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ON MEROMORPHIC MAPPINGS INTO TAUT COMPLEX ANALYTIC SPACES

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In this paper, we study a certain difference between meromorphic mappings and holomorphic mappings into taut complex analytic spaces. We prove in §2 that, for any complex analytic space X, there exists a unique proper modification $\{M(X) \xrightarrow[\pi]{} X\}$ of X with center $\mathrm{Sg}(X)$ which is minimal with respect to the property that $M(X)$ is normal and, for any T-meromorphic mapping $f: X \to Y$ (see Definition 1.3) into a complex analytic space Y, there exists a unique holomorphic mapping $\tilde{f}: M(X) \to Y$ such that $\tilde{f} = f \circ \pi$ on $M(X)$ except some nowhere dense complex analytic set, where $\mathrm{Sg}(X)$ denotes the set of all singular points of X.

Using the above result, we can prove that the group of all biholomorphic mappings of X onto itself is finite for a compact complex analytic space X which is bimeromorphic with a compact taut complex analytic space (§3). It is proved that, if there exists an open surjective holomorphic mapping of a complex analytic manifold onto a normal complex analytic space X, then $M(X) = X$ (§2), which means that every T-meromorphic mapping of X into a complex analytic space extends holomorphically to X. Moreover, we give some sufficient conditions for a normal complex analytic space X of complex dimension 2 to be $M(X) = X$ (§3).

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In this paper, complex analytic spaces are always assumed to be reduced and connected.

§ 1. Preliminaries

Let X and Y be complex analytic spaces. We denote by $\mathrm{Hol}(X, Y)$

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the set of all holomorphic mappings of X into Y. A sequence $\{f_i\}_{i>0}$ in $\mathrm{Hol}(X, Y)$ is said to be compactly divergent on X if, for any compact sets K in X and L in Y, there exists some i_0 such that $f_i(K) \cap L = \phi$ for all integer $i > i_0$.

DEFINITION 1.1. A complex analytic space X is said to be *taut*, if $\mathrm{Hol}(Y, X)$ is normal for any complex analytic space Y, i.e., any sequence in $\mathrm{Hol}(Y, X)$ contains a subsequence which is either uniformly convergent on every compact set in Y or compactly divergent on Y.

Let X and Y be complex analytic spaces. A meromorphic mapping $f: X \to Y$ is by the definition of Remmert a set-valued function with $f(x) \subset Y$ for any $x \in X$ such that the restriction $f_{|X-A}$ of f to $X - A$ for some nowhere dense complex analytic set A in X is a holomorphic mapping and the graph

$$\Gamma_f = \{(x, y) \in X \times Y : x \in X \quad \text{and} \quad y \in f(x)\}$$

is a complex analytic set in $X \times Y$ which coincides with the closure of $\{(x, f(x)) \in X \times Y : x \in X - A\}$ in $X \times Y$ and, in addition, the canonical projection $p: \Gamma_f \to X$ is proper.

By virtue of Theorem 4.1 in [3], we have easily

PROPOSITION 1.2. *Any meromorphic mapping of a complex analytic manifold M into a taut complex analytic space is holomorphic on M.*

This is also valid for meromorphic mappings into hyperbolic complex analytic spaces in the sense of S. Kobayashi [11] as is shown by the argument in [12].

DEFINITION 1.3. A meromorphic mapping $f: X \to Y$ of a complex analytic space X into a complex analytic space Y is said to be *T-meromorphic* on X if, for any $x \in X$, there exist an open neighborhood U of x in X and a taut local complex analytic set V in Y such that $f(U) \subset V$ and so $f_{|U}: U \to V$ is meromorphic.

PROPOSITION 1.4. *Any T-meromorphic mapping of a complex analytic manifold M into a complex analytic space is holomorphic on M.*

This follows immediately from Proposition 1.2.

REMARK 1.5. By the theorem of resolution of singularities by Hironaka [5] and Kwack's theorem [12], for a nowhere dense complex ana-

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lytic set A of a complex analytic space X, every holomorphic mapping of $X - A$ into a compact taut complex analytic space is considered to be T-meromorphic on X.

Now, let X be a complex analytic space and $F = \{f_i\}_{i \in I}$ be a family of holomorphic mappings f_i of X into the complex analytic space $Y_i (i \in I)$. We define an equivalence relation R_F on X such that

$x \ R_F \ y$ in X if and only if $f_i(x) = f_i(y)$ in Y_i for all $i \in I$. Then, the quotient space X/R_F has the topology with the property that a mapping f of X/R_F into a topological space Y is continuous if and only if $f \circ p: X \to Y$ is continuous, where $p: X \to X/R_F$ is the canonical projection.

In [2], H. Cartan gave the following

PROPOSITION 1.6. (1) *For any relatively compact set K in X, there exists a finite subset J of I such that the equivalence relation R_F on X coincides with the equivalence relation R_{F_J} on K defined by $F_J = \{f_i\}_{i \in J}$.*

(2) *If there exists a proper holomorphic mapping $f_{i_0}: X \to Y_{i_0}$ for some $i_0 \in I$, then X/R_F has a structure of a complex analytic space with the property that the canonical projection $p: X \to X/R_F$ is holomorphic and any mapping f of X/R_F into another complex analytic space Y is holomorphic if and only if $f \circ p: X \to Y$ is holomorphic.*

Now, let T be the class of all taut complex analytic spaces. Then, it satisfies the following conditions (P_1) and (P_2) (c.f. [7], p. 314):

(P_1) The product of two spaces in T is contained in T.

(P_2) If X is a complex analytic space and if, for each point $x \in X$, there exists a proper holomorphic mapping f of X into some Y in T such that x is an isolated point of $f^{-1}f(x)$, then X is contained in T.

For a complex analytic space X, assume that there exists a proper holomorphic mapping of X into a taut complex analytic space. Let R_T be the equivalence relation on X defined by the family of all holomorphic mappings of X into taut complex analytic spaces, which we call the taut proper relation on X. By the theorem of H. Cartan ([2], p. 12), we have

THEOREM 1.7. *In the above situation, the quotient complex analytic space X/R_T satisfies the conditions:*

(1) *The canonical projection $p: X \to X/R_T$ is proper surjective and holomorphic.*

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(2) *For any holomorphic mapping f of X into a taut complex analytic space Y, there exists a holomorphic mapping $\hat{f}: X/R_T \to Y$ such that $f = \hat{f} \circ p$ on X.*

(3) *X/R_T is taut.*

(4) *Each fiber of $p: X \to X/R_T$ is connected.*

(5) *If X is normal, then X/R_T is also normal.*

§ 2. Existence and Properties of $M(X)$

Let X be a complex analytic space. We denote by $M_T(X)$ the set of all T-meromorphic mappings defined on X.

