

# Preliminary investigation of the gravitomagnetic effects on the lunar orbit

X. M. Deng

Purple Mountain Observatory, Nanjing 210008, China  
email: dengxuemei403@163.com

**Abstract.** Gravitomagnetic effects are studied in this paper. Starting from the metric in the BCRS and then using the matching method, the metric in the GCRS are derived. Furthermore, we give some estimation of the order of the gravitomagnetic effects on the lunar orbit.

**Keywords.** reference systems, Moon

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## 1. Introduction

The magnetic field is produced by the motion of electric-charge. The close analogy between the Einstein's field equation and the Maxwell's equation led many authors to investigate whether or not the mass current would produce what is so-called the gravitomagnetic field. But, first of all, we must clarify the definition of the gravitomagnetic field. Generally, one of the Maxwell's equation has the following form

$$\nabla \times \vec{B} = \vec{J} + \frac{\partial \vec{E}}{\partial t} \quad (1.1)$$

where not only the electric-charge but also the variation of  $\vec{E}$  with respect to time can produce the magnetic field. So, not only the mass current (the spins of celestial bodies) but also the potentials's change with time such as the third body's effect can produce the gravitomagnetic field. Recently, Murphy *et al.* (2007a, 2007b) and Kopeikin (2007) have a go-round about whether the Lunar Laser Ranging (LLR) is currently capable to detect the gravitomagnetic effect. We will give some results of our work about this problem.

## 2. Our works

In our model, the Sun and the Earth are massive point-particles. The Moon is considered as a massless test particle. Based on the International Astronomical Union (IAU) resolution for the relativistic reference systems in 2000, we employ the barycentric celestial reference systems (BCRS) and the geocentric celestial reference systems (GCRS) with two PPN parameters  $\beta$  and  $\gamma$  and the harmonic gauge (Klioner and Soffel, 2000). The metric respectively in the BCRS and GCRS is

$$\begin{aligned} g_{00}(t, \vec{x}) &= -1 + 2\epsilon^2 w(t, \vec{x}) - 2\beta\epsilon^4 w^2(t, \vec{x}) + \mathcal{O}(\epsilon^5), \\ g_{0i}(t, \vec{x}) &= -2(1 + \gamma)\epsilon^3 w^i(t, \vec{x}) + \mathcal{O}(\epsilon^5), \\ g_{ij}(t, \vec{x}) &= \delta_{ij}(1 + 2\epsilon^2 \gamma w(t, \vec{x})) + \mathcal{O}(\epsilon^4), \end{aligned} \quad (2.1)$$

and

$$\begin{aligned} G_{00} &= -1 + 2\epsilon^2 W(T, \vec{X}) - 2\epsilon^4 \beta W^2(T, \vec{X}) + \mathcal{O}(\epsilon^5), \\ G_{0a} &= -2\epsilon^3 (1 + \gamma) W^a(T, \vec{X}) + \mathcal{O}(\epsilon^5), \end{aligned}$$

$$G_{ab} = \delta_{ab}(1 + 2\epsilon^2\gamma W(T, \vec{X})) + \mathcal{O}(\epsilon^4), \quad (2.2)$$

where  $\epsilon = 1/c$ . Procedures we adopt are as the following:

- (a) Derive the potentials  $w$  and  $w^i$  in  $g_{\mu\nu}$  under the BCRS;
  - (b) Match the potentials between the BCRS and the GCRS and obtain  $W$  and  $W^a$  in the GCRS;
  - (c) Derive the equation of motion for the moon in the GCRS by using the geodesic principle;
  - (d) Derive the ranging time from the light equation;
  - (e) Compute the observable quantity – proper laser ranging time.
- Among them, (d) and (e) are in workings.

### 3. Discussion

Based on our current works, the gravitomagnetic acceleration produced by the Earth's spin is

$$\vec{a}_{GMES} = (1 + \gamma)G \left[ \frac{\vec{V} \times \vec{J}_{\oplus}}{r^3} + 3\frac{\vec{r} \times \vec{V}}{r^5}(\vec{r} \cdot \vec{J}_{\oplus}) \right] \sim 10^{-16} \vec{a}_{New}, \quad (3.1)$$

the gravitomagnetic acceleration caused by the Sun's angular momentum is

$$\vec{a}_{GMSS} \sim -2(1 + \gamma)\frac{G}{R^3}(\vec{V} \times \vec{J}_{\odot}) \sim 10^{-14} \vec{a}_{New}, \quad (3.2)$$

and the gravitomagnetic acceleration due to the third body's effect from the Sun is

$$\vec{a}_{GMSP} \sim (1 + \gamma) \left[ 2\frac{Gm_{\odot}}{R^3}v^2\vec{r} - 3G\frac{\vec{r} \times \vec{J}_{\odot}}{R^5}(\vec{R} \cdot \vec{v}) \right] \sim 10^{-11} \vec{a}_{New}, \quad (3.3)$$

where the Newtonian term is

$$\vec{a}_{New} = -\frac{Gm_{\oplus}}{r^3}\vec{r}. \quad (3.4)$$

and  $v$  is the Earth's velocity around the Sun; the Moon's velocity around the Earth is  $V$ ; the mass of the Sun is  $m_{\odot}$ ; the mass of the Earth is  $m_{\oplus}$ ; the distance between the Sun and the Earth is  $R$ ; the distance between the Earth and the Moon is  $r$ ;  $J_{\oplus}$  and  $J_{\odot}$  are respectively the spins of the Earth and the Sun.

Although  $\vec{a}_{GMSP}$  has some effect on the Earth-Moon distance on the order of a millimeter, which could be measured by the next generation LLR, it is a coordinate-dependent quantity instead of an observable one. To remove the residual gauge freedom, we will calculate the proper time for the LLR in the next move.

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