperature increases, reaches a maximum, and then begins to fall again. See Figure 1 Millar & Marlborough (1998) for a plot of the temperature distribution for  $\gamma$  Cas along the equatorial plane with and without the inclusion of the diffuse field.

**P. Harmanec:** It is certainly encouraging to see that the disk temperature of  $\gamma$  Cas estimated by your method agrees well with an independent estimate by Hony et al. (1999). Now,  $\gamma$  Cas was observed to show two separate periods of the long-term V/R variations. The cycle was 3-6 years during this episode, and 4.0 years during the other one. Dr. Okazaki told us earlier that with his one-armed oscillation model, the cycle length depends on the disk temperature. I believe there are enough observational data from both episodes and it would be very useful to: 1) check if there is evidence for different disk temperatures during the two episodes and 2) how the cycle lengths compare with the temperatures predicted by Okazaki's model.

**C. Millar:** Our models are 2 dimensional and axisymmetric. It would require substantial modification to the code to simulate the effect of a discrete density enhancement. Furthermore, it is my understanding that Okazaki's models used an isothermal disk temperature and to my knowledge he does not predict the change in temperature due to the density enhancement.

**J. Telting:** You've shown that the disk temperature may be about twice as low as previously assumed. This may have implications for the source function, free-free emissivity, and hydrogen ionization fraction in the disk. Could you comment on how your results will affect the disk density values as previously derived from IR continuum measurements?

**C. Millar:** Our models determine self-consistently the source function and free-free emissivity with no *a priori* assumptions regarding the degree of ionization. At present, we do not calculate the IR continuum flux nor match the IR lines. We simply tested the parameters used by Waters et al. (1987) in order to determine a self-consistent temperature distribution. The average of our temperature distributions are  $\sim \frac{1}{2}$  the values of the isothermal envelopes assumed by Waters. See Millar & Marlborough (1999a).