Let $\pi: M \to X$ be a proper holomorphic mapping of a complex analytic space M onto X. A proper modification $\{M \xrightarrow[\pi]{} X\}$ of X with center A means that A and $\pi^{-1}(A)$ are nowhere dense complex analytic sets in X and M respectively and $\pi_{|M-\pi^{-1}(A)}$ is a biholomorphic mapping of $M - \pi^{-1}(A)$ onto $X - A$. A proper modification $\{M \xrightarrow[\pi]{} X\}$ of X with center $\mathrm{Sg}(X)$ is called a resolution of singularities of X if M is a complex analytic manifold.

Let X be a complex analytic space and let U' be an open set in X such that there exists a resolution of singularities $\{M \xrightarrow[\phi]{} U'\}$. Since $f \circ \phi$ has a holomorphic extension $\phi^* f$ to M for each $f: X \to Y_f$ in $M_T(X)$ by Proposition 1.4, we may consider $\phi^* M_T(X) = \{\phi^* f: M \to Y_f\}_{f \in M_T(X)}$ to be a family of holomorphic mappings defined on M. Clearly, $\phi^* M_T(X)$ contains a proper holomorphic mapping $\phi = \phi^* id: M \to U'$ for the identity mapping $id: X \to X$ in $M_T(X)$. Thus, $\phi^* M_T(X)$ defines an equivalence relation $R_M := R_{\phi^* M_T(X)}$ on M as stated in §1. We have the following

LEMMA 2.1. *Let U be a relatively compact open set in U'. Then, there exists a T-meromorphic mapping $f: X \to Y_f$ such that a holomorphic mapping $\phi^* f: M \to Y_f$ defines an equivalence relation on M which coincides with R_M on $\phi^{-1}(U)$.*

Proof. Since $\phi^{-1}(U)$ is relatively compact in M, there exist by Proposition 1.6 finitely many T-meromorphic mappings $f_1: X \to Y_1 := Y_{f_1}, \cdots, f_n: X \to Y_n := Y_{f_n}$ such that $\{\phi^* f_1: M \to Y_1, \cdots, \phi^* f_n: M \to Y_n\}$ defines R_M on $\phi^{-1}(U)$. As is easily seen, $f := (f_1, \cdots, f_n): X \to Y_f := Y_1 \times \cdots \times Y_n$ is T-meromorphic and satisfies the desired condition.

THEOREM 2.2. *Let X be a complex analytic space. Then, there ex-*

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ists a unique proper modification $\{M(X) \xrightarrow[\pi]{} X\}$ *of* X *with center* $\mathrm{Sg}(X)$ *such that the following holds:*

(1) $M(X)$ *is normal.*

(2) *For any T-meromorphic mapping* f *of* X *into a complex analytic space* Y, *there is a holomorphic mapping* $\tilde{f}: M(X) \to Y$ *such that* $\tilde{f} = f \circ \pi$ *on* $M(X)$ *except some nowhere dense complex analytic set.*

(3) *If another proper modification* $\{M' \xrightarrow[\pi']{} X\}$ *of* X *has the above properties* (1) *and* (2), *then there is a holomorphic mapping* $\phi: M' \to M(X)$ *such that* $\pi' = \pi \circ \phi$ *on* M'.

Proof. (a) Construction of $M(X)$. By Hironaka's theorem of resolution of singularities [5], we can take an open covering $\{U_i\}_{i\in I}$ of X such that (i) there exists an open covering $\{V_i\}_{i\in I}$ of X such that each V_i is a relatively compact open taut complex analytic subspace of U_i, and (ii) each U_i has a resolution of singularities $\{M_i \xrightarrow[\phi_i]{} U_i\}$.

We consider the equivalence relation $R_i := R_{M_i}$ on M_i defined by $\phi_i^* M_T(X)$ as in Lemma 2.1 for each $i \in I$. Let $\phi_i^{-1}(V_i)/R_i$ be the quotient complex analytic space of $\phi_i^{-1}(V_i)$ by the relation R_i on $\phi_i^{-1}(V_i)$, and let $\beta_i: \phi_i^{-1}(V_i) \to \phi_i^{-1}(V_i)/R_i$ be the canonical projection (c.f. Proposition 1.6(2)). We denote by $N(\phi_i^{-1}(V_i)/R_i) \xrightarrow[\gamma_i]{} \phi_i^{-1}(V_i)/R_i$ the normalization of $\phi_i^{-1}(V_i)/R_i$.

We have a holomorphic mapping $\alpha_i: \phi_i^{-1}(V_i)/R_i \to V_i$ by Proposition 1.6(2) such that $\phi_i = \alpha_i \circ \beta_i$ on $\phi_i^{-1}(V_i)$ for each $i \in I$. By Lemma 2.1, we can choose a T-meromorphic mapping $f_i: X \to Y_i$ such that $\{\phi_i^* f_i: M_i \to Y_i$ and $\phi_i: M_i \to U_i\}$ induces on $\phi_i^{-1}(V_i)$ the equivalence relation R_i on M_i for each $i \in I$. Then, a proper holomorphic mapping

$$s_{i,j} := (\phi_i, \phi_i^* f_i, \phi_i^* f_j): M_i \longrightarrow U_i \times Y_i \times Y_j$$

induces also the equivalence relation R_i on $\phi_i^{-1}(V_i)$ and there exists a holomorphic homeomorphism $t_{i,j}$ of $\phi_i^{-1}(V_i)/R_i$ onto a complex analytic subvariety of $V_i \times Y_i \times Y_j$ with $s_{i,j} = t_{i,j} \circ \beta_i$ on $\phi_i^{-1}(V_i)$ for any $i, j \in I$. Now, we suppose that $V_i \cap V_j$ is not empty. Since ϕ_i maps $M_i - \phi_i^{-1}(\mathrm{Sg}(X))$ biholomorphically onto $U_i - \mathrm{Sg}(X)$, we see that

$$s_{i,j}(\phi_i^{-1}(V_i \cap V_j)) = u_{i,j} \circ s_{j,i}(\phi_j^{-1}(V_i \cap V_j))$$

and

$$t_{i,j}(\alpha_i^{-1}(V_i \cap V_j)) = u_{i,j} \circ t_{j,i}(\alpha_j^{-1}(V_i \cap V_j)) \, ,$$

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where $$u_{i,j}(z, y, x) := (z, x, y) \in [V_i \cap V_j] \times Y_i \times Y_j$$
$$\text{for} \quad (z, y, x) \in [V_i \cap V_j] \times Y_j \times Y_i \ (i, j \in I) .$$

