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THE CENTRALISER OF THE INJECTIVE TENSOR PRODUCT

WEND WERNER

The aim of this note is to obtain an intrinsic product formula for the centraliser of the injective tensor product of a couple of Banach spaces, $Z(X\widehat{\otimes}_\epsilon Y)$. More precisely, we are going to prove that

$$Z(X\widehat{\otimes}_\epsilon Y) = C^b(Z_X/\mathfrak{J}_X \times_k Z_Y/\mathfrak{J}_Y).$$

Here, the spaces $Z_X/\mathfrak{J}_X$ and $Z_Y/\mathfrak{J}_Y$ depend only on X and Y, respectively, and $\times_k$ denotes the topological k-product.

A counterexample used to demonstrate that the k-product cannot be avoided serves also as an answer to a question posed by W. Rueß and D. Werner concerning the behaviour of M-ideals on $X\widehat{\otimes}_\epsilon Y$.

1. INTRODUCTION

Let X be a Banach space, B_X its unit ball and denote by ex K the set of extreme points of some subset $K \subseteq X$. Suppose for the moment that X is a real space and put

$$Z(X) := \{T \in L(X) \mid \forall p \in \text{ex } B_{X'}\ \exists a_T(p) \in \mathbb{R} \qquad T'p = a_T(p)p\}.$$

In the operator norm, $Z(X)$ is a commutative C^*-algebra. (For the definition in the complex case see the following section.)

The aim of the present note is to obtain an intrinsic product formula for $Z(X\widehat{\otimes}_\epsilon Y)$, that is, an expression which does not resort to any properties of the injective tensor product as such. More precisely, we are going to show that the equation

$$Z(X\widehat{\otimes}_\epsilon Y) = C^b(Z_X/\mathfrak{J}_X \times_k Z_Y/\mathfrak{J}_Y),$$

holds within the frame of Banach algebras. Here, the spaces $Z_X/\mathfrak{J}_X$ and $Z_Y/\mathfrak{J}_Y$ depend only on X and Y, respectively, and $\times_k$ denotes the topological k-product.

A related formula was obtained in [**24**], where it was shown that

$$Z(X\widehat{\otimes}_\epsilon Y) = [Z(X) \otimes Z(Y)]^-.$$

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Here, the closure has to be taken with respect to the strong operator topology on the space $X\widehat{\otimes}_\varepsilon Y$. For a different approach to this result see [**3**]. (For some similar results in more special situations the reader is referred to [**3, 5, 13, 14, 22, 23**] where, however, sometimes a slightly different notation is used.)

Let us indicate the source of interest in $Z(X)$. First, its subalgebras appear quite naturally whenever X is represented as a space of (all continuous) sections in a Banach bundle, and in fact the whole algebra gives rise to such a representation which in some sense is maximal. (See [**4**]; in light of this property of $Z(X)$, the above equation can be used to obtain a maximal bundle representation of the injective tensor product without any of the restrictions on the involved Banach spaces as in [**3**] — but we won't touch this here.) The interest in Banach bundles in turn is manifold, see for example [**4, 9, 11**]. In [**6**] this concept has recently become a tool in the biholomorphic classification of domains in infinite dimensional Banach spaces. (Note, however, that the pertinent definitions frequently differ.) Second, in the theory of non associative Banach algebras, $Z(A)$ quite often provides a concept of centroid, which seems to be more manageable than the pure algebraic definition. For a more recent application of this sort see [**17**]. For the question of how $Z(X)$ looks like in some more concrete examples, the reader is referred to the following section.

Let us explain how this paper is organised: The following section collects some necessary notation as well as two auxiliary results. To one of them, a theorem of Stone-Weierstraß type, we briefly sketch some further applications. In the third section we state and prove our main theorem. We finally present an example in Section 4 that serves for two purposes: First, it provides a counterexample to a more ambitious conjecture in connection with the main result. On the other hand, it answers a question of W. Rueß and D. Werner posed in [**20**].

2. Notation and useful results

We begin with

DEFINITION 1: The Banach algebra *Mult* X consists of all those operators T for which each $p \in \operatorname{ex} B_{X'}$ is an eigenvector of T' with eigenvalue $a_T(p)$.

Those $T \in$ *Mult* X that possess a natural adjoint in *Mult* X, that is for which there exists $T^* \in$ *Mult* X with $a_{T^*}(p) = \overline{a_T(p)}$ for all $p \in \operatorname{ex} B_{X'}$, are said to belong to the centraliser, denoted by $Z(X)$.

