or, the sum to be secured for annuity of £1,

$$\frac{1}{(1-\mathrm{A})\div a-1+v}$$

Finally, if the annuity to the borrower is not to be held as due, but to make the first payment at the end of a year, the investment of the lender for post obit of £1 must then be v, with which, after paying the first premium for assurance, π, he buys the annuity (not one due) of $\frac{v-\pi}{a}$; and the annuity payable to the borrower is therefore

$$\frac{v-\pi}{a}-(\pi+v\pi)=(v-\pi)\left(1+\frac{1}{a}\right)-1,$$

from which resulting formulæ may be easily deduced.

I am, Sir,

Your most obedient Servant,

Aberdeen, 4th May, 1857. H. A. S.

ON MR. ALEXANDER GLEN FINLAISON'S TABLES FOR ALLOWANCES IN SICKNESS.

To the Editor of the Assurance Magazine.

SIR,—Since the publication, by order of the House of Commons, of Mr. Alexander Glen Finlaison's Tables for Allowances in Sickness, I have used them in preference to other data, because, subject to a few criticisms with which I shall presently trouble you, I think them more satisfactory than any we previously possessed. They do not, however, give the money values of such allowances after the age of 70; and as it is found that some Benefit Societies, albeit unwisely, contract for grants extending over the whole of life, I have been led to compute, and I now submit to you, an extension of the Heavy Labour Table (see p. 116) to meet that case. In doing this, and in examining carefully Mr. Finlaison's Reports, I have noticed some peculiarities in his methods of procedure on which I shall offer some comments, in a spirit of great respect for a gentleman who has devoted much labour and ingenuity to the performance of a very useful and difficult task.

The first thing that strikes one, in looking over the Reports, is, that Mr. Finlaison employs one table of mortality in computing his allowances in sickness (given at page 21, 1854), and another (given at page 31, 1854) in computing the values of annuities, pensions, and assurances on death. The former table enormously overrates the probabilities of life; and this appears to me the most satisfactory of several reasons assigned for employing it in conjunction with the tables of average sickness, which probably underrate the liability with which they deal. The result is, a measure of protection in the *single premium* for allowance in sickness; but unfortunately, when that single premium is converted by the same table of mortality into a periodical contribution extending over the whole duration of the benefit, *the protection disappears.* It would have been better, in my judgment, to discard the mortality table altogether. As it is, we shall have, in estimating the position of a Society acting on these premiums, to value the contributions

https://doi.org/10.1017/S2046165800023388 Published online by Cambridge University Press

for sickness by one table, and the contributions for pensions, &c. by another; for if we calculate the former by the table applicable to the latter, the result will be an immediate deficit.

In accounting for the small rate of mortality which is shown in the returns of Friendly Societies, Mr. Finlaison hardly does justice to his subject. Differing from previous writers, who attribute the anomaly to the quality of the lives, he attributes it solely to the shortness of the period over which the observations extend. Surely the former view is right; for a short observation, regarding the question as one only of time, is as likely to err on the side of excess as of defect.

An unfortunate illustration of our author's opinion is taken from the mortality on a railway. "The infinitesimal number of deaths recorded * * * would appear to be incredibly small, if ascribed to the general body. Yet nobody will believe that fewer railway travellers die than are buried out of the same number of other persons who stay at home." Presuming that, in the latter case, the number buried stands for the number who die, I entirely differ from this opinion. The proportion of persons who die while travelling is unquestionably far less than the proportion who die elsewhere, and for the very simple reason, that the former case includes only sudden deaths, while the sick and dying stop at home. It is the strongest possible example of the effect of selection, and proof that the mortality depends on the quality of the lives.

In Friendly Societies there is probably no material selection at entry, for the measures taken to exclude bad lives are usually far from stringent. The selection probably arises from the enormous number of withdrawals. I imagine that if a man falls into bad circumstances, more especially if he falls into bad habits, one of the first things he does is to "drop his club," and that here we shall find the only solution of our difficulty.

The table of mortality from which Mr. Finlaison's annuities, pensions, and assurances on death are calculated is formed at one period of life from the returns of the Friendly Societies of the metropolitan province, at another from the returns of all England and Wales, and at two others from the experience of the Government Male Annuitants. It seems matter of regret that, instead of thus constructing a composite table from data selected simply because they agree with other trustworthy tables, he did not use one of those tables at once. We had already more than enough.

Some trouble may be saved to those who desire to follow Mr. Finlaison's methods of calculation, if I point out that, in deducing from the rate of sickness the value of an allowance in sickness, he introduces an element which I have not found in other writers. Mr. Ansell and Mr. Neison represent the value of a weekly allowance in sickness, extending over one year, by the average number of weeks' sickness multiplied into the value of £1 due six months hence. Mr. Finlaison further multiplies into this product the probability of being alive at the end of six months. At the first glance, this looks like a closer approximation to accuracy; but I think it is really the reverse. The rate of sickness is stated, I imagine, on an average of those who live over the year and those who die in the year; and if the cessation of liability to sickness, by reason of death during the year, is to be taken into account, the intensity of the rate of sickness must be increased, or the contributions will not cover the claims. In other words, Mr. Finlaison's mode of calculation is admissible only where the datum is the number of persons constantly sick—not where it is the average amount of sickness to

https://doi.org/10.1017/S2046165800023388 Published online by Cambridge University Press

be endured during the year, by a number of persons now taken under observation.