Then, the holomorphic mapping

$$t_{j,i}^{-1} \circ u_{j,i} \circ t_{i,j} : \alpha_i^{-1}(V_i \cap V_j) \longrightarrow \alpha_j^{-1}(V_i \cap V_j)$$

induces a biholomorphic mapping $\lambda_{j,i}: \pi_i^{-1}(V_i \cap V_j) \to \pi_j^{-1}(V_i \cap V_j)$, where $\pi_i := \alpha_i \circ \gamma_i$ on $N(\phi_i^{-1}(V_i)/R_i)$ $(i, j \in I)$. We see easily that $\pi_i \circ \lambda_{i,j} = \pi_j$ on $\pi_j^{-1}(V_i \cap V_j)$ and $\lambda_{i,j} \circ \lambda_{j,k} = \lambda_{i,k}$ on $\pi_k^{-1}(V_i \cap V_j \cap V_k)$ $(i, j, k \in I)$. Now, as the required normal complex analytic space $M(X)$, we take the quotient (complex analytic) space of $\bigcup_{i \in I} N(\phi_i^{-1}(V_i)/R_i)$ by the equivalence relation defined by the identification of points x in $\pi_j^{-1}(V_i \cap V_j)$ and y in $\pi_i^{-1}(V_i \cap V_j)$ with $\lambda_{i,j}(x) = y$ $(i, j \in I)$.

Then, the family $\{\pi_i : N(\phi_i^{-1}(V_i)/R_i) \to V_i\}_{i \in I}$ induces a proper surjective holomorphic mapping $\pi: M(X) \to X$ such that $\{M(X) \xrightarrow[\pi]{} X\}$ is a proper modification of X with center $\mathrm{Sg}(X)$.

(b) Proof of property (2) for $M(X)$. Let $f: X \to Y$ be a T-meromorphic mapping. Then, by Proposition 1.6(2), there exists $f_i \in \mathrm{Hol}(M_i/R_i, Y_i)$ such that $f \circ \phi_{i|\phi_i^{-1}(V_i)} = f_i \circ \beta_i$ on $\phi_i^{-1}(V_i)$ for each $i \in I$. So, $\{f_i \circ \gamma_i : N(\phi_i^{-1}(V_i)/R_i) \to Y\}_{i \in I}$ induces a holomorphic mapping $\tilde{f}: M(X) \to Y$ such that $\tilde{f} = f \circ \pi$ on $M(X)$ except some nowhere dense complex analytic set.

(c) Proof of property (3) for $M(X)$. We have only to prove that a meromorphic mapping $\pi^{-1}: X \to M(X)$ is T-meromorphic on X. Indeed, in this case, if $\{M' \xrightarrow[\pi']{} X\}$ is a proper modification of X satisfying the conditions (1) and (2) in Theorem, there exists a holomorphic mapping $\phi: M' \to M(X)$ such that $\phi = \pi^{-1} \circ \pi'$ on M' except some nowhere dense complex analytic set and then $\pi \circ \phi = \pi'$ on M' which asserts the property (3) for $M(X)$.

Now, let $x \in X$. There exists some V_i $(i \in I)$ and a T-meromorphic mapping $f_i: X \to Y_i$ such that $x \in V_i$ and $g_i := (\alpha_i, \alpha_i^* f_i): \phi_i^{-1}(V_i)/R_i \to V_i \times Y_i$ is a holomorphic homeomorphism onto a complex analytic subvariety of $V_i \times Y_i$, where $\alpha_i^* f_i : \phi_i^{-1}(V_i)/R_i \to Y_i$ is a naturally induced holomorphic mapping from $\phi_i^* f_i : M_i \to Y_i$ (cf. Proposition 1.6(2)). On the other hand, since $f_i: X \to Y_i$ is T-meromorphic, there exists a taut open neighborhood U' of x in X and a taut local complex analytic set V in Y_i such that $f_i(U') \subset V$. We see easily that $U := U' \cap V_i$ is a taut complex analytic space. Thus, $g_i: \alpha_i^{-1}(U) \to U \times V$ is a holomorphic

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homeomorphism onto a complex analytic subvariety of $U \times V$. By the fact that the class of all taut complex analytic spaces satisfies the conditions (P_1) and (P_2) in §1, $\alpha_i^{-1}(U)$ is taut and hence its normalization $\pi_i^{-1}(U)$ is also taut. Since $\pi: M(X) \to X$ is induced from $\{\pi_i: N(\phi_i^{-1}(V_i)/R_i) \to V_i\}_{i\in I}$, we conclude that $\pi^{-1}: X \to M(X)$ is T-meromorphic on X.

(d) The uniqueness of $\{M(X) \xrightarrow[\pi]{} X\}$ up to complex analytic isomorphisms follows from the property (3) of $M(X)$.

PROPOSITION 2.3. *If V is a taut open complex analytic subspace of a complex analytic space $X, \pi^{-1}(V)$ is also taut for the proper modification $\{M(X) \xrightarrow[\pi]{} X\}$ of X in Theorem* 2.2.

Proof. By virtue of [1], it suffices to show that $\mathrm{Hol}(\varDelta, \pi^{-1}(V))$ is normal for the unit disc $\varDelta = \{|z| < 1\}$ in the complex plane C. Let $F = \{f_n\}_{n>0}$ be a sequence in $\mathrm{Hol}(\varDelta, \pi^{-1}(V))$. Then, $\{\pi \circ f_n \in \mathrm{Hol}(\varDelta, V)\}_{n>0}$ is normal, because V is taut. Assume that F is not compactly divergent on $\varDelta$. Since $\pi: M(X) \to X$ is proper, we may assume that there exists $\lim_n \pi \circ f_n = g$ in $\mathrm{Hol}(\varDelta, V)$. As in the proof of Theorem 2.2, any $x \in \varDelta$ has an open neighborhood U in $\varDelta$ such that, for an open neighborhood D of $g(x)$ in V, (i) $\pi^{-1}(D)$ is taut, and (ii) $\{f_{n|U}\}_{n>n_0} \subset \mathrm{Hol}(U, \pi^{-1}(D))$ for some n_0. By the diagonal argument, we see that F has a subsequence which is uniformly convergent on every compact set in $\varDelta$. This shows that $\mathrm{Hol}(\varDelta, \pi^{-1}(V))$ is normal.

REMARK 2.4. We can prove also that, if X is a (complete) hyperbolic complex analytic space in the sense of S. Kobayashi, then $M(X)$ is also (complete) hyperbolic.

