

## SESSION VII

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In recent years with the advance of large fusion devices, very high temperature plasmas ( $>1$  keV) with long confinement times ( $>$  tens of msec) are produced in the laboratory, offering the possibilities of studying many basic plasma processes of astrophysical interest in laboratory plasmas. In the following, I wish to review some of the recent progress in fusion plasma physics bearing on astrophysical and space plasma processes:

- (i) turbulent relaxation towards the state of minimum magnetic energy;
- (ii) resonant radial diffusion of energetic ions;
- (iii) wave acceleration of electrons to sustain a current in a plasma.

#### I. TURBULENT RELAXATION OF UNSTABLE PLASMAS TOWARDS THE STATE OF MINIMUM MAGNETIC ENERGY IN PINCHES

In toroidal confinement schemes, there are basically two types of discharges characterized by the degree of twist of magnetic field lines, termed "rotational transform,"  $\iota/2\pi$ , which is the average change of the poloidal angle following the field line once around the torus,  $\Delta\bar{\theta}/2\pi$ . In a pinch experiment a toroidal magnetic field  $B_0$  is produced by external coils and then a toroidal current  $I$  is induced. For high currents, the magnetic field line is tightly twisted,  $\iota/2\pi > 1$  and the plasma column is unstable to kink modes, much like a tightly twisted rubber band to form loops. After an initial phase of violent instability, however, the plasma remarkably relaxes to a quiescent state which depends only on the pinch ratio  $\Theta = 2I/aB_0$ , where  $a$  is the plasma radius. For  $\Theta$  exceeds a certain value, the toroidal magnetic field plasma at the plasma edge is reversed in the quiescent state.

Recently, J. B. Taylor proposed an elegant theory,<sup>1</sup> based on the

theory of minimum energy state first developed by Woltjer<sup>2</sup> and successfully explained many of the observations. The essence of the theory is that the plasma seeks a state of minimum magnetic energy

$$W_B = \int_V dV B^2 / 8\pi$$

subject to the constraint that the toroidal and poloidal flux linkage or magnetic helicity over the total volume of the plasma  $V_0$ :

$$K_0 = \int_V dv \vec{A} \cdot \vec{B}$$

be invariant, where  $\vec{A}$  is the vector potential. This is greatly relaxed over the ideal MHD constraint which requires  $K = \int dv \vec{A} \cdot \vec{B}$  to be conserved on each line of force, because of the effect of finite resistivity. By varying  $W$  subject to the constancy of  $K$ :  $\delta(W_B - \lambda K_0) = 0$ , one finds the minimum energy state is a force-free state governed by  $\nabla \times \vec{B} = \lambda \vec{B}$  depending on only one parameter  $\lambda$ . A particular solution is

$$B_r = 0, B_\theta = \alpha J_1(\lambda r), B_z = \alpha J_0(\lambda r),$$

for which  $\lambda a/2 = \theta = 2I/aB_0$ . If  $\theta < 1.2$ , then the toroidal field is reversed in good agreement with experimental observation. This field-reversed quiescent state is maintained over a period of several milliseconds, much longer than the classical skin time for magnetic field diffusion:  $\tau_s = 4\pi a^2/\eta c^2$ , where  $\eta = mv_{ei}/ne^2$  is the plasma resistivity and  $v_{ei}$  is the electron-ion collision frequency, suggesting that perhaps some kind of turbulent dynamo action is at play, counterbalancing the resistive diffusion. This is presently an active area of theoretical and experimental investigation and the understanding of these processes will undoubtedly help us to understand better the phenomenon associated with solar flare and magnetic stars.

## II. RESONANT RADIAL DIFFUSION OF ENERGETIC-IONS FROM TOKAMAKS BY TRAPPED ION-INDUCED INTERNAL KINKS

Tokamaks, unlike pinches, have strong toroidal magnetic fields so that the field line is only weakly twisted:  $1/2\pi < 1$ . However, due to a thermal instability of ohmic heating during which the current tends to peak where the temperature is high resulting in a current distribution peaked at the center, and a local region near the center where  $1/2\pi > 1$ . The kink instability can then develop in this localized region and is called "internal kink." Nonlinearly this instability causes the electron temperature and current in the unstable central region to be flattened so that  $1/2\pi \sim 1$  at center. This unstable buildup and relaxation of the electron temperature is observed in the "sawtooth"-like variation of x-rays emitted from a tokamak plasma.<sup>3</sup>

