

A theoretical model of torsional oscillations from a flux transport dynamo model

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Abstract. Assuming that the torsional oscillation is driven by the Lorentz force of the magnetic field associated with the sunspot cycle, we use a flux transport dynamo to model it and explain its initiation at a high latitude before the beginning of the sunspot cycle.

Keywords. Flux transport dynamo, magnetic field, sunspot cycle

1. Introduction

The small periodic variation in the Sun's rotation with the sunspot cycle, first discovered on the solar surface by Howard & LaBonte (1980), is called torsional oscillations. Helioseismology has now established its existence throughout the convection zone (see Howe *et al.* 2005 and references therein). Its amplitude near the surface is of order 5 m s^{-1} or about 1% of the angular velocity. Apart from the equatorward-propagating branch which moves with the sunspot belt, there is also a poleward-propagating branch at high latitudes. One intriguing aspect of the equatorward-propagating branch is that it begins a couple of years before the sunspots of a particular cycle appear and at a latitude higher than where the first sunspots are seen. The top panel of Fig. 1 shows the torsional oscillations at the solar surface with the butterfly diagram of sunspots. If the torsional oscillation is caused by the Lorentz force of the dynamo-generated magnetic field as generally believed, then the early initiation of this oscillation at a higher latitude does look like a violation of causality! Our main aim is to explain this which could not be explained by the earlier theoretical models (Durney 1980; Covas *et al.* 2000; Bushby 2006; Rempel 2006). The details of our work can be found in a recent paper (Chakraborty, Choudhuri & Chatterjee 2009a, hereafter CCC). Please note that this paper has an erratum (Chakraborty, Choudhuri & Chatterjee 2009b).

2. Theoretical model

The flux transport dynamo model first developed by Choudhuri, Schüssler & Dikpati (1995) appears to be the most promising model for explaining the sunspot cycle. We use the model presented by Chatterjee, Nandy & Choudhuri (2004). Some details of the model with the basic equations can be found in Choudhuri (2011). In order to model

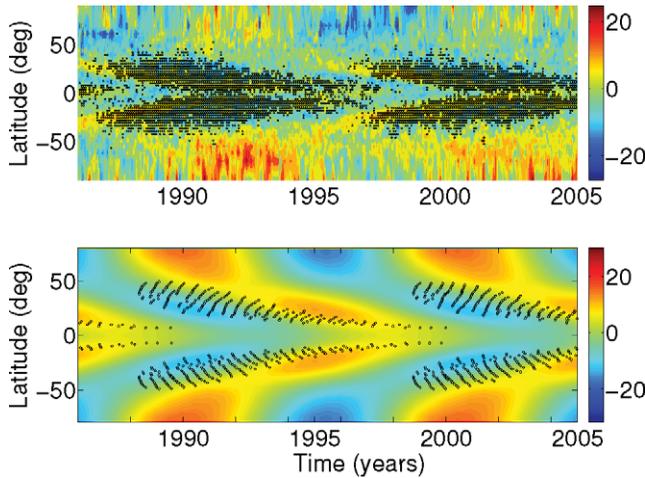


Figure 1. The time-latitude plot of torsional oscillation on the solar surface with the butterfly diagram of sunspots superposed on it. The upper panel is based on observational data of surface velocity v_ϕ measured at Mount Wilson Observatory (courtesy: Roger Ulrich). The bottom panel is from our theoretical simulation.

torsional oscillations, in addition to the basic equations of the dynamo, we simultaneously have to solve the Navier–Stokes equation in the form

$$\rho \left\{ \frac{\partial v_\phi}{\partial t} + D_v[v_\phi] \right\} = D_\nu[v_\phi] + (\mathbf{F}_L)_\phi, \quad (1)$$

where $D_v[v_\phi]$ is the term corresponding to advection by the meridional circulation, $D_\nu[v_\phi]$ is the diffusion term, and $(\mathbf{F}_L)_\phi$ is the ϕ component of the Lorentz force. If the magnetic field is assumed to have the standard form

$$\mathbf{B} = B(r, \theta, t)\mathbf{e}_\phi + \nabla \times [A(r, \theta, t)\mathbf{e}_\phi], \quad (2)$$

then the Lorentz force is given by the Jacobian

$$4\pi(\mathbf{F}_L)_\phi = \frac{1}{s^3} J \left(\frac{sB_\phi, sA}{r, \theta} \right), \quad (3)$$

where $s = r \sin \theta$. On the basis of flux tube simulations suggesting that the magnetic field in the tachocline should be of order 10^5 G (Choudhuri & Gilman 1987; Choudhuri 1989; D’Silva & Choudhuri 1993), it is argued by Choudhuri (2003) that the magnetic field has to be intermittent in the tachocline. Hence the full expression of Lorentz force involves a filling factor as explained by CCC.

Our theoretical model incorporates a hypothesis proposed by Nandy & Choudhuri (2002), which is essential for explaining the early initiation of the torsional oscillation at high latitudes. According to this Nandy–Choudhuri (NC) hypothesis, the meridional flow penetrating in stable layers below convection zone causes formation of toroidal field in high latitude tachocline. Sunspots form a few years later when this field is advected to lower latitudes and brought inside convection zone. We also assume that the stress of the magnetic field formed in the tachocline is carried upward by Alfvén waves propagating along vertical flux concentrations conjectured by Choudhuri (2003).

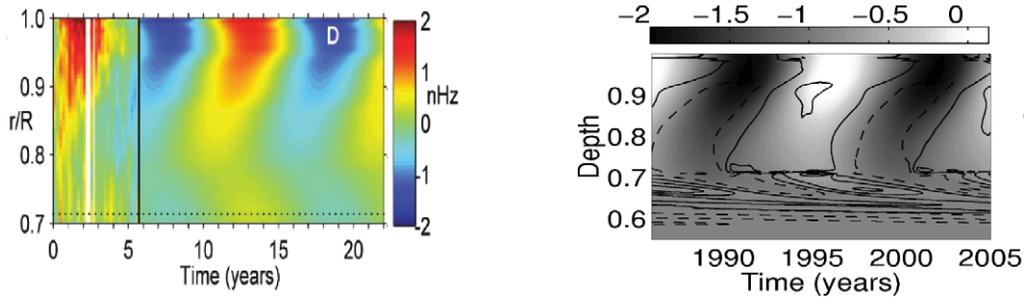


Figure 2. The depth-time plot of torsional oscillations at latitude 20° . The left panel from Vorontsov *et al.* (2002) is based on SOHO observations, whereas the right panel from CCC is based on our theoretical simulation. The solid and dashed lines in the right panel indicate the Lorentz force (positive and negative values respectively).

3. Results of simulation

The incorporation of the NC hypothesis in our theoretical model causes magnetic stresses to build up at higher latitudes before sunspots of the cycle appear, leading to the early initiation of torsional oscillations. The bottom panel of Fig. 1 shows theoretical results of torsional oscillations at the surface with the theoretical butterfly diagram. This bottom panel can be compared with the observational upper panel in Fig. 1. Our theoretical model also gives a satisfactory account of the evolution of torsional oscillations within the convection zone. The depth-time plot of torsional oscillations at a certain latitude given in Fig. 3 of CCC compares favourably with the observational plot given in Fig. 3(D) of Vorontsov *et al.* (2002). This is reproduced in Fig. 2 for completeness.

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