PROPOSITION 2.5. *Let $f: X \to Y$ be a holomorphic mapping of a complex analytic space X into another Y such that $f^{-1}(\mathrm{Sg}(Y))$ is nowhere dense in X. Then, there is a holomorphic mapping $\hat{f}: M(X) \to M(Y)$ such that $\pi_Y \circ \hat{f} = f \circ \pi_X$ on $M(X)$ for the proper modifications $\{M(X) \xrightarrow[\pi_X]{} X\}$ of X and $\{M(Y) \xrightarrow[\pi_Y]{} Y\}$ of Y as in Theorem* 2.2.

Proof. Since $\pi_Y^{-1}: Y \to M(Y)$ is T-meromorphic on $Y, \pi_Y^{-1} \circ f: X \to M(Y)$ is also T-meromorphic on X. By the property (2) in Theorem 2.2, there exists a holomorphic mapping $\hat{f}: M(X) \to M(Y)$ with $\pi_Y \circ \hat{f} = f \circ \pi_X$ on $M(X)$.

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REMARK 2.6. Let X be a taut complex analytic space with a resolution of singularities $\{M \xrightarrow[\phi]{} X\}$ of X. By Theorem 1.7, for the taut proper relation R_T on $M, M/R_T$ is a normal taut complex analytic space and we have a commutative diagram

$$\begin{array}{ccccc} & \overset{\beta}{\swarrow} & M & \overset{\mu}{\searrow} & \\ M(X) & & \downarrow \phi & & M/R_T \\ & \underset{\pi}{\searrow} & X & \underset{\tau}{\swarrow} & \end{array}$$

for the canonically defined holomorphic mappings. Since $M(X)$ is taut (Proposition 2.3), there exists a holomorphic mapping $s: M/R_T \to M(X)$ such that $s \circ \mu = \beta$ on M by Theorem 1.7. On the other hand, we see easily that $\{M/R_T \xrightarrow[\tau]{} X\}$ is a proper modification of X with center $\mathrm{Sg}(X)$ and $\tau^{-1}: X \to M/R_T$ is T-meromorphic. So, there exists a holomorphic mapping $t: M(X) \to M/R_T$ such that $\tau \circ t = \pi$ on $M(X)$ by the property (2) in Theorem 2.2. From the fact that $\{M(X) \xrightarrow[\pi]{} X\}$ and $\{M/R_T \xrightarrow[\tau]{} X\}$ are both proper modifications of X with center $\mathrm{Sg}(X)$, we see that $\pi \circ s = \tau$ on M/R_T, whence $t: M(X) \to M/R_T$ is biholomorphic, i.e., $\{M(X) \xrightarrow[\pi]{} X\}$ $= \{M/R_T \xrightarrow[\tau]{} X\}$.

PROPOSITION 2.7. *Let N be a connected complex analytic manifold with a nonconstant proper holomorphic mapping $\phi: N \to X$ into a taut complex analytic space X. Then, for the quotient complex analytic space N/R_T of N, we have $M(N/R_T) = N/R_T$, i.e., the canonical holomorphic mapping $\pi: M(N/R_T) \to N/R_T$ is biholomorphic.*

Proof. By Proposition 2.5, we have a holomorphic mapping $\tilde{\gamma}: N \to M(N/R_T)$ with $\pi \circ \tilde{\gamma} = \gamma$ on N for the canonical holomorphic mapping $\gamma: N \to N/R_T$. Since N/R_T is taut, $M(N/R_T)$ is also taut by Proposition 2.3. Then, by the property (2) in Theorem 1.7, there exists a holomorphic mapping $s: N/R_T \to M(N/R_T)$ such that $s \circ \gamma = \tilde{\gamma}$ on N. From the fact that $\{M(N/R_T) \xrightarrow[\pi]{} N/R_T\}$ is a proper modification of N/R_T, we see that $\pi: M(N/R_T) \to N/R_T$ is biholomorphic.

PROPOSITION 2.8. *Let X be a normal complex analytic space such that there exists an open surjective holomorphic mapping ϕ of a complex analytic manifold N onto X. Then, it holds that $M(X) = X$, i.e., every T-meromorphic mapping defined on X is holomorphic on X.*

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Proof. Without loss of generality, we may assume that X and N are both connected. By Proposition 2.5 we have a holomorphic mapping $\tilde{\phi}: N \to M(X)$ such that $\pi \circ \tilde{\phi} = \phi$ on N for the proper modification $\{M(X) \xrightarrow[\pi]{} X\}$ of X, because $M(N) = N$ obviously. We can prove that $\tilde{\phi}: N \to M(X)$ is surjective. Let $p \in M(X)$. If $p \in M(X) - \pi^{-1}(\mathrm{Sg}\,(X))$, for $x \in N$ with $\phi(x) = \pi(p) \in X$, $\tilde{\phi}(x) = p \in M(X)$ by the definition of $\tilde{\phi}: N \to M(X)$. We suppose that $p \in \pi^{-1}(\mathrm{Sg}\,(X))$. Then there exists a point $x \in N$ and a relatively compact open neighborhood U of x in N such that $\phi(x) = \pi(p) \in X$. Since $V: = \phi(U)$ is an open neighborhood of $\phi(x) = \pi(p)$ in X, there exists a sequence of points $\{p_n \in \pi^{-1}(V - \mathrm{Sg}\,(X))\}_{n>0}$ such that, for some $\{x_n \in U\}_{n>0}, \phi(x_n) = \pi(p_n)$ in X for all $n > 0$ and $\lim_n p_n = p \in M(X)$. Then, $\tilde{\phi}(x_n) = p_n$ in $M(X)$ for all $n > 0$. Since $U \subset N$ is relatively compact in N, we may assume that $\lim_n x_n = x_0 \in N$ and then $\tilde{\phi}(x_0) = \lim_n \tilde{\phi}(x_n) = \lim_n p_n = p \in M(X)$. Thus $\tilde{\phi}: \mathrm{N} \to M(X)$ is surjective. We see easily that $\pi: M(X) \to X$ is open, because $\phi: N \to X$ is open surjective and $\tilde{\phi}: N \to M(X)$ is surjective. By the Remmert's theorem for open holomorphic mappings, $\pi: M(X) \to X$ is of finite-fibers. It means that $\pi: M(X) \to X$ is biholomorphic by the uniqueness of normalizations of complex analytic spaces.

