CORRESPONDENCE.

LIMITED PAYMENT POLICIES. MORTALITY AND SPECIAL RESERVES.

To the Editors of the Journal of the Institute of Actuaries.

Sirs,—In the discussion on Mr. Todhunter's paper submitted in March there were remarks by various speakers, particularly Mr. R. G. Salmon, on the variations in the mortality rates under different classes of assurance, and Mr. Salmon called in question the value of certain figures I had supplied to Mr. Todhunter, of which only the final results and not the details were given. As the matter is one which appears to be creating a good deal of interest at the present time, it is possible that further details of the investigation, which I made three years ago, may be considered of sufficient value to be inserted in the *Journal*, and I accordingly send short summaries of the actual and expected deaths, grouped in different ways. The figures are based on the experience of the Scottish Life Assurance Company, and although the numbers involved are small when compared with those included in modern investigations, yet the results seem to me to be remarkably regular throughout the groups, and therefore to be worthy of confidence. The experience was extracted by policies—or rather by medical examinations—and included only male lives insured at the ordinary rates under participating policies in the various classes effected during the thirty years 1881 to 1910, and it terminated with the anniversaries in 1911 of existing policies. Reassurances from other Offices were excluded, as it was found that the rates of mortality amongst them were considerably higher than under direct policies. The expected deaths were based throughout on the $O^{[M]}$ Table.

In the summaries arranged by ages at entry and durations the percentages become increasingly good as the class of policy proceeds from whole life to limited payment and to endowment assurance. Mr. Salmon referred in the discussion to the high ratios found in the whole life class at the older ages in another investigation upsetting the comparison of the total results, and while this will no doubt always be the case to a certain extent where a larger body of whole life policyholders reaches the higher ages, the figures in the third summary, arranged by ages attained, indicate throughout heavier percentages for the whole life class. The differences are not great up to age 40, but thereafter the other classes, especially the endowment assurances, seem to show a greater staying power. Another point made in the discussion was that the better mortality shown amongst limited payment and endowment assurance policies might be due to these classes being more recent, and accordingly showing more the modern improved mortality amongst policyholders generally. This may, perhaps, have had something to do with the very light mortality shown above in the endowment assurances, but as regards the limited payment class, the policies were on the average older than the

I.—*Summary arranged by Ages at Entry.*

Ages at Entry	Whole Life			Limited Payments			Endowment Assurances		
	Actual Deaths	Expected Deaths	Percentages	Actual Deaths	Expected Deaths	Percentages	Actual Deaths	Expected Deaths	Percentages
15–34	57	98·79	58	61	115·05	53	130	246·09	53
35–54	101	125 76	80	59	87·46	67	53	109·25	49
55 and over	31	31·13	100	8	6 34	126	1	·51	197
Total	189	255·67	74	128	208·85	61	184	355·85	52

II.—*Summary arranged by Durations.*

Durations	Whole Life			Limited Payments			Endowment Assurances		
	Actual Deaths	Expected Deaths	Percentages	Actual Deaths	Expected Deaths	Percentages	Actual Deaths	Expected Deaths	Percentages
0– 4	51	72·51	70	27	45·19	60	81	149·78	54
5– 9	45	63·03	71	30	49·79	60	49	110·75	44
10–14	37	48·24	77	25	43·69	57	31	58·13	53
15–19	29	38·21	76	26	34·95	74	16	25·69	62
20–24	19	24·93	76	14	26·83	52	6	10·05	60
25–29	8	8·75	91	6	8·40	71	1	1 16	86
Total	189	255·67	74	128	208·85	61	184	355·85	52

III.—*Summary arranged by Ages attained.*

Ages Attained	Whole Life			Limited Payments			Endowment Assurances		
	Actual Deaths	Expected Deaths	Percentages	Actual Deaths	Expected Deaths	Percentages	Actual Deaths	Expected Deaths	Percentages
Up to 29	9	17·28	52	7	13 90	50	36	74·63	48
30–39	27	48·69	55	28	49·98	56	70	138·77	50
40–49	45	64·46	70	38	66·28	57	58	96·26	60
50–59	50	61·76	81	39	55·93	70	19	41·29	46
60–69	33	41·71	79	14	19·30	73	1	4·89	20
70–79	22	18·87	117	2	2·84	70	..	..	. .
80–89	3	2·91	103	...	·62	...	...	...	.
Total	189	255·67	74	128	208·85	61	184	355·85	52

corresponding whole life policies, owing to a large proportion of the business of the office in its early years having been done under the former class.

I had not an opportunity before the date of the meeting of

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looking fully into the method suggested by Mr. Todhunter for making the special reserve required for limited payment policies, as compared with that proposed by me in the *Transactions of the Faculty of Actuaries* (vol. vi, page 93) to which Mr. Todhunter refers in the footnote at page 263, and I may, perhaps, be allowed to make the following remarks.

The formula given by me for the requisite addition to the net premium reserve was (using Mr. Todhunter's notation)

$$\{{}^{\prime}\mathrm{P}_x(1-\kappa)\mathbf{a}^{(j)}_{x+n} - \mathrm{P}_x\mathbf{a}_{x+n}\} - \{{}_t{}^{\prime}\mathrm{P}_x(1-\kappa)\mathbf{a}^{(j)}_{x+n\overline{t-n|}} - {}_t\mathrm{P}_x\mathbf{a}_{x+n\overline{t-n|}}\}$$

which, making obvious substitutions, is practically identical with Mr. Todhunter's "commutation-basis" formula. The main difference is that Mr. Todhunter restricts his formula to the case of the office premiums being valued by the rates of mortality and interest used in obtaining the limited payment from the whole life premiums, while I showed that the formula was perfectly general when the office premiums are valued by the rates of mortality and interest expected to be experienced by the office in the future.

The Text-Book formula gives reserves much too high throughout, and it seems almost a pity that Mr. Todhunter departs from his original formula and recommends a modification which, although very ingenious, not only entails the use of hypothetical office premiums, but reproduces the Text-Book formula after the premiums are paid up. This weighs heavily on policies by a small number of payments, and particularly (as Mr. Todhunter himself points out) on single payment business, the strain in the latter case being 4 per-cent to 5 per-cent of the sum assured, as shown in the tables appended to my note. The use of the correct formula involves a certain amount of additional work at a valuation, but in these days of classified registers and calculating machines, the time taken is in my opinion much more than counterbalanced by the truer results obtained.

Yours faithfully,

ALEX. FRASER.

19 *St. Andrew Square, Edinburgh,*
8 *September* 1915.

[With reference to the concluding paragraph of Mr. Fraser's letter, Mr. Todhunter states that the "commutation-basis" formula, given in his paper as representing on certain assumptions the true value of a limited-payment policy, was intended to be used, not as a practical valuation formula, but as a standard of comparison—with due regard to the validity of the underlying assumptions—for the other formulas.

The formula merely expresses the fact that a limited-payment

policy may be regarded as a combination of two contracts, viz.: an ordinary whole-life policy and a premium-commutation contract, and although it could no doubt be used for valuation purposes as suggested by Mr. Fraser in his Transactions of the Faculty note, a question would arise as to the basis on which the premium-commutation contract should be valued. That basis should be a "future experience" basis—not necessarily the basis employed in commuting the premium—and it might be considered that the only course consistent with the principles on which the general valuation is made would be to employ the basis used in valuing other (non-profit) contracts. But the effect of this would, in general, be to reproduce the Text-Book formula.—EDS. *J.I.A.*]

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