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ABSTRACT

The dust production rates of comets on various orbits with perihelia between 0.1 and 0.5 a.u. and semimajor axes between 5 and 40 a.u. are calculated. A new model is used taking into account the screening of solar radiation by the dust and the multiple scattering of the photons. Screening and multiple scattering effects tend to compensate each other over a range of moderate optical thicknesses.

A new model has been developed to investigate the feedback of the light scattering by the dust coma on the production rate of cometary gas and dust (Hellmich, 1979). It takes full account of the radiative transfer of the incident solar light. A terse description of the model is given and first results of sample calculations of the total dust and gas production rates of comets are presented.

Only the innermost region (out to a few hundred nuclear radii,  $R_k$ ) of the dust coma is important for the screening of the solar radiation and the production of higher order scattering. The hydrodynamic interaction of the gas dragging the dust particles from the cometary surface takes place over the first few tens of  $R_k$  (Probstein, 1969). The dust density increases strongly just above the nuclear surface. The solution of Probstein for initially subsonic outflow of the gas-dust mixture was generalized to accommodate a particle size distribution.

In order to stay with the one-dimensional hydrodynamic solution for the dust flow the simplification of an isothermal nuclear surface had to be used. This results in a radially symmetric density distribution of the dust. The influence of radiation pressure on the grains can be neglected out to distances of interest for the radiative transfer. The absorption and scattering of the gas coma is neglected. Mie theory is applied to calculate the scattering matrices of the dust grains.

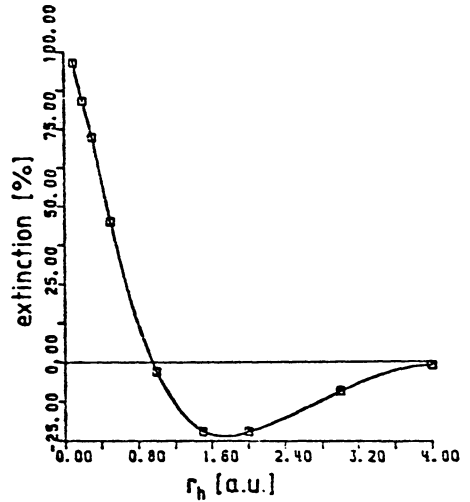
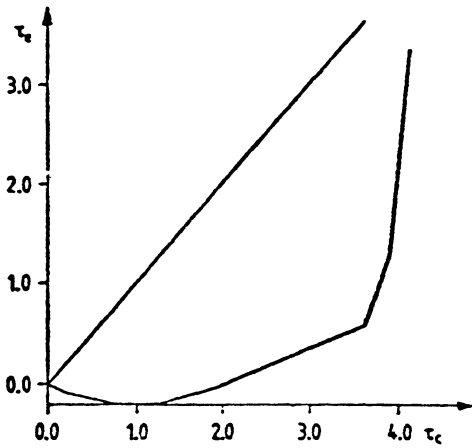


Fig. 1. The effective optical thickness of the dust coma,  $\tau_e$ , is displayed versus the optical thickness of the dust column,  $\tau_c$ , between nucleus and sun. The straight line indicates the linear relationship  $\tau_e = \tau_c$ .

Fig. 2. The effective extinction of the dust coma of the "standard" comet with varying heliocentric distance is shown.

An iterative scheme is used to calculate the gas and dust production rate of a comet at a given heliocentric distance,  $r_h$ :

- 1) The energy balance on the surface is calculated using an estimated extinction of the dust coma. This yields the surface temperature and the resulting gas and dust production.
- 2) The density distribution of the dust grains is calculated using generalized Probstein hydrodynamics.
- 3) The extinction of this dust coma is determined taking multiple scattering into account. This yields an altered energy flux,  $F_N$ , onto the nucleus

$$F_N = F_{\odot} \cdot e^{-\tau_e}$$

$F_{\odot}$  is the unattenuated solar radiation flux at the position of the comet,  $\tau_e$  is the effective optical thickness of the dust coma. The extinction in percent is given by  $100 \cdot (1 - e^{-\tau_e})$ . The newly determined flux,  $F_N$ , is used for a new energy balance on the surface in step 1). This iteration converges rather fast. The details of the radiative transfer calculations are described elsewhere (Hellmich, 1979).

The model calculations confirm that the effects of multiple scattering reduce the screening of the direct sunlight by the dust. They can even overcompensate the extinction and result in an increase of the

energy reaching the surface (see Fig. 1). Here, the effective optical depth is shown as function of the optical depth of the dust column between the nucleus and the sun,  $\tau_c$ .  $\tau_e$  is negative for moderate  $\tau_c$ . Only if  $\tau_c > 4$  the enhancement by multiple scattering becomes unimportant and  $\tau_e \sim \tau_c$ . For moderate  $\tau_c$  a simple-minded approximation of the screening effects using single scattering leads to an underestimation of the production rate by more than an order of magnitude. For these  $\tau_c$  the neglect of any screening leads to more realistic results.