PROPOSITION 2.9. *Let X be a complex analytic space with the proper modification $\{M(X) \xrightarrow[\pi]{} X\}$ of X in Theorem 2.2. Let f be a T-meromorphic mapping of an open set U in X into a complex analytic space Y such that, for some nowhere dense compact complex analytic set A of U, $f_{|U-A}$ is holomorphic on $U - A$. Then, there exists a holomorphic mapping $\tilde{f}: \pi^{-1}(U) \to Y$ such that $\tilde{f} = f \circ \pi$ on $\pi^{-1}(U - A)$.*

Proof. Let $\Gamma_f \subset U \times Y$ be the graph of f. Then, there exists a biholomorphic mapping $\phi: U - A \to \Gamma_f - pr^{-1}(A)$ for the canonical projection $pr: \Gamma_f \to U$. We can consider a quotient complex analytic space $Z: = (X - A) \cup \Gamma_f/\sim$ by the identification between $x \in U - A$ and $\phi(x) \in \Gamma_f - pr^{-1}(A)$. This is possible, because Z with the quotient topology is a Hausdorff space. Moreover, we have a holomorphic mapping $g: X - A \to Z$ with $g(x) = \psi(x) \in Z$ $(x \in X - A)$ for the canonical projection $\psi: (X - A) \cup \Gamma_f \to Z$. Since $\psi_{|\Gamma_f}: \Gamma_f \to \psi(\Gamma_f)$ is a biholomorphic mapping onto an open set $\psi(\Gamma_f)$ of Z by the definition of the complex analytic structure on Z, $g: X \to Z$ is T-meromorphic on X. So, there

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exists a holomorphic mapping $\tilde{g}: M(X) \to Z$ such that $\tilde{g} = g \circ \pi$ on $M(X)$. It is easy to see that $\tilde{g}(\pi^{-1}(U)) \subset \psi(\Gamma_f)$ in Z. Then, for the natural projection $pr_Y: \Gamma_f \to Y$, $\tilde{f}: = pr_Y \circ \psi^{-1} \circ \tilde{g}: \pi^{-1}(U) \to Y$ is the desired holomorphic mapping.

REMARK 2.10. Let $f: X \to Y$ be a meromorphic mapping of a complex analytic space X into a hyperbolic complex analytic space Y. Let $\Gamma_f \subset X \times Y$ be the graph of $f: X \to Y$, and let $p_X: \Gamma_f \to X$ and $p_Y: \Gamma_f \to Y$ be the canonical projections. Then, we can define a meromorphic mapping $g: X \to \Gamma_f$ by $g(x) = \{(x, y): y \in f(x)\} \subset X \times Y$ for $x \in X$. We shall prove that $g: X \to \Gamma_f$ is T-meromorphic. From this, we can conclude that the assertion (2) in Theorem 2.2 and Proposition 2.9 hold for arbitary meromorphic mapping into any hyperbolic complex analytic space. For our purpose, it suffices to show that, for any point $x \in X$, there exists an open neighborhood U of x in X such that $p_X^{-1}(U) \subset \Gamma_f$ is taut. Now, we choose an open neighborhood U of x in X such that U is complete hyperbolic. Then, $p_X^{-1}(U)$ is complete hyperbolic, because (i) $p_X^{-1}(U) \subset U \times Y$ is hyperbolic, (ii) $p_X^{-1}(x)$ is a compact hyperbolic complex analytic space for any $x \in U$, and (iii) U is complete hyperbolic. It is easy to see that a complete hyperbolic complex analytic space is taut (c.f. [9]). This completes the proof.

§ 3. Applications

Let X and Y be complex analytic spaces. We denote by Mero (X, Y), Aut (X) and $\mathrm{Aut}_m(X)$ the set of all meromorphic mappings of X into Y, the group of all biholomorphic mappings of X onto X and the group of all bimeromorphic mappings of X onto X respectively.

We suppose that Y is taut. By Theorem 2.2, there exists a bijective mapping α: Mero $(X, Y) \to$ Hol $(M(X), Y)$ such that $\alpha(f) = f \circ \pi$ on $M(X)$ except some nowhere dense complex analytic set for any $f \in$ Mero (X, Y), where $\{M(X) \xrightarrow[\pi]{} X\}$ is the proper modification of X constructed in Theorem 2.2. If Y is a compact taut complex analytic space, then we have a natural bijective mapping β: Hol $(X - \mathrm{Sg}(X), Y) \to$ Hol $(M(X), Y)$ from Remark 1.5.

Now, Proposition 2.3 asserts that, if Y is taut, then $M(Y)$ is also taut for the proper modification $\{M(Y) \xrightarrow[\pi]{} Y\}$ of Y. This shows that $\pi^{-1} \circ f: Y \to M(Y)$ is T-meromorphic on Y for any $f \in \mathrm{Aut}_m(Y)$. By Theo-

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rem 2.2, there exists a holomorphic mapping $\gamma(f): M(Y) \to M(Y)$ such that $\pi \circ \gamma(f) = f \circ \pi$ on $M(Y)$ except some nowhere dense complex analytic set for any $f \in \mathrm{Aut}_m(Y)$. We see easily that $\gamma(f) \in \mathrm{Aut}(M(Y))$ since $f \in \mathrm{Aut}_m(Y)$ and then $\gamma: \mathrm{Aut}_m(Y) \to \mathrm{Aut}(M(Y))$ is a group-isomorphism. We have

PROPOSITION 3.1. *Let X be an irreducible taut complex analytic space of complex dimension n. Then, $\mathrm{Aut}_m(X)$ admits a structure of a real Lie group of real dimension $\leqq 2n + n^2$. In particular, if X is compact, it is a finite group.*

Proof. By Proposition 2.3, $M(X)$ is taut. Proposition 3.1 follows easily from Satz 1.3 and 5.2 in [7].

COROLLARY 3.2. *Let X be a compact complex analytic space which is bimeromorphic with a compact taut complex analytic space $\tilde{X}$ by $\phi: X \to \tilde{X}$. Then, $\mathrm{Aut}(X)$ is a finite group.*

Proof. The bimeromorphic mapping $\phi: X \to \tilde{X}$ induces an injective group-homomorphism $\phi^{\#}: \mathrm{Aut}(X) \to \mathrm{Aut}_m(\tilde{X})$ with $\phi^{\#}(f) = \phi \circ f \circ \phi^{-1} \in \mathrm{Aut}_m(\tilde{X})$ for any $f \in \mathrm{Aut}(X)$. Corollary 3.2 is a direct result of Proposition 3.1.

