scope of the semigroups considered by replacing the endomorphisms with completely positive (CP) maps. Of course Arveson was one of the originators of the theory of completely positive maps [**1**]. Some physical motivation of the importance of 'CP-semigroups' is nicely presented at the beginning of Part 3, via a discussion of the heat semigroup associated with a Riemannian manifold; the generators of CP-semigroups in many ways act like the Laplacian operator. However, CP-semigroups are clearly interesting in their own right, and it is nice that the author has devoted so much space here to their theory. A main point is that every CP-semigroup may be 'dilated' to an E_0-semigroup that has nice properties. Part 4 is titled 'Causality and dynamics'. It discusses 'eigenvalue lists', namely sequences of eigenvalues of positive trace class operators associated with the causal structure we mentioned earlier, and uses these to produce nontrivial 'past' and 'future' E_0-semigroups. Part 5, the final part, discusses several fascinating examples, including Powers's examples based on quasi-free states, and the probabilistic examples of Tsirelson, related to 'off-white noise'.

The book has clearly been written with the goal of teaching the subject; the exposition is crisp, clear, and will be accessible to graduate students. Nonetheless, the author's sophistication, profundity, and broad culture sparkles throughout the 434 pages. The book is handsomely bound in the bright yellow Springer Monographs in Mathematics series. It has a fine index, and the reader will greatly appreciate the trouble the author takes to describe what is going on, relate how a result or formula ought to be viewed, and so on. There are many new results, and significant reformulations and simplifications of the theory. It is superbly written for the most part; there are a few minor imperfections, such as the misspellings that occur from time to time throughout the book. In addition to being the essential reference for those working on E_0-semigroups, this magnificent book will be useful and inspirational to a wide range of mathematicians and mathematical physicists.

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Duren, P. *Harmonic mappings in the plane* (Cambridge University Press, 2004), 0 521 64121 7 (hardback), £40.

Harmonic mappings are plane bijective maps of the form

$$w = u + \mathrm{i}v = f(x, y), \tag{1}$$

where u and v are harmonic (but in general not conjugate) functions of (x, y). Let us quote from the introduction to the book (p. xi).

> In many instances the properties of analytic univalent functions serve as models for generalizations to harmonic mappings, but other results are peculiar to analytic functions and do not extend to more general harmonic maps. On the other hand some results for harmonic mappings have no counterpart for conformal maps. This is particularly true of the connections with minimal surfaces.

Let us look at these connections, which are developed in the last two chapters of the book. A surface S in $\mathbb{R}^3$ is called a minimal surface if, for each sufficiently small simple closed curve C on S, the portion of S enclosed by C has the minimum area among all surfaces spanning C. Minimal surfaces can be constructed physically by dipping a loop of wire into soap solution. Because of surface tension, the resulting soap film will assume the shape that minimizes surface area (p. 161).

The book contains many useful diagrams and in particular some pretty pictures of minimal surfaces (e.g. pp. 161, 162, 171, 181).

Locally a minimal surface can always be projected onto one of the coordinate planes. A surface that has such a projection onto a plane domain D is called a non-parametric surface over D. A general surface S can be regarded as an equivalence class of differentiable mappings $X = \Phi(U)$ from a domain D in $\mathbb{R}^2$ onto a set Ω in $\mathbb{R}^3$. Here $U = (u, v)$ and $X = (x, y, z)$ denote points in $\mathbb{R}^2$ and $\mathbb{R}^3$, respectively. The surface is regular if the Jacobian matrix

$$\begin{pmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \end{pmatrix}$$

has rank 2 at every point of D, so that it is possible locally to express one of the coordinates in terms of the other two. It is possible to choose isothermal parameters so that in the map angles between curves in the parameter plane are the same as the angles between the image curves on S. In this case the distance element $\mathrm{d}s$ on the surface takes the simple form

$$\mathrm{d}s^2 = \lambda^2(\mathrm{d}u^2 + \mathrm{d}v^2).$$

We now have the following theorem.

Theorem (p. 165). *Let a regular surface S be expressed in terms of isothermal parameters. Then the coordinates (x_1, x_2, x_3) of the position vector X in S are harmonic functions of the parameters (u, v) if and only if S is a minimal surface.*

The rather complicated necessary and sufficient conditions for the harmonic functions x_k, $k = 1, 2, 3$, to give rise to a minimal surface will be found in § 9.3.

