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Power series expansions of algebraic functions

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In this thesis I study algebraic functions $y(X)$, that is, zeros of polynomials $F(Y)$ in $K[X][Y]$.

It is well known that, in characteristic zero, the function y can be expanded as an extended power series in a fractional power of X,

$$y(X) = \sum_{h=-N}^{\infty} c_h X^{h/r}$$

where r is no greater than the degree of f. I study this situation in detail describing how to get the Puiseux expansion using a simple and explicit algorithm. The analysis differs slightly from previous works allowing a purely formal proof of Puiseux's result when generalised to the situation where the coefficients of f are Dirichlet series or exponential polynomials.

The second part of this thesis deals with the Eisenstein constant of an algebraic series $\sum_{h\geqslant 0} c_h X^h$ with rational coefficients, that is, the constant $c \in \mathbf{N}$ given by Eisenstein's theorem so that, for some integer c_*, each $c_* c^h c_h \in \mathbf{N}$. I provide a survey of results relating to Eisenstein's theorem and discuss various techniques for determining c.

In the third part of this thesis I focus on the case where the ground field is the finite field $\mathbf{F}_\mathbf{p}$. I survey results concerning algebraic power series in particular with relation to diagonals of rational functions and finite automata. I consider the questions "when is an algebraic power series the reduction modulo p of an algebraic series with rational coefficients?" and "what happens when an algebraic series with rational coefficients cannot be reduced modulo p?". I show that $y(X)$ may expand as a series which is not a Puiseux series and which I call an *anti-power series*. I discuss the structure of these anti-power series and the relation between them and the Eisenstein constant.

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I survey results concerning Hilbert's Irreducibility theorem in relation with the Puiseux' expansion.

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