AN OSCILLATOR MODEL FOR STELLAR VARIABILITY

T. NAKAHARA and Y. TANAKA
Faculty of Education, Ibaraki University, Bunkyo, Mito 310, Japan

Abstract. The dependence of coupling constants in a coupled oscillator model is examined with simplified methods. The Lyapunov exponents are preliminary introduced for the model. The behaviors of oscillator model are examined in a parameter plane. So-called the Arnold's tongues for phase-locking states are observed in fractal patterns.

1. Introduction

Two-mode coupling oscillator model for stellar variability has investigated in the view of nonlinear dynamics (Seya et al. 1989). The equations of coupled oscillator model are described as follows;

$$\begin{aligned}\frac{d^2x_1}{dt^2}=&-\sigma_1^2x_1+\sigma_1^2\{[(1/2)C_{111}x_1+C_{112}x_2]x_1\\&+\varepsilon_1(1-\alpha_1^2x_1^2)(dx_1/dt)+(1/2)C_{122}x_2^2\},\\\frac{d^2x_2}{dt^2}=&-\sigma_2^2x_2+\sigma_2^2\{[(1/2)C_{222}x_2+C_{212}x_1]x_2\\&+\varepsilon_2(1-\alpha_2^2x_2^2)(dx_2/dt)+(1/2)C_{211}x_1^2\},\end{aligned}\tag{1}$$

where the coupling constants C_{ijk} are evaluated by Takeuti (1985) for a classical cepheid model. It is featured that the van der Pol's damping term is introduced. Seya et al. (1989) show the complicated behaviors of Eq. (1) as the change of coefficients in damping terms. The behaviors seem to be phase-locking, quasi-periodicity or chaos. Using these equations, we examine the dependence of behaviors on coupling constants by rather simple manner. We simplify the constants as $C_{ijk} = C_i$ for all j,k. The evaluated values of C_{1jk} and C_{2jk} by Takeuti (1985) equal $2 \sim 4$ and 6, respectively. Thus the value of C_1 is varied from 2 to 6 and that of C_2 is fixed at 6.0. It should be worthwhile to research the tendency of solutions for wide values of C_1 and C_2. The set of angular frequencies is chosen as $\sigma_1^2 = 1.0$ and $\sigma_1^2 = 1.9$.

2. Results

First, we follow the results by Tanaka et al. (1991a) that the Lyapunov exponents are preliminary introduced in order to distinguish the solutions of Eq. (1). The computed results is shown in Table I. In these case, all λ_1 is nearly zero and λ_2 is zero and negative, which mean that quasi-periodicity or phase-locking occur. We also examined time developments and projected orbits on a phase plane for the same parameter and confirmed the state of

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solutions in the Table. We should note that the other phase-locking states appear as the values of coupling constants and coefficients of damping terms are varied. The frequency ratios cover from 2/3 to 1/1 as Farey's series, while indistinguishable (quasi-periodic, chaotic and convergent) and divergence cases are also observed. These have been observed as the ratio of angular frequency is changed (Tanaka et al. 1990).

TABLE I

The samples of the Lyapunov exponents and the period ratios. The coupling constants and parameters of nonlinear damping term are also given. $\sigma_1^2 = 1.0$, $\sigma_2^2 = 1.9$, $\varepsilon_1 = 0.1$. Q-P means the quasi-periodicity.

Parameters					Lyapunov exponents				
C_1	C_2	ε_2	α_1^2	α_2^2	λ_1	λ_2	λ_3	λ_4	P_2/P_1
6.0	6.0	0.3	1600	1600	0.000	0.000	−0.148	−0.845	Q-P
6.0	6.0	0.3	1600	800	0.001	−0.016	−0.141	−0.872	7/10
6.0	6.0	0.4	1600	800	0.000	−0.021	−0.149	−1.195	5/7
6.0	6.0	0.3	1600	2400	0.000	−0.031	−0.113	−0.844	3/4
2.0	6.0	0.1	800	800	0.000	−0.063	−0.064	−0.329	4/5

Next, the behaviors of coupled oscillator are studied by Tanaka et al. (1991 b) on the parameters plane, (C_1, C_2) where we shall see fractal patterns. Numerical integration are carried out by the mixed Euler and Heun method with a personal computers. The first diagram is divided into two regions, divergent or non-divergent. The divergent region (D) is defined by $A = x_1^2 + x_2^2 > 10$. When A becomes greater than 10, the pixel at (C_1, C_2) is colored by the number of iteration. When A keeps to be small after the 500 iteration, the set of (C_1, C_2) is seemed as non-divergent region (ND) and colored black. In the diagram, D region is patterned by complex stripes. When α_1^2 becomes large and α_2^2 small, ND region tends to expand. They also illustrate the structure of ND region. Although phase-locking, quasi-periodicity and chaos are included in ND region, they only distinguish the phase-locking and color according to counted ratios. We seem that these regions are so-called the Arnold's tongues, which appear in the circle map. We can understand that the coupled oscillator model behaves as a simple sine map which is the simplified form of forced oscillator.

3. Concluding Remarks

We have shown the model coupling oscillator models for stellar variability by using simplified sets of constants. The route to chaos in the coupled oscillator are period doubling and quasi-periodicity which depends upon the control parameters.

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