Ohmic heating alone, however, is insufficient to heat a tokamak plasma to the temperatures required for thermonuclear ignition because of the decreasing plasma resistivity with increasing temperature and the amount of current limited by  $i/2\pi < 1$ . Thus, auxiliary heating schemes employing neutral beam or radio frequency waves are needed. In neutral beam heating, energetic neutral atoms ( $>40$  keV) were injected into a tokamak plasma which, upon ionization and charge exchange, become energetic ions in the plasma. These energetic ions are then slowed down by Coulomb collisions with background plasma which in turn are heated by the energetic ions. The neutral beam injection has been quite successful as a way of plasma heating and a record in temperature of 7 keV was attained in Princeton Large Torus (PLT) several years ago with neutral beam injection of several megawatt of power into the plasma.<sup>4</sup>

Recently up to 7 MW of neutral beam power is injected almost perpendicular to the magnetic field into a plasma in the Princeton Divertor Experiment (PDX). Above certain injection power, the efficiency of plasma heating is reduced. One reason for the decreased efficiency is the observed ejection of energetic ions from the tokamak, associated with the observed oscillation of the poloidal magnetic field and electron temperature, at the  $\nabla B$  drift frequency of the energetic ions around the torus ( $\approx 20$  kHz), so-called "fish-bones."<sup>5</sup> This oscillation has been interpreted as an induced internal kink mode destabilized by the resonance with the drift of trapped energetic ions,

$$\omega \approx \omega_d = cE/eBR ,$$

where  $E$  is the energy of the ions,  $R$  is the major radius. The resonance between the wave and the drift then causes the energetic ions to diffuse radially outward, successively interacting with different poloidal modes coupled by toroidicity, each peaked at different radii, eventually ejected out of the tokamak. White et al.<sup>6</sup> have obtained very good agreement between theory and experiment. This is similar to the mechanism for the building up of the ring current belt of energetic protons ( $>40$  kW) in the earth's magnetosphere.<sup>7</sup> During magnetic storms the energetic ions from solar wind come into the magnetosphere when the particle drift frequency once around the earth is in resonance with the frequency of external magnetic perturbation associated with the magnetic storm. Such interaction would preserve the invariants of the magnetic moment  $\mu = mv^2/2B$  and  $J = \int mv_{\parallel} dl$  while diffusing the ions radially inward. Such  $\mu$   $J$ -conserving drift-resonant diffusion process can simply be described with the drift kinetic equation for the guiding centers in a magnetic field  $\vec{B} = \nabla\psi \times \nabla\theta$ .<sup>8</sup> For total energy  $H = (1/2)mv_{\parallel}^2 + \mu B + e\phi$  implicitly expressed as function of  $\mu, J, \psi, \theta$ , through

$$J(\mu, H, \psi, \theta) = \int dl [2m(H - \mu B - e\phi)]^{1/2} .$$

The drift equations of motion in  $\psi$  (radial) and  $\theta$  (azimuthal) are Hamiltonian equations

$$\dot{\psi} = -\frac{c}{e} \frac{\partial H}{\partial \theta} \Big|_{\mu, J, \psi} \quad \dot{\theta} = \frac{c}{e} \frac{\partial H}{\partial \psi} \Big|_{\mu, J, \theta} .$$

The Vlasov equation in the drift approximation for the guiding center distribution  $F(\mu, J, \psi, \theta, t)$  is

$$\frac{\partial F}{\partial t} + \frac{c}{e} \left[ \frac{\partial H}{\partial \psi} \frac{\partial F}{\partial \theta} - \frac{\partial H}{\partial \theta} \frac{\partial F}{\partial \psi} \right] = 0 .$$

For a perturbed magnetic and electric field  $\delta B$  and  $\delta \phi$  with conservation of  $J$ :  $\delta J = 0$ , there is a corresponding

$$\delta H = \int \frac{d\ell}{v_{\parallel}} (\mu \delta B + e \delta \phi) / \int d\ell / v_{\parallel} = \langle \mu \delta B + e \delta \phi \rangle .$$

For perturbations periodic in  $\theta$ , we may Fourier analyze

$$\delta H = \sum_n \delta H_n \exp(in\theta - i\omega t)$$

and

$$\delta F = \sum_n \delta F_n \exp i(n\theta - \omega t) ,$$

and linearized Vlasov equation gives

$$\delta F_n = \left( \frac{c}{e} H_n \frac{\partial F_0}{\partial \psi} \right) / (n\omega_d - \omega) .$$

The ensemble averaged distribution  $F_0(\mu, J, \psi)$  then evolves according to quasilinear equation

$$\frac{\partial F_0}{\partial t} = \frac{\partial}{\partial \psi} D \frac{\partial F_0}{\partial \psi} ,$$

where

$$D(\mu, J, \psi) = \frac{c^2}{2} \sum_n |H_n|^2 \pi \delta(\omega - n\omega_d) .$$

Note that the diffusion coefficient is nonvanishing only for resonant

particles with  $n\omega_d(\mu, E, \psi) = \omega$ . For spatially overlapping modes with the same frequency, large scale radial excursion is possible only for particles with  $\omega_d$  weakly depending on  $\psi$ , which is indeed the case in PDX tokamak.

