4TH ASTIN COLLOQUIUM

SUBJECTS FOR DISCUSSION

1. *Theory of extreme values and application to insurance problems*

The statistical theory of extreme values is well known from its use in the development of tests of significance based upon the range of observed variates. Other applications have been devised, particularly in problems involving natural phenomena such as floods, droughts etc., but it would appear that there are a number of problems arising from insurance, particularly non-life, to which the theory may be usefully applied. Two publications by E. J. Gumbel namely, "Statistics of Extremes", Columbia University Press 1958, and "Statistical Theory of extreme values and some practical applications", U.S. Dept. of Commerce, National Bureau of Standards, Applied Mathematics, Series No. 33, provide a wide theoretical approach, a number of practical applications and an extensive bibliography, and the following brief note outlines one possible approach. It is hoped that by providing these references the value of the discussions at the 4th Colloquium will be materially increased.

It has been shown that the asymptotic distribution of extreme values falls into one of 3 classes depending on the form of the underlying distribution. Where this is of exponential type, i.e. unlimited to the right and convergent at least as rapidly as the exponential function, the distribution function of the extreme value has the form $\Phi = exp\{-e^{-y}\}$ This case, referred to generally as the first asymptotic, has been extensively treated by Gumbel and it would appear to be appropriate for a very wide range of practical insurance distributions.

The simplest model to adopt as a starting point is to assume that the largest claims arising from a portfolio in successive equal exposed periods are known and that monetary values and other factors have been unchanged over the period. It is also assumed that the probability of claim has remained constant. The value of $y = \alpha_n (x-u_n)$ where α_n and u_n are parameters easily determined from the series of extreme values can then be found. The differential of Φ then gives the probability that the extreme value has the value between y and $y + dy$ and the expectation of the extreme value follows directly.

If now the premium for an excess of loss reinsurance is required it is necessary to find the expected value of the largest value, next largest value etc. . . . The distributions of the extreme m'th value are known in terms of the observed m'th values but for the simple model considered the distribution of the m'th extreme in terms of the highest value is required. For this purpose it seems reasonable to use the same distribution as for the largest value, i.e. $exp\ \{-e^{-y}\}$ with $y_m = y_1 + \log m$ so that the required expectation becomes

$$\phi\ (y) + \frac{2^2}{1!}\{\phi\ (y) + log\ 2\}^2 + \ldots .$$

which reduces to e^{-y}

https://doi.org/10.1017/S051503610000180X Published online by Cambridge University Press

The cost of claims in excess of Y then becomes $\int_{y}^{\infty} (y - \Upsilon)\, e^{-y}\, dy = \frac{1}{\alpha} e^{-y_1}$ which means that the expected excess loss premium can be estimated directly from the series of extreme values. This technique avoids the difficulty with those which are based on the whole claim distribution where it becomes necessary to make assumptions about the form of the tail of the distribution if sensible results are to be achieved.

There are a number of obvious improvements to be made; firstly the situation of varying claim frequency requires study; secondly the problem of varying numbers exposed; thirdly the adjustments for changes in the value of money; fourthly to develop the formal derivation for the distribution of m'th values in terms of the largest value; fifthly the efficiency of the method compared with others.

Finally the question of other applications should be considered. For example it is well known that the numerical results of some problems of risk theory are almost entirely dependent on the behaviour of the tail of the distribution of claims. This class of problem would appear to be appropriate for investigation by means of extreme value theory.

2. *Actuarial Control of Non-Life Accounts*

A high proportion of ASTIN activity has been devoted to the theoretical problems arising from the special distributions arising from claims and it is felt that it would be a useful contribution if our discussions were directed to some of the practical implications of the theoretical work that has been done. The subject is very wide and would appear to lend itself to being considered under three main headings, i.e.

(a) Problems arising from claims.
(b) Problems arising from estimates of reserves for unexpired risks.
(c) Contingency and other reserves.

It is proposed therefore to devote separate sessions to each of the above topics with a final session devoted to general questions which may embrace all three topics.

The following notes are intended as a guide to some aspects of the topics which would appear to be worth detailed consideration.

(A) *Claims*

The substantial growth in liability claims, with a long settlement time raises some interesting parallels with the valuation of life business and it is pertinent to enquire whether more can be done in the direction of what may be termed collective estimation. Not only would control of the business from a management aspect be improved, but the considerable labour now expended in repeated valuation of long duration claims might be reduced. To be successful it is necessary to devise methods which enable major factors to be isolated or approximated and the following outline is suggested as a possible model for development.