Clearly, when X is a real space, both algebras coincide. Note that both algebras are function algebras and that $Z(X)$ is a CK-space for a suitable compact K. For a more detailed presentation of this topic see [**4**].

Suppose that X is a closed subspace of C_0L, the space of all continuous functions on the locally compact space L vanishing at infinity, and let

$$\begin{aligned}Mult(X, C_0L) &:= \{f \in C^bL \mid fX \subseteq X\},\\ Z(X, C_0L) &:= \{f \in Mult(X, C_0L) \mid \overline{f} \in Mult(X, C_0L)\}.\end{aligned}$$

We further denote by

$$\mathfrak{F}(X, C_0L)$$

the set of equivalence classes which are obtained by

$$l \sim k \iff f(l) = f(k) \qquad \forall f \in Z(X, C_0L).$$

The reader should observe that $Z(X, C_0L)$ is always a closed subalgebra of $Z(X)$. Furthermore, when X is canonically embedded into the space C_0Z_X, where $Z_X := \overline{\mathrm{ex}\, B_{X'}}^{w^*} \setminus \{0\}$, then $Z(X) = Z(X, C_0Z_X)$ as well as $\mathfrak{F}(X, C_0Z_X) = \mathfrak{F}_X$. A result similar to the following can be found in [**10**, Theorem 13.2].

Theorem 2. *Let X be a closed subspace of C_0L. Then $f \in C_0L$ belongs to X if and only if*

$$f_{|F} \in X_{|F} \qquad \forall F \in \mathfrak{F}(X, C_0L).$$

The proof of this theorem is nothing but a slight modification of the argument Glicksberg gave in order to prove Bishop's version of the classical Stone-Weierstraß theorem (see for example [**10**]), and in fact, if X is a function algebra then Theorem 2 reduces to Bishop's theorem. (Note that in this case $\mathfrak{F}_X$ is the maximal antisymmetric decomposition of X's Shilov boundary.) We therefore omit it. Instead, let us see what is going on for special spaces:

Corollary 3.

(i) *A C^*-algebra A is commutative if and only if its centroid separates the points in the w^*-closure of the set of pure states of A.*

(ii) *A compact convex set K in a LCTVS is a Bauer simplex if and only if the order bounded operators on $A(K)$ separate the points in $\overline{\mathrm{ex}\, K}$.*

(iii) *Denote by $(Z_X)_\sigma$ the quotient space obtained from identifying points of the form γp with $|\gamma| = 1$. Then X is a C_σ-space if and only if $Z(X)$ separates the points of $(Z_X)_\sigma$.*

Let us briefly sketch the proofs : For (i), one has to use the fact that for C^*-algebras $Z(A)$ coincides with the centroid of A, [**12**]. In the unital case, this is of course a special case of Théorème 11.3.1 in [**7**]. To see why (ii) holds, one has to take into account that an operator T on $A(K)$ is order bounded if and only if $T \in Z(A(K))$, see [**1**, II Section 7], and that the Bauer simplices represent precisely the sets $M_1^+(C)$ for some compact space C [**1**, II Section 4]. The statement of (iii), the proof of which follows readily from Theorem 2 and [**15**, p.218], should be compared to the central results of [**18**] and [**21**], where two other classes of L^1-preduals are classified in a similar way. To see this connection (and for the sake of preparing the counterexample announced in the introduction), we need

THEOREM 4. ([2]) *Denote by* $(\mathrm{ex}\, B_{X'})_\sigma$ *the space obtained from* $\mathrm{ex}\, B_{X'}$ *by identifying points of the form* γp *with* $|\gamma| = 1$, $p \in \mathrm{ex}\, B_{X'}$. *Then the sets of the form*

$$(\mathrm{ex}\, B_{X'} \cap J^\circ)_\sigma,$$

where J *runs through the* M*-ideals of* X, *form the closed sets of a topology called the* structure topology *of* X.

Recall that a subspace J of a Banach space X is called an M-ideal, if and only if for some subspace J^* of X'

$$X' = J^\circ \oplus_1 J^*.$$

The point here is that the functions a_T (introduced in Definition 1) correspond to the bounded structurally continuous functions (see [**4**, Chapter 3]). Now, in [**18**] the L^1-preduals with the property that the elements of $Z(X)$ separate the points of $(\mathrm{ex}\, B_{X'})_\sigma$ have been characterised, whereas in [**21**] it was shown that a Banach space is G-space if and only if $(\mathrm{ex}\, B_{X'})_\sigma$ is Hausdorff.