### III. ELECTRON ACCELERATION BY LOWER HYBRID WAVES

Present tokamak operation is pulsed in nature because the toroidal current is driven by an inductive electric field, which is undesirable for an electricity-generating power plant requiring more or less continuous operation. In an attempt to operate a steady-state tokamak, it has been proposed that radio frequency waves can be used to drive a current by imparting wave momentum directly to electrons. Recent experiments on PLT and other devices showed that a plasma current of several hundred kiloamperes, carried mostly by energetic electrons can be sustained for several seconds by lower hybrid (LH) waves of a few hundred kilowatts of rf power.<sup>9</sup> There are several interesting features of the PLT experiment:

1. The launched LH waves have very high phase-velocity,  $\omega/k_{\parallel}$ , along the magnetic field. Typically the energy spectrum of the wave peaks at  $\omega/k_{\parallel} \approx c/2$ , where  $c$  is the speed of light.
2. Hard x-ray emission ( $E > 30$  keV) shows marked increase with RF on, with maximum energy comparable to  $m/2 (\omega/k_{\parallel})^2$  and mean energy about 150 keV. The mean energy as well as the maximum energy of electrons increase with increasing phase velocity.
3. Concurrent with the appearance of the energetic electrons, there is enhanced plasma wave radiation near the plasma frequency, which is much higher than the launched LH frequency.

The acceleration of the electrons to high energy is accomplished by the combined action of the initial ohmic electric field which can accelerate electrons to energies about 10 keV before the LH wave is launched and the LH waves with a minimum phase velocity in resonance with these accelerated electrons can further accelerate them to the maximum phase velocity  $\sim c/2$ . This process can be described by solving the Fokker-Planck, quasilinear equations

$$\frac{\partial f(v_{\parallel}, t)}{\partial t} = \frac{\partial}{\partial v_{\parallel}} D \frac{\partial f}{\partial v_{\parallel}} + \frac{e}{m} E_{\parallel} \frac{\partial f}{\partial v_{\parallel}} + C(f) ,$$

where

$$D = \frac{\pi e^2}{m} \sum |\tilde{E}_{\parallel}|^2 \delta(\omega - k_{\parallel} v_{\parallel})$$

is the quasilinear diffusion coefficient due to waves,  $E_{\parallel}$  is the ohmic

electric field, and  $C(f)$  is the Coulomb collision term. Eventually a steady-state is established when the collisional drag balances the wave acceleration. The resulting distribution, however, has very large anisotropy with average parallel energy  $T_{\parallel}$  much greater than perpendicular  $T_{\perp}$ :  $T_{\parallel} \gg T_{\perp}$ , because the electrons are accelerated by the LH waves primarily along the magnetic field. This anisotropic distribution of energetic electrons can destabilize plasma waves  $\omega = \omega_p k_{\parallel}/k$  via the anomalous Doppler resonance  $\omega + \Omega = k v_{\parallel}$ ,<sup>9</sup> where  $\Omega = eB/mc$  is the electron gyrofrequency, typically larger than the plasma frequency  $\omega_p$ .

As a result of this instability, lower phase velocity but higher-frequency plasma waves are excited resulting in copious plasma radiation observed during the experiment. The unstable plasma waves in turn scatter the energetic electrons in pitch angle to increase their perpendicular energies, leading to enhanced cyclotron radiation in x-mode. When this process becomes sufficiently strong, relaxation oscillation in cyclotron radiation at the fundamental frequency, (ordinary mode measuring  $T_{\perp}$ ) out of phase with that at the 2nd harmonic (x-mode measuring  $T_{\parallel}$ ) along the burst of radiation at plasma frequency, qualitatively described by the nonlinear theory using the moment equations of the quasilinear equation for  $T_{\parallel}$ ,  $T_{\perp}$ , and  $W = |\tilde{E}|^2/\delta\pi$  the energy density of the plasma waves.<sup>10</sup> Similar processes may also occur in the Bow shock of the earth's magnetosphere.

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