PROPOSITION 3.3. *Let X be a compact taut complex analytic space and A a nowhere dense complex analytic set in X. If f is a holomorphic mapping of $X - A$ into itself, then there exists a positive integer n such that $g := f^n = f \circ f \circ \cdots \circ f: X - A \to X - A$ is a holomorphic retraction, i.e., $g \circ g = g$ on $X - A$. Moreover, if $f: X - A \to X - A$ is surjective, it is biholomorphic on $X - A$.*

Proof. By Remark 1.5, $f: X \to X$ is meromorphic on X. Therefore, there is a holomorphic mapping $\tilde{f}: M(X) \to X$ such that $\tilde{f} = f \circ \pi$ on $M(X)$ except some nowhere dense complex analytic set, where $\{M(X) \xrightarrow[\pi]{} X\}$ is the proper modification of X as in Theorem 2.2. We put $Y := \tilde{f}(M(X)) \subset X$. It is easy to see that $Y \cap A$ is nowhere dense in Y and $f(X - A) \subset Y - A$. Since Y is a compact taut complex analytic space and $f_{|Y-A}: Y - A \to Y - A$, it suffices to show that this Proposition is true for $f_{|Y-A}: Y - A \to Y - A$.

Succeeding this process, we have a finite sequence

$$X - A \xrightarrow[f]{} Y - A_0 \xrightarrow[f_1]{} Y_1 - A_1 \xrightarrow[f_2]{} \cdots\cdots \xrightarrow[f_n]{} Y_n - A_n$$

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of compact taut complex analytic subvarieties $Y \supset Y_1 \supset \cdots\cdots \supset Y_n$ in X and holomorphic mappings $f_1 := f_{|Y-A_0}: Y - A_0 \to Y_1 - A_1$, $f_i := f_{|Y_{i-1}-A_{i-1}}: Y_{i-1} - A_{i-1} \to Y_i - A_i$ $(i > 1)$ such that $A_0 := Y \cap A$ and $A_i := Y_i \cap A$ $(i > 0)$ are nowhere dense in Y and Y_i $(i > 0)$ respectively and such that $f_{|Y_n-A_n}: Y_n - A_n \to Y_n - A_n$ is surjective. Thus, if the second part in Proposition 3.3 is proved, then $f_{|Y_n-A_n}: Y_n - A_n \to Y_n - A_n$ is biholomorphic and so bimeromorphic on Y_n by Remark 1.5. It means that $(f_{|Y_n})^p = id$ in $\mathrm{Aut}_m(Y_n)$ for some positive integer p by Corollary 3.2 and then $f^{p(n+1)}: X - A \to X - A$ is a holomorphic retraction.

Now, we shall prove that, if $f: X - A \to X - A$ is surjective, it is biholomorphic on $X - A$. We suppose that $f: X - A \to X - A$ is surjective. Since X is compact, $f^{-1}(\mathrm{Sg}(X) \cap (X - A))$ is nowhere dense in $X - A$. Thus, we have a T-meromorphic mapping $\pi^{-1} \circ f: X \to M(X)$ because of (i) Remark 1.5, and (ii) the fact that $M(X)$ is taut. By Theorem 2.2, there exists a holomorphic mapping $\tilde{f}: M(X) \to M(X)$ such that $\pi \circ \tilde{f} = f \circ \pi$ on $M(X)$ except some nowhere dense complex analytic set. Since $\pi^{-1}(A)$ is nowhere dense in $M(X)$ and $f: X - A \to X - A$ is surjective, $\tilde{f}: M(X) \to M(X)$ is surjective and so biholomorphic on $M(X)$ by Satz 5.2 in [7]. There exists a positive integer m such that $\tilde{f}^m = id$ on $M(X)$ by Corollary 3.2, and then $f^m = id$ on $X - A$. This means that $f \in \mathrm{Aut}(X - A)$.

PROPOSITION 3.4. *Let X be a compact irreducible complex analytic space and Y a compact taut complex analytic space whose universal covering does not contain compact complex analytic subvarieties of positive complex dimension. Then, $\{f \in \mathrm{Hol}(X - A, Y): f(x) = y \in Y\}$ is finite for any nowhere dense complex analytic set A in X and $x \in X - A$ and $y \in Y$.*

Proof. Let $\{M(X) \xrightarrow[\pi]{} X\}$ be the proper modification of X as in Theorem 2.2. By Remark 1.5 and Theorem 2.2, for any $f \in \mathrm{Hol}(X - A, Y)$ there exists a holomorphic mapping $\tilde{f}: M(X) \to Y$ such that $\tilde{f} = f \circ \pi$ on $M(X)$ except some nowhere dense complex analytic set. Thus, we see that, if $f \in \mathrm{Hol}(X - A, Y)$ and $f(x) = y \in Y$ for $x \in X$, then $\tilde{f}(\pi^{-1}(x)) = \{y\}$ in Y. On the other hand, $\{\psi \in \mathrm{Hol}(M(X), Y): \psi(\pi^{-1}(x)) = \{y\} \subset Y\}$ is finite by Satz 5.4 in [7] and then $\{f \in \mathrm{Hol}(X - A, Y): f(x) = y \in Y\}$ is finite for $x \in X - A$ and $y \in Y$.

Now, we shall give some sufficient conditions for a normal complex analytic space X with only one singular point to be $M(X) = X$.

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PROPOSITION 3.5. *Let X be a taut normal complex analytic space with an isolated singularity $p \in X$. Suppose that there exists a resolution of singularity $\{M \xrightarrow[\phi]{} X\}$ of X such that $d_M(x, y) = 0$ for any points x and y in $\phi^{-1}(p)$, where d_M is the Kobayashi pseudodistance on M* (cf. [11]). *Then $M(X) = X$, i.e., $\pi: M(X) \to X$ is biholomorphic for the proper modification $\{M(X) \xrightarrow[\pi]{} X\}$ of X as in Theorem* 2.2.

Proof. Since $\phi: M \to X$ is a proper holomorphic mapping onto a taut complex analytic space X, we can consider the taut proper relation R_T on M. By Theorem 1.7, we have a quotient complex analytic space $X' := M/R_T$ of M by the taut proper relation R_T on M such that there exists a holomorphic mapping $\tau: X' \to X$ which satisfies the condition $\phi = \tau \circ p$ on M for the canonical projection $p: M \to X'$. By Remark 2.6, $\{X' \xrightarrow[\tau]{} X\}$ is a proper modification of X with center $\mathrm{Sg}(X) = \{p\}$ and $\{M(X) \xrightarrow[\pi]{} X\} = \{X' \xrightarrow[\tau]{} X\}$. Therefore, it suffices to show that $\tau: X' \to X$ is biholomorphic.