The connection with minimal surfaces is one reason why it is important to study harmonic maps. Let us now look at the analogies between harmonic and analytic univalent maps.

A complex-valued harmonic function f in a domain D can be written in the form

$$f = h + \bar{g}, \tag{2}$$

where h and g are analytic in D.

One of the striking differences between analytic and harmonic maps concerns the behaviour at the boundary. An analytic mapping from the unit disc Δ onto a Jordan domain D extends to a homeomorphism of the closures, from $\bar{\Delta}$ onto $\bar{D}$. By contrast, a harmonic homeomorphism may well map a boundary arc of Δ onto a single point of $\bar{D}$ (Chapter 3).

It is also possible to obtain sharp bounds for the distortion and coefficients for the class C_{H} of convex harmonic maps, normalized so that

$$h(0) = g(0) = g'(0) = 0 \quad \text{and} \quad h'(0) = 1 \tag{3}$$

in (2). One can also obtain qualitatively similar bounds for the class S_{H}^0 of harmonic maps, normalized as in (3), onto arbitrary simply connected domains, but the known bounds are no longer sharp. However, there exists a 'harmonic Koebe function' with

$$h(z) = \frac{z - \frac{1}{2}z^2 + \frac{1}{6}z^3}{(1-z)^3}, \qquad g(z) = \frac{\frac{1}{2}z^2 + \frac{1}{6}z^3}{(1-z)^3}$$

in (2), which is conjectured to give sharp bounds for the coefficients a_n and b_n of h and g, respectively. The only sharp bound known so far is $|b_2| \leqslant \frac{1}{2}$.

Clunie and Sheil-Small proved that the conjectured bounds also hold for typically real functions and starlike mappings (Chapter 6) and also obtained bounds for the general class S_{H}^0.

The author presents all the above results and many more in his usual clear and concise way. There is a good list of references, but there is no detailed reference to the classical representation of Weierstrass and Enneper in the 1960s, 'which appears to have been the start of the study of minimal surfaces'.

I predict a growing interest in the study of harmonic mappings both for the beauty of the results already obtained and for the many open problems and conjectures. For all students in this field Duren's book will be essential reading. It will also be the classic reference book in this area.

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Voisin, C. *Hodge theory and complex algebraic geometry*, Volumes I, II (transl. from the French original by L. Schneps) (Cambridge University Press, 2002, 2003), 0 521 80260 1 (hardback), £55 ($80), 0 521 80283 0 (hardback), £60 ($85).

During a period in which algebraic geometry was crying out for new techniques, Hodge produced a deep insight, realizing that harmonic forms could be used both to understand old results from a clearer perspective and to produce new results including the Hodge index theorem, one of the inspirations for the Atiyah–Singer theorem and its consequences. His existence proof was analytic in nature and thus combined differential and algebraic geometry and has encouraged many others to use analytic techniques in geometry. Some of the prominent names that have done so more recently are Yau, Donaldson, Hamilton and Perelman. Hodge theory itself has continued to develop and, in particular, work of Deligne and Griffiths over a significant part of the second half of the last century greatly increased its scope. Griffiths, in particular, often wrote in an expository style and this helped others to understand the results and as a consequence Hodge theory has become an essential tool in diverse areas.

During the 1990s other expository accounts were written but these two volumes give the most complete account to appear so far. They include a great deal of background material often seamlessly woven into the proofs of results that were very new when the course on which they are based was given in Paris in 1998–2000. A motivated reader who knows basic facts about differential forms and algebraic topology should have little difficulty in learning a great deal by reading these volumes; one can even start afresh at a number of points. The whole is written with the authority of the foremost expert on Hodge theory; some further striking results have been found by Voisin since these volumes were written but the background to these recent papers is all here. She described some of her new results at the Hodge centenary meeting held in Edinburgh in 2003. Even more recently she has found striking and simple examples of compact Kähler manifolds not homotopically equivalent to a projective variety, thus solving a very longstanding problem.

Due to the origin of these volumes, there are a number of exercises even in the more advanced chapters and hence they could easily form the basis for reading courses or seminars as well as for individual learning. Both volumes start with fairly detailed introductions, each about 15 pages long. Anyone wishing to find out what the theory is about in a general way or to understand how the material is arranged should spend some time reading these chapters. The first volume is divided into four parts: 'Preliminaries', 'The Hodge decomposition', 'Variations of Hodge structure' and 'Cycles and cycle classes'. The chapters in the part entitled 'Preliminaries' are on