MASON, L. J. and WOODHOUSE, N. M. J. *Integrability, self-duality and twistor theory* (London Mathematical Society Monographs (New Series) No. 15, Clarendon Press, Oxford, 1996), x + 364pp., 0 19 853498 1, (hardback) £45.

Integrable systems of PDEs have come a long way for a subject where the object of study lacks a definition. In medicine it is common to define a condition by saying that to qualify a patient must exhibit (say) four out of six given symptoms and perhaps we should do something analogous to define integrability of a system of PDEs, the symptoms being things like: the existence of nonlinear "superpositions" of solutions; existence of "sufficient" constants of the motion; existence of a "Lax pair"; satisfying analogues of the Painlevé condition for ODEs; etc. The unifying idea behind this book is the suggestion that one particular feature that is particularly important in integrability is the existence of a "twistor construction" for the system. The authors' hope (I am perhaps putting words into their mouths here) is that, if one knew how to say this correctly, it might serve as a single feature characterising integrable systems.

The chain of ideas which has lead to this is as follows. One example of an integrable system is the self-dual Yang-Mills equations for a connection on a vector bundle over $\mathbb{R}^4$. One has a twistor construction for this system (the Ward correspondence) which puts solutions of this system in one-to-one correspondence with a certain set of holomorphic vector bundles over a subset of complex projective 3-space. It is now known that many popular integrable systems of PDEs are equivalent to the Yang-Mills equations with some symmetry group imposed. (The discovery that the KdV and nonlinear Schrödinger equations arise this way is joint work of one of the authors in 1989). This process provides something like a classification scheme for a large family of integrable systems and the authors have done us a favour by presenting this large body of information in a unified way.

The twistor construction for Yang-Mills may also undergo a "symmetry reduction", leading to a "twistor construction" for the reduced system. The nature of this varies widely depending on the signature of the original metric on $\mathbb{R}^4$ and the symmetry group, but it leads to some family of objects in holomorphic geometry being in one-to-one correspondence with the solutions of the system. The authors show how many features of the integrable system can be seen and understood in terms of the twistor constructions.

This is a subject which needed a book firstly to draw together a rather large range of ideas and secondly to gather together systematically a lot of things spread through the literature (and a few useful things that have not previously made it into print). The authors have on the whole done a very good job; doubtless the readiness to retreat into coordinate and index notation will be more popular with mathematical physicists than with mathematicians, but parts of the book are rather easier for a mathematician to approach. Anybody working in integrable systems or in twistor constructions will want a copy of this book or at least want it in their Library.

T. BAILEY

KANAMORI, A. *The higher infinite: large cardinals in set theory from their beginnings* (Perspectives in Mathematical Logic, Springer-Verlag, Berlin–Heidelberg–New York–London–Paris–Tokyo–Hong Kong, 1994), xxiv+536 pp., 3 540 57071 3, £77.50.

For some books it is easy to predict a longevity and a wide readership. This is one of them. The first in a several volume series, the book comprehensively provides an account of the theory of large cardinals from their beginnings to the early 70's, and several of the more important offshoots leading up to the frontiers of current research.

As such it will be invaluable for graduate students and others coming to the field, both for the evolutionary view that it espouses of the various historical confluences to the theory, and the clear insights and perspectives it throws on the pathways to those frontiers.

The first of the seven chapters deals then with those beginnings and several strands of earlier work on large cardinal properties, as well as Gödel's notion of his universe of constructible sets, L, and Scott's seminal result that V could not be L if there was a σ-complete 2-valued measure on an uncountable cardinal number. This phylogenetic account is then developed in the subsequent text, and forms the basis of chapters on measurability, and its relation to embeddings of inner models of set theory; to model theoretic consequences of that measurability, and indiscernibles for first order structures; and to extensions of such embedding properties in terms of large cardinal axioms (that these days are often interpreted as embeddings of the universe V into some inner model of the ZFC axioms with particular properties). A chapter is devoted to the set theory of the reals. This serves to introduce forcing (although the basics of this technique are assumed of the reader). The keywork here was that of Solovay who showed the consistency of all sets of reals being Lebesgue measurable, if one assumed the consistency of an inaccessible cardinal. Forcing and large cardinal connections with properties of the real continuum are laid out. Descriptive set theoretical representations of sets of real numbers are given here (which will be needed for later work in the Chapter on Determinacy). Some may feel that what is commonly called 'Set Theory of the Reals' and the so-called cardinal invariants are given short shrift here (although this reviewer is not amongst them). The reader will find little on inner model theory nor a general survey of forcing consistency results. They are promised in a subsequent volume.

In the eager rush of mathematics that often stylises 'developmental' texts history is often trampled underfoot and, if mentioned, is all too often at the mercy of the *Weltanschauung* of the author, or hastily disposed of in footnotes or an embarrassed appendix. But here there is a sensitive interpretation of the notions of past mathematicians and we hear how their own views coloured their work and the subject's development. The text is bookended by an Introduction which gives an overview of that evolution and its formative influences, and at the end by an appendix in which philosophical discussion has been coralled. The avowed purpose of the latter is to pre-empt, or perhaps defuse, attempts to 'over-metaphysicize' the discussion on mathematical truth, existence, and such concepts that are thrown into sharp relief in the light of any discussion of set theory as a foundation, and even more so when in the blinding glare of large cardinal hypotheses. Quite rightly, it is judged that the autonomy of set theory *qua* mathematical practice is a justification in itself, and that set theory provides an open-ended framework for the interpretation of mathematical systems rather than (a now rather simple view) a reductionist 'foundation' for all mathematics.

The exposition is intelligent and well-paced; misprints are extremely few; as a source book it is a compendium of references, well indexed, and it will become literally the reference book, a Baedeker, for the enquiring student of the subject. It should therefore be on every University Library's mathematical shelf.

P. D. WELCH

SHELAH, S. *Cardinal arithmetic* (Oxford Logic Guides Vol. 29, Clarendon Press, Oxford, 1994), xxxi+481 pp., 0 19 853785 9, (hardback) £65.

Since its inception cardinal arithmetic, or rather the cardinal exponentiation function, has been problematic. Cantor, the founder of set theory, showed (1874) that $2^{\aleph_0} > \aleph_0$, or more generally that $2^{\aleph_\alpha} > \aleph_\alpha$, but was unable to prove the Continuum Hypothesis, that $2^{\aleph_0} = \aleph_1$. Since 1963 we now know why: Cohen showed by his method of forcing that it was consistent with ZFC (the widely accepted axioms of Zermelo-Fraenkel set theory, with the axiom of Choice) that $2^{\aleph_0}$ could be almost anything (the caveat being due to the only other restriction on $2^{\aleph_\alpha}$ known – due to König (1927) – that $cf(2^{\aleph_\alpha}) \neq cf(\alpha)$). (Here cf, or cofinality, of λ is in fact the size of a smallest

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