Let us finally point out that the version of the Stone-Weierstraß theorem which is valid in the context of function modules on some compact space K (see for example [**11**]), can also be obtained using Theorem 2.

The reason we are interested in Theorem 2 at this place is

COROLLARY 5. *Denote by* $\mathfrak{F}_X$ *the set of equivalence classes on* Z_X *defined by*

$$p \sim q \iff \Phi' p = \Phi' q \qquad \forall \Phi \in Z(X).$$

The algebra $Z(X, C_0 L)$ *consists exactly of those* $f \in C^b L$ *which are constant on each* $F \in \mathfrak{F}_X$.

We have to fix some further notation: Let T be a Hausdorff space. The space $k(T)$ is the set T together with the topology in which a set is open if and only if its intersection with the compact subsets of T is (relatively) open. $k(T)$ belongs to the class of k-spaces, which means that its topology is generated by the compact subsets of $k(T)$. In the same vein, the mapping

$$k(f) : k(T_1) \to k(T_2)$$

differs from $f : T_1 \to T_2$ by change of topologies only, and it is continuous whenever f is. We will also follow the convention to write

$$T_1 \times_k T_2 := k(T_1 \times T_2).$$

The most exhaustive reference on this topic known to the author is [8]. The following lemma contains the topological ingredients of the proof of Theorem 7. Since we couldn't locate one in the literature, we include a proof.

LEMMA 6. *Suppose that $T_{1,2}$ are Hausdorff spaces, that T_1 is locally compact, and that there are given equivalence relations $R_{1,2}$ on $T_{1,2}$ with appertaining quotient maps $\pi_{1,2}$ such that T_i/R_i is Hausdorff and the space $(T_1 \times T_2)/(R_1 \times R_2)$ is a k-space. Then*

$$(T_1 \times T_2)/(R_1 \times R_2) \cong T_1/R_1 \times_k T_2/R_2,$$

where the homeomorphism is given by $k(H)$ with

$$H([(t_1, t_2)]) = ([t_1], [t_2]).$$

Here, $[\cdots]$ refers to the formation of equivalence classes in either of the equivalence relations.

PROOF: By definition of the respective topologies, H and hence $k(H)$ are continuous. Thus we are left with showing that the map $k(H)^{-1} = k\big(H^{-1}\big)$ is continuous, which is the same as showing that H^{-1} is continuous when restricted to compact subsets K of $T_1/R_1 \times T_2/R_2$. By assumption on T_i/R_i, we may think of K as having the form $K = K_1 \times K_2$ with K_i compact in T_i/R_i. Denote by π_{12} the quotient mapping that belongs to the relation $R_1 \times R_2$ on $T_1 \times T_2$. By [8, 3.3.28],

$$\psi := \pi_1 \times \pi_2|_{\pi_1^{-1}(K_1) \times \pi_2^{-1}(K_2)}$$

is a quotient map and so the continuity of $H^{-1}|_K$ follows from the fact that

$$H^{-1}|_K \psi = \pi_{12}|_{\pi_1^{-1}(K_1) \times \pi_2^{-1}(K_2)}$$

is continuous. ∎

3. Main theorem and proof

THEOREM 7. *For Banach spaces X and Y we have*

$$Z\big(X \widehat{\otimes}_\varepsilon Y\big) \cong C^b(Z_X/\mathfrak{Z}_X \times_k Z_Y/\mathfrak{Z}_Y),$$

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where the (algebraic) isomorphism between these spaces can be chosen so that the operator $\sum T_i \otimes S_i$, *which is in* $Z(X\widehat{\otimes}_\varepsilon Y)$, *may be identified with the map* $\sum a_{T_i} \otimes a_{S_i}$.

Note that the quotient spaces involved need not be completely regular. Therefore the Gelfand space of $Z(X\widehat{\otimes}_\varepsilon Y)$ has to be written

$$\beta\varrho(Z_X/\mathfrak{F}_X \times_k Z_Y/\mathfrak{F}_Y),$$

where ϱT denotes the complete regularisation of T, which in our case is nothing but the weak C^bT topology of T.

