Foundations of Linear Algebra, by A. I. Mal' cev; 2nd edition (of 1956) translated by Thomas Craig Brown, edited by J. B. Roberts. San Francisco, W. H. Freeman, 1963. xi + 304 pages. $7.50.

This book is a masterful presentation of its subject matter; it differs from most Russian textbooks in that problems and exercises are included in the present work. There are numerical exercises the reader can use to verify that he has assimilated the material, routine exercises of various types that test understanding of the definitions, and some exercises in every chapter that contain significant results not covered elsewhere in the text.

In accordance with modern practice, the language used by the author to expound linear algebra is chiefly geometric: invariant subspaces; spectral decomposition; symmetric transformation; symplectic space. While not encyclopedic, the book does cover its material very thoroughly, and is also satisfying in the inclusion of comforting motivations for all the results presented. This fact allows the author to omit a bibliography; in fact he mentions the names of no more than eight or ten mathematicians in the entire book. The translators have rendered the author' s thoughts faithfully.

The eight chapters are: Matrices; Linear Spaces; Linear Transformations; Polynomial Matrices; Unitary and Euclidean Spaces; Bilinear and Quadratic Forms; Linear Transformations of Bilinear-metric Spaces; Multilinear Functions: Tensors.

Chapters 1 and 2 are elementary. Chapter 3, Linear Transformations, includes a section "Root subspaces" that could be used as a foundation for the derivation of the canonical form of a linear transformation over an algebraically closed field. If E is the identity, and A is a linear transformation, a root subspace of A corresponding to the proper value ρ is the set of all vectors x annihilated by some iterate of the transformation $A - \rho E$. [In point of fact, the author bases his discussion of the Jordan form on the theory of elementary divisors. In exercises, and in sections that discuss the canonical form of a linear transformation of a vector space over a field that is not algebraically closed, the author uses more powerful and general methods.] One of the author' s canonical forms seems to be slightly novel; it is the form

$$\begin{array}{cccc} A & E & & \\ & A & E & \\ & & \ddots & \\ & & & A \end{array},$$

where A is a square matrix (box) with irreducible characteristic polynomial, and E is the identity matrix of the same dimension.

Besides conventional material, Chapter IV includes a discussion of "Matrices permutable with given matrices", and "Matrices permutable with every matrix that is permutable with A". The proofs in these sections are of moderate length, and in a pleasant style. The author mentions that a transformation permutable with every endomorphism of the n-dimensional space must be scalar; the same conclusion follows if "endomorphism" is replaced by "isomorphism".

The author's treatment of inner product spaces, of the structure of unitary, symmetric, antisymmetric, and complex-symmetric transformations is thorough. Besides this, the chapter on linear transformations of bilinear metric spaces is superb, and gives a clear exposé as well as a satisfying motivation of the Wellstein theory. The last 50-page chapter on multilinear functions and tensors is more than an introduction to the subject; it is a revelation of some of its aspects.

The translators have appended a brief index.

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<u>Linear Algebra and Matrix Theory</u>, by Evar D. Nering. John Wiley and Sons, Inc., New York, 1963. xi + 289 pages.

The chapter titles are: I Vector spaces. II Linear transformations and matrices. III Determinants, eigenvalues, and similarity transformations. IV Linear functionals, bilinear forms, quadratic forms. V Orthogonal and unitary transformations, normal matrices. VI Selected applications of linear algebra. The applications in Chapter VI are a feature of this book. They include: vector geometry (with some mention of convex sets), finite cones and linear inequalities, linear programming, the finite sampling theorem in communications theory, spectral decomposition of a linear transformation, systems of linear differential equations, small oscillations of mechanical systems, and representations of finite groups by matrices. This material accounts for one quarter of the book.

The author states that "the underlying spirit of this treatment of the theory of matrices is that of a concept and its representation". This theme is kept constantly before the reader. It is emphasized that matrices can and do represent different things in different contexts, and that formal manipulation of matrices without an understanding of the underlying concepts can lead to disaster.