We shall show that $\tau: X' \to X$ is injective. Then $\tau: X' \to X$ is an open holomorphic mapping by Remmert's theorem on open holomorphic mappings, which means that $\tau: X' \to X$ is biholomorphic since τ is an open bijective holomorphic mapping between normal complex analytic spaces X' and X. Now, if $\tau: X' \to X$ is not injective, $\tau^{-1}(p)$ contains, at least, two distinct points of X' because $\tau_{|X' - \tau^{-1}(p)}: X' - \tau^{-1}(p) \to X - \{p\}$ is biholomorphic. This means that there exists a holomorphic mapping f of M into a taut complex analytic space Y such that $f_{|\phi^{-1}(p)}: \phi^{-1}(p) \to Y$ is non-constant since $X' = M/R_T$. On the other hand, every holomorphic mapping g of M into a taut complex analytic space is constant on $\phi^{-1}(p)$ because $d_M = 0$ on $\phi^{-1}(p)$ (cf. [11]). This is a contradiction. Thus, we proved that $\tau: X' \to X$ is injective. This completes the proof.

COROLLARY 3.6. *Let X be a normal complex analytic space of complex dimension* 2 *with a rational or elliptic isolated singularity $p \in X$. Then, $M(X) = X$ for the proper modification $\{M(X) \xrightarrow[\pi]{} X\}$ of X as in Theorem* 2.2.

Proof. Let $\{U' \xrightarrow[\phi]{} U\}$ be a resolution of singularity of an open neighborhood U of p in X. Taking a taut open neighborhood of p in U, we may assume that U is taut. Moreover, we may assume that $\phi^{-1}(p)$

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is connected and is the union of irreducible nonsingular curves (cf. [13]). If p is a rational (resp. elliptic) singularity of X, then each irreducible curve of $\phi^{-1}(p)$ is a rational (resp. rational or elliptic) curve (cf. [13]). Let $d_{U'}$ be the Kobayashi pseudodistance on U' and d the Kobayashi pseudodistance on $\phi^{-1}(p)$. Since the Kobayashi pseudodistances on compact Riemann surfaces of genus < 2 are trivial and $\phi^{-1}(p)$ is connected, d is trivial, i.e., $d = 0$ on $\phi^{-1}(p)$. Let $i: \phi^{-1}(p) \to U'$ be the natural injection. It is well-known that $d_{U'}(i(x), i(y)) \leqq d(x, y) = 0$ for any points x and y in $\phi^{-1}(p) \subset U'$, which means that $d_{U'} = 0$ on $\phi^{-1}(p) \subset U'$. By Proposition 3.5, $M(U) = U$ for the proper modification $\{M(U) \xrightarrow[\pi']{} U\}$ of U as in Theorem 2.2.

From this, we see that every T-meromorphic mapping defined on X is extended holomorphically to X and then $M(X) = X$ by the uniqueness of $\{M(X) \xrightarrow[\pi]{} X\}$ of X in Theorem 2.2.

PROPOSITION 3.7. *Let V be a normal irreducible complex analytic space of complex dimension* 2 *with an isolated singularity $p \in V$. If V is taut and* $\dim_R \mathrm{Aut}(V) \geqq 2$ *for the real Lie group* $\mathrm{Aut}(V)$, *then* $M(V) = V$ *for the proper modification* $\{M(V) \xrightarrow[\pi]{} V\}$ *of V as in Theorem* 2.2.

Proof. Since V and $M(V)$ are taut (cf. Proposition 2.3) and irreducible, $\mathrm{Aut}(V)$ and $\mathrm{Aut}(M(V))$ are both real Lie groups. Moreover, $\mathrm{Aut}(V)$ is a compact real Lie group, because V is taut and $f(p) = p$ in V for all $f \in \mathrm{Aut}(V)$. By Proposition 2.5, we have a group-homomorphism $\gamma: \mathrm{Aut}(V) \to \mathrm{Aut}(M(V))$ such that $\pi \circ \gamma(f) = f \circ \pi$ on $M(V)$ for any $f \in \mathrm{Aut}(V)$. We can easily see that $\gamma: \mathrm{Aut}(V) \to \mathrm{Aut}(M(V))$ is continuous. Therefore, $\mathrm{Aut}(V)$ is isomorphic to a compact real Lie subgroup H of $\mathrm{Aut}(M(V))$ as a real Lie group. Since $f(\pi^{-1}(p)) = \pi^{-1}(p)$ for any $f \in H$, the quotient group H/G is considered as an effective transformation group on $\pi^{-1}(p)$ for a subgroup $G: = \{f \in H : f = id \text{ on } \pi^{-1}(p)\}$ of H. Then it is a finite group, because $\pi^{-1}(p)$ is a compact taut complex analytic space. Thus, we have $\dim_R \mathrm{Aut}(V) = \dim_R H = \dim_R G$.

Now, we assume that $\dim_C \pi^{-1}(p) \quad 1$. Then, $(M(V) - \mathrm{Sg}(M(V))) \cap \pi^{-1}(p) \neq \phi$. We consider an unitary isotropy representation of G at a suitable point $x \in (M(V) - \mathrm{Sg}(M(V))) \cap \pi^{-1}(p)$, that is, a homomorphism $\chi: G \to U(C^2)$ defined by assigning the differential of f at x for each $f \in G$, where $U(C^2)$ is the unitary group in $GL(C^2)$ with respect to some

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fixed hermitian product on C^2. Using this, we conclude that $\dim_R G = \dim_R \operatorname{Aut}(V) = 1$, because χ is injective and $f_{|\pi^{-1}(p)} = id$ on $\pi^{-1}(p)$ for any $f \in G$ (c.f. [8]). This contradicts the assumption of Proposition 3.7. We proved that $\dim_C \pi^{-1}(p) = 0$ and then $\pi: M(V) \to V$ is biholomorphic as easily seen.

Proposition 3.7 was communicated to the author by Professor H. Fujimoto, who has proved a more general result that, for any taut normal irreducible complex analytic space X of complex dimension n with an isolated singularity $p \in X$, if $\dim_C \pi^{-1}(p) = m$ for the proper modification $\{M(X) \xrightarrow[\pi]{} X\}$ of X, then $\dim_R \operatorname{Aut}(X) \leqq (n - m)^2$.

We can give some examples of non-taut projective algebraic varieties which are bimeromorphic with compact taut complex analytic spaces, using the taut proper relation on complex analytic spaces (see §1).