To study the influence of the parameters of the problem we have used a "standard" comet with a single size of olivine grains of a radius of  $1\mu$ , an albedo of the nucleus  $A = 0$ ,  $R_k = 3$  km, and a gas to dust production ratio by mass of one. Such a model comet has an increased gas and dust production between  $r_h = 1$  to 3.5 a.u. (see Fig. 2). The maximum enhancement is 25%. For small heliocentric distances  $r_h < 0.5$  a.u. the production is suppressed, as expected.

From these results it is obvious that the influence of the dust screening on the total gas and dust production of a comet on its orbit around the sun is not as strong as could be expected from a simple estimate using the single scattering approach. We have calculated the total production of the standard comet for different orbits with perihelia between 0.1 and 0.5 a.u. and semimajor axes from 5 to 40 a.u. (Table I). Contributions from heliocentric distances  $r_h > 4$  were neglected. The results with (MS) and without (NO) multiple scattering are compared. Increasing eccentricity of the orbit leads to a minor decrease of production in both cases. The production increases with decreasing perihelion by a factor of 3 if the dust screening is not considered. If

Table I

semimajor axis [a.u.]	p = 0.1 a.u.		p = 0.3 a.u.		p = 0.5 a.u.	
	MS $m_d$ [g]	NO $m_d$ [g]	MS $m_d$ [g]	NO $m_d$ [g]	MS $m_d$ [g]	NO $m_d$ [g]
5.0	$1.2 \cdot 10^{14}$	$4.5 \cdot 10^{14}$	$1.3 \cdot 10^{14}$	$2.2 \cdot 10^{14}$	$1.2 \cdot 10^{14}$	$1.5 \cdot 10^{14}$
10.0	$1.2 \cdot 10^{14}$	$4.5 \cdot 10^{14}$	$1.2 \cdot 10^{14}$	$2.2 \cdot 10^{14}$	$1.2 \cdot 10^{14}$	$1.4 \cdot 10^{14}$
20.0	$1.1 \cdot 10^{14}$	$4.5 \cdot 10^{14}$	$1.2 \cdot 10^{14}$	$2.1 \cdot 10^{14}$	$1.1 \cdot 10^{14}$	$1.4 \cdot 10^{14}$
40.0	$1.1 \cdot 10^{14}$	$4.5 \cdot 10^{14}$	$1.2 \cdot 10^{14}$	$2.1 \cdot 10^{14}$	$1.1 \cdot 10^{14}$	$1.4 \cdot 10^{14}$

Table II

semimajor axis [a.u.]	Sekanina-Miller albedo 0.8 p = 0.3 a.u.	
	MS $m_d$ [g]	NO $m_d$ [g]
5.0	$5.3 \cdot 10^{13}$	$4.6 \cdot 10^{13}$
10.0	$5.2 \cdot 10^{13}$	$4.5 \cdot 10^{13}$
20.0	$5.1 \cdot 10^{13}$	$4.4 \cdot 10^{13}$
40.0	$5.1 \cdot 10^{13}$	$4.4 \cdot 10^{13}$

it is, the production for a comet passing the sun with  $p = 0.5$  is the same as for one with  $p = 0.1$ . A smaller perihelion (between 0.5 and 0.1 a.u.) does not increase the total production of gas and dust during one revolution of a comet if the screening by dust is taken into account. The production of a comet with  $p = 0.5$  is slightly decreased by screening if compared to the no scattering case.

How effective the screening and therefore the multiple scattering

influence is, depends strongly on the dust particles involved. The scattering efficiency per mass of the grains increases with smaller size. We have also made some calculations using a Sekanina and Miller (1973) particle size distribution (Table II). The distribution function is cut off at  $1\mu$  particles. For this elaborate calculation an albedo  $A = 0.8$  was chosen. The larger nuclear albedo decreases the production by a factor of 5. Since the dust mass is distributed over a rather large range of grain sizes we find an increase in production for orbits with  $p = 0.3$  if compared to the no scattering case. The optical thickness of the dust column is only small or moderate so that we have some enhancement of the production rate over the whole orbit of the comet.

Currently we are using more dispersed parameter values for our calculations to investigate how important the screening will be if a stronger contribution of submicron particles should be present in the cometary dust. Qualitatively it is clear that a larger portion of submicron particles would choke the gas and dust production even at heliocentric distances considerably larger than 0.5 a.u. The same holds true if the dust to gas ratio were increased over one and/or the size of the nucleus were bigger, or if with the same  $\dot{m}_d/\dot{m}_g$  ratio the density of the dust particles is near  $1 \text{ g cm}^{-3}$  instead of 4 as taken for obsidian.

#### SUMMARY

For the above mentioned parameters - essentially excluding submicron dust grains - the total dust and gas production of comets is hardly changed by screening in the dust coma for comets with perihelia  $p > 0.5$  a.u. For comets reaching  $p = 0.1$  a.u. the decrease in total dust production is approximately a factor 3. Multiple scattering in the dust coma compensates most of the screening of the direct sunlight by the dust grains for moderate optical thickness and yields even an increase in flux reaching the nucleus for small  $\tau$ .

However, future calculations introducing submicron particles will show that choking of the gas production can be severe and lead to a smaller overall dust production of comets. Calculations based on the single scattering approximation tend to overestimate the choking by more than an order of magnitude. The results are very sensitive to the choice of the dust material.

#### REFERENCES

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