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The primitive soluble permutation groups of degree less than 256

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Primitive permutation groups have long been objects of interest: in 1872 Jordan [5], listed them up to degree 17 (but made a number of omissions, as pointed out by Miller in several papers from 1894 to 1900). In 1970 the primitive permutation groups were still only known up to degree 20. Sims [7] verified this list by machine and later extended it to degree 50. Harada and Yamaki [4] determined the primitive permutation groups of degree 64 that have soluble socle. More recently, Dixon and Mortimer [3] have determined the primitive permutation groups of degree less than 1000 that have insoluble socle. The central topic of this thesis is to determine and provide electronic access to the *soluble* primitive permutation groups of degree less than 256. This is motivated partly by the work of Dixon and Mortimer and partly because of the anticipated usefulness of such a list in a soluble quotient algorithm currently under development.

The primitive soluble permutation groups are in one-to-one correspondence with the irreducible soluble linear groups over finite prime fields. There is a large body of theory due to Jordan [2] and Suprunenko [8] which describes the structures of maximal irreducible soluble subgroups of general linear groups over arbitrary fields (hereafter called *JS-maximals*). This theory is extended as necessary to construct explicitly the JS-maximals of $GL(n,p)$ for a variety of pairs (n,p), where p is prime.

The early chapters of the thesis examine in detail the JS-maximals of $GL(2,p)$. These investigations lead to a theorem which lists exactly one member of each conjugacy class of irreducible soluble subgroups of $GL(2,p)$.

The later chapters deal with the problem of listing the irreducible soluble subgroups of $GL(n,p)$ for $n > 2$ and $p^n < 256$. Some of these cases are dealt with by *ad hoc* arguments. A theorem is obtained for listing certain irreducible soluble subgroups of $GL(q,p)$, where q is any prime.

The two remaining cases are $GL(6,2)$ and $GL(4,3)$ of which the latter is more difficult. The relevant lists for these groups are compiled by a combination of theory

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and computation. An algorithm for listing the irreducible subgroups of several of the JS-maximals is presented, and is implemented in the group theory system Cayley [1]. The irreducible subgroups of the remaining JS-maximals are determined by theoretical techniques similar to those developed for $GL(2,p)$.

The provision of electronic access to the list of groups obtained is discussed. There are 370 conjugacy classes of irreducible soluble subgroups of $GL(n,p)$ for $n > 1$, p prime, and $p^n < 256$. A Cayley library [6] which provides access to one group from each such class is described. The library also contains on-line help and some procedures for manipulating the groups; in particular, there is a procedure for converting the groups to the primitive soluble permutation groups to which they correspond. The library will be released as part of Cayley Version 3.8.

The thesis concludes with a note on work in progress on the irreducible soluble subgroups of $GL(8,2)$ (that is, the primitive soluble permutation groups of degree 256). The preparation of a list of these groups will involve a refinement of the algorithm used for $GL(4,3)$ and $GL(6,2)$; the original algorithm is not feasible for $GL(8,2)$ because of the large order of one of the JS-maximals.

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