CORRESPONDENCE

[The first three letters follow Mr. Walton's comments on my review (May, 1968) of texts of the Scottish Mathematics Group. A letter also came from Mr. T. N. Duffy, but, for reasons of space, I have selected the first three written; Mr. Duffy's careful analysis supports them. E.A.M.]

To the Editor, *The Mathematical Gazette*

DEAR SIR,

Perhaps it is because of the intrinsic difficulty of devising a school geometry course that discussion about it is apt to generate more heat than light. At the very least we are faced with the task of introducing the richness and complexity of Euclidean space to pupils who are immature. But also we want to do it in a mathematical way, relating one result to another by reasoning of a deductive kind: we want to avoid if we possibly can the method of stating results *ex cathedra* which is so harmful to the growth of mathematical ability. And we would like a development which leaves plenty of scope for exploration and (intellectual) experiment on the part of the pupil.

In *Modern Mathematics for Schools*, which Mr. Walton attacks so warmly, the S.M.G. copes with these difficulties in an original way. (Regrettably I can claim no part of the credit for this.) The outline and philosophy of the approach are given in the notes to the teacher, particularly those facing 1–60, 1–68 (3rd reprint), 3–113, 3–128, and 4–88, and are hard to summarize briefly. The relevant quotation in your May 1966 review catches the spirit well. Essentially, two powerful and intuitively appealing assumptions are made quite early on about the rectangle, which is a shape familiar to every child in the industrialized Western world. Then, through a development abounding in inventive detail, the learner is encouraged to think out the consequences of these assumptions.

Practical activities with shapes and tracing paper are aids to this thinking-out process, crutches to be taken up or put aside according to the needs of the young learner: they do not constitute "measurement" in any recognizable sense of the word. The usual properties of the various standard figures come quickly to light in a simple and memorable way and at the same time the mental operations of "turning over," "turning round" and "sliding" prepare the way for later study of reflection, rotation and translation.

There seems no need to ask for space to comment on all Mr. Walton's individual points. Are teachers really confused about the number of sides of a rectangle? The discussion in 1–74 to 1–76 is entirely concerned with four-sided figures and is well illustrated with diagrams: the misprints were put right in 1966, and the quote from Book 5 is merely brief revision notes. The thinking about bilateral symmetry of a circle is based on that of the isosceles triangle, see 4–130, 3–130. The kite tiling follows from the basic assumptions and gives a memorable way of thinking about angle sum: other less memorable ways are provided in the same question.

It seems to me that the serious error on Mr. Walton's part is to suppose that the activities with shapes and tracing paper constitute measurement. I hope I have made it clear that this is not so.

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To the Editor, *The Mathematical Gazette*

DEAR SIR,

With reference to the letter by Mr. R. D. Walton in the February 1969 issue of the Gazette I make the following comments on two of the points he raises.

(i) On the question of bilateral symmetry of the circle Mr. Walton makes reference to the following two properties of the circle, which we shall call property 1 and property 2.

property 1: A diameter perpendicular to a chord bisects the chord.
property 2: A diameter is an axis of bilateral symmetry.

On this topic his attack on the Scottish Mathematics Group is based on his assertion "... it is necessary to prove that a diameter perpendicular to a chord bisects the chord in order to establish that the diameter is an axis of bilateral symmetry." If by this he means that the only way to prove property 2 is as a corollary to a proof of property 1 then his assertion is false. It is very easy to prove property 2 independently of property 1 (either in transformation terminology or in Euclidean terminology) and then deduce property 1 as a corollary.

(ii) With reference to tiling with congruent kites Mr. Walton's attack on the Scottish Mathematics Group is based on his assertion "The angle sum of a kite must be previously known to be 360° before we are entitled to produce a diagram of a tiling without gaps". This assertion is false. Those who have worked carefully through the geometry of Books 1 and 2 will realize that the rectangular tiling in Fig. 1 can be used to justify tiling (without gaps) with congruent kites independently of the fact that the sum of the angles of a kite is 360°.

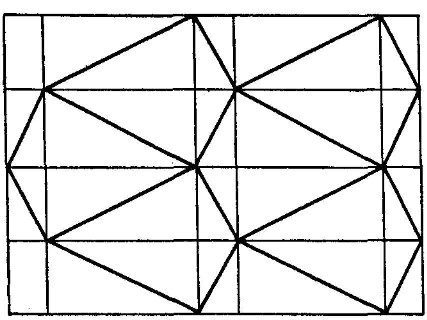

FIG. 1

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There are clear-cut answers to all the other criticisms made by Mr. Walton, but space does not permit me to develop them.