For example: suppose the laws to be, that between any two consecutive years of age one man in a hundred will die, and one out of fifty living men will be constantly sick. Then, on the usual theory of equal decrements during the year (which theory our author follows), the aggregate amount of sickness endured by a thousand men will be *

$$990 \times 52 \times \cdot02 = 1029\cdot6 \text{ weeks}$$
$$\text{added to } 10 \times 26 \times \cdot02 = \quad 5\cdot2 \text{ weeks}$$
$$1034\cdot8 \text{ weeks.}$$

And this amount of sickness, divided by 1000, gives 1·0348 weeks, answering to the average sickness undergone by each person, as observed and recorded by Mr. Finlaison.

Now if, working with the proportion constantly sick, we take $52 \times \cdot02 \times \cdot995$ (the probability of living six months), we correctly obtain 1·0348 as the expectation of sickness for one year. But if, working with the average sickness, we multiply 1·0348 by ·995, we obviously commit an error. Still, as the error is not very material, I have followed Mr. Finlaison's plan in computing the table appended to this paper; my object being to extend, rather than to amend, the materials with which he has supplied us.

Another peculiarity in Mr. Finlaison's procedure is, that he reckons the value of an annuity payable monthly, whether temporary or for the whole duration of life, to be the annuity payable yearly $+ \cdot5$; from which it follows, that a deferred annuity is of the same value whether payable yearly or monthly. This is a little departure from accuracy, but really quite near enough; and I am glad to see so good a precedent for dealing boldly with a nicety about which we often give ourselves much trouble to very little purpose. My own practice, shorter still, is to discard altogether, in small Friendly Society matters, the consideration of weekly or monthly payments of premiums or pensions. The disregarded elements go to balance each other, and the ultimate error, where appreciable at all, is on the side of safety. We do not sufficiently bear in mind the distinction in this respect between annuities certain and contingencies depending upon life. In the former, every additional decimal brings us nearer to the truth; in the latter, the decimals are not to be depended upon at all. It seems a lamentable waste of labour, that a table made to measure, like the composite table before us, should be calculated by logarithms to eight places, while the units in the resulting annuities are but approximations to the real values. Five-figure logarithms, which may be read off at sight, are, in my judgment, quite sufficient for our ordinary purposes.

I may now present a comparative statement of the single premium for an allowance of £1 per week in sickness, extending over the whole duration of life, as shown at several ages by various observations, in juxtaposition with values which I have obtained by combining the materials collected by Ansell, Neison, Ratcliff, and Finlaison, and with Dr. Farr's values founded on Mr. Edmunds's hypothesis that the proportion of persons constantly sick is double the proportion of deaths.

* For simplicity, I take the year as 52 weeks. Note, however, that Dr. Farr, more accurately, takes it as 52·18 weeks.

https://doi.org/10.1017/S2046165800023388 Published online by Cambridge University Press

Single Premium for an Allowance of £1 *a week in Sickness for the whole duration of Life.*

	Age 20.	Age 30.	Age 40.	Age 50.	Age 60.	Age 70.
Neison 3 per cent.	54·96	68·73	88·33	114·68	151·61	186·10
Ratcliff 3 per cent.	39·73	49·03	61·12	76·27	95·20	
Finlaison, Heavy Labour, 3¼ per cent.	37·72	43·09	50·62	59·99	72·87	84·11
Combined Sickness (A, N, R, and F) with combined mortality (same), 3 per cent.	47·28	58·04	73·22	93·50	122·29	152·27
Combined Sickness, with Dr. Farr's Mortality, 3 per cent. . .	41·20	50·02	63·00	80·67	104·78	134·10
Mr. Edmunds' Sickness Hypothesis, with Dr. Farr's Mortality, 3 per cent.	37·33	43·61	51·42	60·80	71·80	82·22

The low values resulting at the older ages, from Mr. Finlaison's observations, are accounted for by his rigorous exclusion of all cases of chronic sickness, as explained at page 17 of the 1854 Report. On this account, great caution must be exercised in applying them to Societies which contract for sickness allowances in old age, and which have to pay for chronic disease (although at a reduced rate) as well as for acute disease. The proper advice to be given is, probably, that such allowances be discontinued altogether after the age of 60 or 65, and a pension substituted; but in dealing with existing contracts, we cannot, perhaps, do better than to use Mr. Finlaison's materials, with such margin as, from the experience of the Society under observation, may appear to be necessary.