In general, the total of the claims arising in a given period of time, e.g. an accounting year, is $n \times \bar{C}$, where n is the number of claims and $\bar{C}$ the average value at which the claims will be settled. The number of claims, provided proper attention is paid to definition and provided the portfolio is

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divided to look after differential frequencies in classes of business, the proportions of which are varying, will normally be a variable susceptible to statistical control via the claim frequency, i.e. n/E where E is a suitable measure of the exposure. The average future settlement $\overline{C}$ will, in general, be a slowly varying function since it is essentially the mean value of the distribution of claims by amounts. Provided proper methods of estimation of $\overline{C}$ are available, it is quite possible that a "collective" estimate of $\overline{C}$ will be less liable to error than estimates of claims based on the summation of estimates of individual claims.

Thus it might well prove satisfactory in practice to develop a system of accounting in which factors are developed for (a) outstanding claims K^0 and (b) intimated claims K^I. Then if O_0 are the number of outstanding claims at the beginning of the year, I the number of intimated claims and S the actual amount of claims settled, then the "expected outstanding" at the year end would be found by

$$^{E}O_1 = O_0 \times K^0_0 + I \times K^I_1 - S$$

This could be compared with $O_1 \times K^0_1$ i.e. the outstanding claims at the year end multiplied by their expected settlement figure and the difference would be the profit or loss on the year.

The central problem is to estimate the K factors. To do this it appears necessary to allow for the following features:—

(a) growth of the business
(b) changes in the value of money
(c) changes in levels of awards
(d) the correlation between amount of claim and time to settle.

A model which appears to be a promising basis is to assume that there is an underlying notional distribution of "intensity of claim" which is such that the intensity distribution of claims arising in a period is invariant.

Thus if $p(s)ds$ is the proportion of claims of intensity between s and $s + ds$, $f(s)$ is the time taken for claims of intensity s to be settled, $\phi(t)$ is an index measuring factors (b) and (c) at time t and $C_t dt$ the claims arising at time t, then, if $S_t dt$ are the claims settled at time t we have

$$S_t dt = \int_{s=0}^{\infty} C_{\{t-f(s)\}}\, p(s)\, ds\, dt$$

and if $^{A}S_t\, dt$ is the amount of claims settled at time

$$^{A}S_t\, dt = \phi(t) \int_{s=0}^{\infty} C_{\{t-f(s)\}}\, sp(s)\, ds\, dt$$

Similar expressions can be developed for outstanding claims, with or without allowance for future changes in the value of money and by the use of mean value theorems or by replacing the integral by a sum of the form $\Sigma H_t C_t$ approximate values found for the parameters K^0 and K^I in terms of observed functions. The essential functions required are:—

(1) No. of intimated claims C_t
(2) No. and amounts of settled claims S_t, $^{A}S_t$
(3) An approximation to $p(s)$

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(4) An approximation to $f(s)$
(5) An approximation to $\phi(t)$

The techniques are, of course, appropriate for analysis of date for rating purposes.

(B) *Unexpired risks reserve*

For many purposes the reserve for unexpired risks is taken as a pro rata portion of the full premium but for a modern portfolio which might consist of policies with high deductibles, excess or stop loss covers, deferred exposures such as professional indemnities etc., this simple method may not be a sufficiently accurate method of assessment. The pro rata method is equivalent to a retrospective method in that the reserve made ignores the current experience; there is a prospective method in which an estimate is made of the claims to emerge from the current risks. A critical survey of the various possibilities, also bringing into account the factors other than claims, e.g. expenses, profits etc., would appear to be of value.

(C) *Contingency and other reserves*

Estimates of claims are essentially expected values and actual results will show a statistical and other fluctuation round the expected values. Statistical fluctuations can be approximately estimated by the use of the theory of risk and it is possible that short term oscillations can be dealt with similarly. However longer term trends and catastrophes fall into a different category and considerable scope exists for the further development of the various results so far obtained. The possible use of extreme value statistics in this field is also worth studying, particularly when dealing with catastrophe covers. It is important that the problems be treated as real management problems and not solely as theoretical investigations.

A closely allied problem relates of course to the question of valuations for solvency and discussions of this subject would be a natural corollary of the study of contingency reserves.

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