PROOF: In the following we shall make use of the fact that $Z_{X\widehat{\otimes}_\varepsilon Y} = Z_X \otimes Z_Y$, which follows from results in [19] and [16]. Our proof consists mainly in showing that

$$\mathfrak{F}_{X\widehat{\otimes}_\varepsilon Y} = \mathfrak{F}_X \otimes \mathfrak{F}_Y.$$

To show this, observe first that for $p \in Z_X$ and $f \in X\widehat{\otimes}_\varepsilon Y$

$$f_p(t) := f(p \otimes t) \qquad t \in Z_Y$$

belongs to Y. Analogously, f^q belongs to X for each $q \in Z_Y$. Representing $Z(X\widehat{\otimes}_\varepsilon Y)$ as a space of bounded continuous functions on $Z_{X\widehat{\otimes}_\varepsilon Y}$ we may define Φ_p with $p \in Z_X$ as above. We have for $e \in X$ with $p(e) = 1$

$$\Phi_p x = \Phi(p, \cdot)p(e)y(\cdot) = [\Phi(e \otimes y)]_p$$

and so, by the Bishop-Phelps Theorem, $\Phi_p \in Mult\,Y$. Since $\overline{\Phi_p} = (\overline{\Phi})_p$ we also have $\Phi_p \in Z(Y)$. In the same way, $\Phi^q \in Z(X)$ for all $q \in Y$. Now let $\xi_{1,2} \otimes \eta_{1,2} \in F \otimes G \in \mathfrak{F}_X \otimes \mathfrak{F}_Y$. Then

$$\Phi(\xi_1 \otimes \eta_1) = \Phi_{\xi_1}(\eta_1) = \Phi_{\xi_1}(\eta_2) = \Phi_{\eta_2}(\xi_1) = \Phi_{\eta_2}(\xi_2) = \Phi(\xi_2 \otimes \eta_2)$$

and thus, each $\Phi \in Z(X\widehat{\otimes}_\varepsilon Y)$ is constant on $F \otimes G$. On the other hand, by definition of $\mathfrak{F}_X$ and $\mathfrak{F}_Y$, two different sets $F_1 \otimes G_1$ and $F_2 \otimes G_2$ in $\mathfrak{F}_X \otimes \mathfrak{F}_Y$ are separated by elements $\Xi \otimes \Psi \in Z(X) \otimes Z(Y) \subseteq Z(X\widehat{\otimes}_\varepsilon Y)$, which settles our claim.

To finish the proof, let τ and π_X denote the quotient maps from $Z_X \times Z_Y$ to $Z_X \otimes Z_Y$ and from Z_X to $Z_X/\mathfrak{F}_X$, respectively. Clearly, the quotient topologies on $Z_{X\widehat{\otimes}_\varepsilon Y}/\mathfrak{F}_{X\widehat{\otimes}_\varepsilon Y}$ induced by $\pi_{X\widehat{\otimes}_\varepsilon Y}$ and $\pi_{X\widehat{\otimes}_\varepsilon Y} \circ \tau$ coincide, and because

$$\pi_{X\widehat{\otimes}_\varepsilon Y} \circ \tau = \pi_X \times \pi_Y,$$

we may use Lemma 6 to obtain (note that the class of k-spaces is stable under the formation of quotient mappings)

$$Z_{X\widehat{\otimes}_\varepsilon Y}/\mathfrak{F}_{X\widehat{\otimes}_\varepsilon Y} \cong (Z_X \times Z_Y)/(\mathfrak{F}_X \times \mathfrak{F}_Y) \cong Z_X/\mathfrak{F}_X \times_k Z_Y/\mathfrak{F}_Y.$$

By Corollary 5 we are done. ∎

The following corollary is essentially known (combine Example 5 on page 155 of [4] with Theorem 4.5 of [3]).

Corollary 8. *Suppose that X and Y are dual spaces. Then*

$$Z(X\widehat{\otimes}_\varepsilon Y) = Z(X)\widehat{\otimes}_\varepsilon Z(Y).$$

Proof: To keep this proof within reasonable limits, we adopt the notation of [**4**, Chapter 4]. It is not very difficult to see that a maximal function module representation of a Banach space X can be obtained by putting $K_X^* := Z_X/\mathfrak{F}_X$, $K_X := \beta K_X^*$, choosing the fibre above $F \in \mathfrak{F}_X$ to be $X_{|F}$ (this is in fact a Banach space) and to be $\{0\}$ elsewhere, and, finally, letting $\|x(F)\| = \|x_{|F}\|$. Theorem 5.13 of [4] then shows that in each dual space X there is an element $e \in X$ such that

$$\|e_{|F}\| = 1 \qquad \forall F \in \mathfrak{F}_X.$$

But $\{F \in Z_X/\mathfrak{F}_X \mid \|x_{|F}\| \geqslant \alpha\}$ is compact for all $x \in X$ and for each $\alpha > 0$ and hence, $Z_X/\mathfrak{F}_X$ is compact. The conclusion follows from this. ∎