Yours sincerely,
W. T. Blackburn

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Dundee.

To the Editor, *The Mathematical Gazette*

Dear Sir,

I would like to make a few comments about Mr. Walton's letter in the February number. I do not have the full series of Scottish texts readily accessible at the moment, but in any case I do not wish to contradict his statements, which do seem to indicate that the wording in some of these texts is rather loose (but in what elementary text is it not, if we leave aside the wholly axiomatic texts originating in the U.S.A.?).

What I wish to recall is the substance of some discussions I had with the late Mr. A. G. Sillitto about the geometry in these texts. I hope this will serve to indicate that there is a logical background behind it, even though it has not always been well expressed. Geoff Sillitto regarded the "four-way fit tile" as basic, but this was intended to be a graphic way of presenting two axioms: the first, the tiling of a plane by parallelograms, which is equivalent to assuming that the plane is a two-dimensional vector space; and the second, the existence of a parallelogram invariant under the operations of the Klein group, each one of which (apart from the identity) interchanges two of the vertices of the parallelogram and at the same time interchanges the other two. It is the difficulty of making the group structure accessible to 12-year-olds, coupled with the failure to specify precisely the operations of the four-way fit, which has led to the difficulties that Mr. Walton points out. One suspects that had Geoff Sillitto lived, he would have seen to it.

The tiling with kites was derived by A. G. S. directly from the tiling with two kinds of rectangles by dividing them into congruent halves; it thus rests squarely on the axioms, and it is quite legitimate to derive the angle-sum from it and not vice-versa.

As to the circle, if we assume, as we are intended to do, that there is an isometry of the plane (reflection) which keeps invariant the points of any given line, then we may define the circle as the set of points consisting of P and all its images in lines through a given point O. This establishes both the equality of all radii and the total bilateral symmetry, from which the theorem about the bisecting diameter of a chord being perpendicular to it can be logically deduced.

I share Mr. Walton's regret about the definition of angle; I would distinguish sharply between *angle* and *corner*, and use the former to mean strictly a measure of rotation, which can itself be defined in terms of the basic axiom about reflection. For myself, if I were to try to set up a logical geometry along these lines, I would prefer to base it explicitly on such an axiom in the form "Given a point P and a vector **a**, there is a mapping of the plane onto itself which preserves P and **a** and reverses another vector **b**; we then say that **b** is perpendicular to **a**". It will be

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found that we need another axiom to say that if **b** is perpendicular to **a**, then **a** is perpendicular to **b**, and one or two other more subtle assumptions will appear as necessary before we can proceed to angles and angle-sums.

It is a tragedy that Geoff Sillitto did not live to work out in detail these ideas which were taking shape in his mind in his last year, and to develop more systematically the geometry which Scotland is now being asked to learn. But I hope I have shown that there was logic in the conception, even if the realization of it has been faulty.

Yours sincerely,
H. Martyn Cundy

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To the Editor, *The Mathematical Gazette*

Dear Sir,

To supplement the proposals made by A. K. Austin in his article "Finite and Infinite Sets" (*Mathematical Gazette*, February, 1968) it should be possible for the sixth form pupil to appreciate that to take the existence of the infinite set as an axiom is parallel to taking the Law of Uniformity as an axiom in Inductive Logic. The ground of logical induction is the relation of Causality. This is a complex relation involving two principles:

Causality—every event has a cause.

Uniformity—under like conditions the same cause always has the same effect. The second is necessary because generalization is impossible without it and it is the nub of the problem presented in the article.

Some would disagree with Mr. Austin's final paragraph:

"This method should help to sustain the pupil's view that mathematics is deductive and not undermine it by suggesting that in some parts of mathematics, deduction is replaced by leaps into the unknown."

We all can argue that mathematics *system* is deductive but its axioms, it must be allowed, have from time to time been "leaps into the unknown". It could also be argued that with the results of the infinite set theory advocated in the article each time they are used is a "leap into the unknown".

Yours faithfully,
P. Quigley

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OBITUARY

CLEMENT VAVASOUR DURELL

"Durell" has been almost a technical term among mathematicians for most of the first half of this century. Born at Fulbourn in Cambridgeshire in 1882, he was educated at Felsted and, as a scholar, at Clare College, Cambridge, being seventh Wrangler in 1903. He began teaching at Gresham's School, Holt, in 1904, and moved almost at once, in 1905,