The differences between the three first lines of the comparative statement are so considerable, that I was led to try the effect of combining the whole data on which they are founded, with the addition of Mr. Ansell's. The result is shown in lines 4 and 5; one of which incorporates the combined sickness with the combined Friendly Societies' mortality, and the other of which incorporates the combined sickness with Dr. Farr's second table of mortality. The difference between these two lines shows the effect which the assumption of a low rate of mortality has in enhancing the value of an allowance in sickness. I had thought of offering you one or both of these tables *in extenso;* but, on consideration, am of opinion that they are not of sufficient interest to call for publication.

The materials which Mr. Finlaison possesses for ascertaining the probability of chronic sickness must be highly valuable: perhaps it is not too much to hope that he may be induced to collate and publish them separately. Insurance Companies will confer another boon on men who, in common with their families, depend on their professional exertions, when, on sufficient data, they can afford the means of providing against permanent sickness—the only ill, legitimately within their province, against which they do not afford protection. The man who has insured his life, or contracted for an endowment or an annuity yet deferred, and who is disqualified by sickness from continuing his premiums and even from maintaining himself, is in sad case, and to this case we can at present apply no remedy.

I have only to add, that the values of allowances in sickness ceasing at age 70, derived from my D and N columns, will differ slightly, at and under

https://doi.org/10.1017/S2046165800023388 Published online by Cambridge University Press

age 62, from those of Mr. Finlaison. An error seems to have crept into his calculations at this point, which runs up the rest of the column.

I am, Sir,
Your obedient Servant,
JOHN A. HIGHAM.

Royal Exchange Assurance,
May 30th, 1857.

Commutation Tables for Allowances in Sickness to Males engaged in Heavy Labour, calculated from the observations of Mr. A. G. Finlaison.

Interest at $3\frac{1}{4}$ per cent.

AGE.	D.	N. For Sickness.	Value of Allowance of £1 a week in Sickness for the whole of Life.	AGE.	D.	N. For Sickness.	Value of Allowance of £1 a week in Sickness for the whole of Life.
15	53967·	1929694·	35·76	57	9097·2	624432·	68·64
16	51983·	1876793·	36·10	58	8606·4	603384·	70·11
17	50055·	1825779·	36·48	59	8131·9	581668·	71·53
18	48181·	1775964·	36·86	60	7676·3	559346·	72·87
19	46363·	1728423·	37·28	61	7238·9	536542·	74·12
20	44598·	1682338·	37·72	62	6820·1	513215·	75·25
21	42890·	1638009·	38·19	63	6415·5	489606·	76·32
22	41241·	1594821·	38·67	64	6022·4	465921·	77·36
23	39651·	1552903·	39·16	65	5639·4	442210·	78·42
24	38124·	1512526·	39·67	66	5264·4	418660·	79·53
25	36656·	1473672·	40·20	67	4895·9	395061·	80·69
26	35244·	1435993·	40·74	68	4536·7	371376·	81·86
27	33885·	1399500·	41·30	69	4187·7	347694·	83·03
28	32576·	1364243·	41·88	70	3849 0	323750·	84·11
29	31314·	1329954·	42·47	71	3524·3	299197·	84·90
30	30097·	1296852·	43·09	72	3214·3	274083·	85·27
31	28925·	1264966·	43·73	73	2918 9	248196·	85·03
32	27795·	1234506·	44·42	74	2639·2	221622·	83 97
33	26706·	1205140·	45·12	75	2373·6	195552·	82 39
34	25657·	1176554·	45 86	76	2121·4	170468·	80·36
35	24646·	1148642·	46 61	77	1882 0	146418·	77·80
36	23672·	1121320·	47·37	78	1656·3	123736·	74·71
37	22728·	1094321·	48·15	79	1441·7	103045·	71·48
38	21814·	1067705·	48·95	80	1241·7	83966·	67·62
39	20929·	1041523·	49·76	81	1056 2	66878·	63·32
40	20071·	1015870·	50 62	82	887·32	52299·	58 94
41	19240·	990318·	51·47	83	733 74	40032·	54·56
42	18437·	964925·	52·34	84	599 20	29739·7	49·63
43	17662·	939982·	53 22	85	479·25	21414 0	44·68
44	16913·	915686·	54·14	86	368·44	14871·7	40·37
45	16191·	891370·	55·05	87	271·01	9938·3	36 67
46	15493·	867443·	55·99	88	190·22	6381·2	33·55
47	14817·	844045·	56 96	89	127·21	3934·1	30·93
48	14164·	820845·	57·95	90	81·43	2326·2	28·57
49	13531·	797734·	58·96	91	49·13	1320·44	26·88
50	12918·	774963·	59·99	92	29·31	716·24	24·44
51	12325·	752468·	61 05	93	16·41	364·24	22 20
52	11751·	730210·	62·14	94	8·58	171·93	20 04
53	11193·	708402·	63·29	95	4·17	73·84	17·70
54	10650·	686976·	64·51	96	1·88	27·35	14·59
55	10120·	665958·	65·80	97	·68	7·73	11 36
56	9602·4	645194·	67·19	98	·16	1·26	7·61

https://doi.org/10.1017/S2046165800023388 Published online by Cambridge University Press