Observe that in the above proof we have essentially profited from the compactness of the space $Z_X/\mathfrak{F}_X$. With a similar reasoning, the above proof transfers to the case of $A(K)$-spaces and unital C^*-algebras.

4. An example

Let us first observe that in general the statement of Corollary 4.2 is wrong : Whenever $L_{1,2}$ are locally compact spaces, then

$$Z(C_0L_1\widehat{\otimes}_\varepsilon C_0L_2) \cong C\beta(L_1 \times L_2)$$

whereas

$$Z(C_0L_1)\widehat{\otimes}_\varepsilon Z(C_0L_2) \cong C(\beta L_1 \times \beta L_2).$$

However, these two spaces are known to be different in general [**8**, 3.12.21].

The following example shows that one cannot dispose of the index k in the statement of Theorem 7: Let $X = \{f \in C_0\mathbb{R} \mid nf(n) = f(1)\ \forall n \in \mathbb{N}\}$. We have $Z_X = \{\pm\delta_k \mid k \in \mathbb{R}\}$ and so $Z(X) = \{f \in C^b\mathbb{R} \mid f_{|\mathbb{N}} = \mathit{constant}\}$. It is also straightforward to check that $X\widehat{\otimes}_\varepsilon X = \{f \in C_0\mathbb{R}^2 \mid mn\,f(m,n) = f(1,1)\}$ as well as $Z(X\widehat{\otimes}_\varepsilon X) = C^b\mathbb{R}^2/\mathbb{N}^2$. We will show that $C^b\mathbb{R}^2/\mathbb{N}^2 \neq C^b(\mathbb{R}/\mathbb{N})^2$. To this end, denote for $m,n \in \mathbb{N}$ by $D_{m,n}$ the (open) disk with radius $(m+n)^{-1}$ centered at (m,n). Let f be any function $f \in C^b\mathbb{R}^2$ that vanishes on $\mathbb{R}^2 \setminus \bigcup_{m,n=1}^{\infty} D_{m,n}$ and attains the value 1 on $\mathbb{N}^2$. Since a neighbourhood of $\mathbb{N}$ always contains a set of the form $\sum_{\mu\in\mathbb{N}}]a_\mu, b_\mu[$ with $\mu \in]a_\mu, b_\mu[$, f cannot be continuous when it is considered as a function on $(\mathbb{R}/\mathbb{N})^2$.

Pursuing the above example a little further, we arrive at

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Proposition 9. *In general, the structure topology on* $\left(\text{ex } B_{X\widehat{\otimes}_\epsilon Y}\right)_\sigma$ *is not the product of the structure topologies of* $(\text{ex } B_X)_\sigma$ *and* $(\text{ex } B_Y)_\sigma$.

This gives an answer to a question posed in [**20**]. Note that, as an equation of sets, we always have

$$\left(\text{ex } B_{X\widehat{\otimes}_\epsilon Y}\right)_\sigma = (\text{ex } B_X)_\sigma \times (\text{ex } B_Y)_\sigma.$$

Proof: In fact, since the space X constructed above is a G-space, one can use [**21**, Theorem 97] and the fact that

$$\text{ex } B_{X'} = \{\pm\delta_k \mid k \in \mathbb{R} \setminus \{2,3,\dots\}\}$$

to see that $(\text{ex } B_{X'})_\sigma$ provided with the structure topology is homeomorphic to $\mathbb{R}/\mathbb{N}$. But then

$$(\text{ex } B_{X'})_\sigma \times (\text{ex } B_{X'})_\sigma \not\cong \left(\text{ex } B_{(X\widehat{\otimes}_\epsilon X)'}\right)_\sigma,$$

since the latter space provided with the structure topology is homeomorphic with $\mathbb{R}^2/\mathbb{N}^2$. ∎

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Department of Mathematics
Universität – Gesamthochschule Paderborn
Fachbereich 17 Postfach 1621
Paderborn
Federal Republic of Germany