PROPOSITION 3.8. *Let X be a compact irreducible complex analytic space and A a nowhere dense complex analytic set in X. Suppose that there exists a holomorphic mapping with discrete fibers $f: X - A \to Y$ into a compact taut complex analytic space Y. Then, there exists a compact taut complex analytic space $\tilde{X}$ and a bimeromorphic mapping $\phi: X \to \tilde{X}$ holomorphic on $X - (A \cup \operatorname{Sg}(X))$.*

Proof. Let $\{M(X) \xrightarrow[\pi]{} X\}$ be the proper modification of X as in Theorem 2.2. Since f is considered to be T-meromorphic on X by Remark 1.5, there exists a holomorphic mapping $\tilde{f}: M(X) \to Y$ such that $\tilde{f} = f \circ \pi$ on $M(X)$ except some nowhere dense complex analytic set. Since $\tilde{f}: M(X) \to Y$ is proper, we can consider a quotient complex analytic space $\tilde{X} = M(X)/R_T$ of $M(X)$ with the canonical projection $p: M(X) \to \tilde{X}$ for the taut proper relation R_T on $M(X)$ by Theorem 1.7. Then, $\tilde{X}$ is obviously compact, normal, irreducible and taut. Since $\pi_{|M(X)-\pi^{-1}(A \cup \operatorname{Sg}(X))}: M(X) - \pi^{-1}(A \cup \operatorname{Sg}(X)) \to X - (A \cup \operatorname{Sg}(X))$ is biholomorphic, $f \circ \pi_{|M(X)-\pi^{-1}(A \cup \operatorname{Sg}(X))} = \tilde{f}_{|M(X)-\pi^{-1}(A \cup \operatorname{Sg}(X))}: M(X) - \pi^{-1}(A \cup \operatorname{Sg}(X)) \to Y$ has discrete fibers and then $p_{|M(X)-\pi^{-1}(A \cup \operatorname{Sg}(X))}: M(X) - \pi^{-1}(A \cup \operatorname{Sg}(X)) \to \tilde{X}$ has discrete fibers. On the other hand, $p: M(X) \to \tilde{X}$ has connected fibers by Theorem 1.7. Therefore, $p_{|M(X)-\pi^{-1}(A \cup \operatorname{Sg}(X))}: M(X) - \pi^{-1}(A \cup \operatorname{Sg}(X)) \to \tilde{X}$ is injective, moreover, $\tilde{X} = p(M(X) - \pi^{-1}(A \cup \operatorname{Sg}(X))) \cup p(\pi^{-1}(A \cup \operatorname{Sg}(X)))$ and $p(M(X) - \pi^{-1}(A \cup \operatorname{Sg}(X))) \cap p(\pi^{-1}(A \cup \operatorname{Sg}(X))) = \phi$ in $\tilde{X}$. The proper mapping theorem asserts that $p(\pi^{-1}(A \cup \operatorname{Sg}(X))) \subset \tilde{X}$ is a nowhere dense

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complex analytic set in $\tilde{X}$. Then, by the theorem of Remmert on open holomorphic mappings, $p_{|M(X)-\pi^{-1}(A\cup\mathrm{Sg}\,(X))}$ is a biholomorphic mapping onto an open set $p(M(X)-\pi^{-1}(A\cup\mathrm{Sg}\,(X)))=\tilde{X}-p(\pi^{-1}(A\cup\mathrm{Sg}\,(X)))$ in $\tilde{X}$. Thus, we obtain a bimeromorphic mapping $\phi:=p\circ\pi^{-1}:X\to\tilde{X}$ which is holomorphic on $X-(A\cup\mathrm{Sg}\,(X))$.

EXAMPLE 3.9. Let π be the canonical projection of $C^3-\{0\}$ onto the complex projective manifold $P_2(C)$ and let $\{x_1,x_2,x_3\}$ be the complex analytic coordinates on C^3 which give also homogeneous coordinates on $P_2(C)$. Let

$$Y_{p,q,r}:=\{ax_1^{rm}+bx_2^{rm}-cx_1^{rp}x_3^{rq}=0\}\quad\text{in}\quad C^3$$

and

$$X_{p,q,r}:=\pi(Y_{p,q,r}-\{0\})\subset P_2(C)\ ,$$

where $a,b,c\in C$ with $abc\neq 0$ in C, m,p,q,r are integers with $p\geqq 0$, $q>0$, $r>0$ and $m=p+q$. Put $Y_r:=Y_{0,1,r}$ and $X_r:=X_{0,1,r}$. Then, the genus of X_r is $(r-1)(r-2)/2$, because X_r is a nonsingular complex projective curve of degree r in $P_2(C)$. In the following, we assume that $r\geqq 4$ and then X_r is taut. We consider holomorphic mappings $\tilde{\phi}(x_1,x_2,x_3):=(x_1^m,x_2^m,x_1^px_3^q)$ of $Y_{p,q,r}$ into Y_r and $\phi:X_{p,q,r}-S\to X_r$ such that $\phi\circ\pi=\pi\circ\tilde{\phi}$ on $Y_{p,q,r}-T$, where $T:=\{x_1=0\}\subset C^3$ and $S:=\pi(T-\{0\})\subset P_2(C)$. We see easily that $\phi:X_{p,q,r}-S\to X_r$ is of finite-fibers. By Proposition 3.8, $X_{p,q,r}$ is a compact taut complex analytic space, because $\dim_C X_{p,q,r}=1$.

Let $X:=\{ax_1^{rm}+bx_2^{rm}-c(x_1h(x_3))^{rn}=0\}$ in C^3 for a non-constant polynomial $h(x_3)$ and positive integers s,t,p,q with $m=p+q$ and $n=p+qs=qt$ (e.g., $p=1$, $q=1$, $s=2$, $t=3$). Let $C_X=X\cap\{x_1h(x_3)=0\}\subset X$. Then, C_X is nowhere dense in X. Consider holomorphic mappings $\tilde{\phi}_1(x_1,x_2,x_3):=(x_1^m,x_2^m,x_1^nh(x_3)^n)$ of $X-C_X$ into Y_r and $\tilde{\phi}_2(x_1,x_2,x_3):=(x_1,x_2,x_1^sh(x_3)^t)$ of $X-C_X$ into $Y_{p,q,r}$ and put $\phi_1:=\pi\circ\tilde{\phi}_1$ and $\phi_2:=\pi\circ\tilde{\phi}_2$. We see easily that a holomorphic mapping $\phi:=(\phi_1,\phi_2):X-C_X\to X_r\times X_{p,q,r}$ is of finite-fibers. By Proposition 3.8, the natural projective compactification $\bar{X}$ of X in $P_3(C)$ is bimeromorphic with a compact taut complex analytic space. On the other hand, $\bar{X}$ is not taut, because there exists a holomorphic mapping $f(z):=(z,(a/b)^{1/rm}z,z_0)$ of the complex plane C into X with $h(z_0)=0$ (c.f. [11], p. 69).

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In this manner, we can construct also higher dimensional, non-taut, projective algebraic varieties which are bimeromorphic with compact taut complex analytic spaces.

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