ON THE KERNELS OF REPRESENTATIONS OF FINITE GROUPS II

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Dedicated to Professor Hiroyuki Tachikawa on his 60th birthday

1. Introduction. About fifteen years ago I. M. Isaacs and S. D. Smith [**9**] gave several character-theoretic characterizations of finite p-solvable groups G with p-length one, where p is a prime number. They proved that for a finite group G with a Sylow p-subgroup P, the following four conditions (a)–(d) are equivalent.

(a) G is p-solvable of p-length one.

(b) Every irreducible complex representation in the principal p-block $B_0(G)$ of G restricts irreducibly to the normalizer $N_G(P)$ of P in G.

(c) Every irreducible complex representation of degree prime to p in $B_0(G)$ restricts irreducibly to $N_G(P)$.

(d) Every irreducible modular representation in $B_0(G)$ restricts irreducibly to $N_G(P)$.

We generalized this to an arbitrary p-block B of a finite group G in our previous paper [**11**]. As a matter of fact, we there gave character-theoretic characterizations of B such that

(1) the defect group D of B is contained in the intersection of the kernels of all irreducible modular representations in B.

The purpose of this note is to complement this in [**11**]. In order to describe it, we need the following notation. Let F be an algebraically closed field of characteristic $p>0$, and $B \leftrightarrow e_B$ a p-block of a finite group G, that is, B is an indecomposable two-sided ideal and a direct summand of the group algebra FG and e_B is a centrally primitive idempotent of FG such that $B = e_B FG = FGe_B$. We write $J(R)$ for the Jacobson radical of a ring R. We denote by $\mathrm{Irr}_i(B)$ the set of all irreducible ordinary characters in B of height i. The other notation is the same as in [**11**]. Recall the definitions of the notation N_B and N_B^*, namely, N_B and N_B^* are respectively the intersections of the kernels of all irreducible complex and modular representations in B. Now we can state our main result.

THEOREM (see [**11**, Theorem]). *Let $B \leftrightarrow e_B$ and $b \leftrightarrow e_b$ be respectively p-blocks of G and N with defect group D which correspond through the Brauer correspondence, that is, $b^G = B$, where $N = N_G(D)$. Then the following conditions are equivalent to* (1)–(7) *in* [**11**, *Theorem*].

(8) $N_b^* \subseteq N_B^*$.

(9) $N_b^* = N_B^* \cap N$.

(10) *The correspondence $\sigma : b \to B$ given by $\sigma(x) = xe_B$ for each $x \in b$ is an isomorphism of F-algebras such that $\sigma(e_b) = e_B$.*

(11) *$J(B)^n = B \,.\, J(b)^n = J(b)^n B = b \,.\, J(B)^n = J(B)^n b$ for any positive integer n and $e_B e_b = e_b e_B = e_B$.*

(12) *The correspondence $\alpha : \mathrm{Irr}(B) \to \mathrm{Irr}(b)$ given by $\alpha(\chi) = \chi_N$ is a bijection.*

(13) *The correspondence $\alpha : \mathrm{Irr}_0(B) \to \mathrm{Irr}_0(b)$ given by $\alpha(\chi) = \chi_N$ is a bijection.*

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(14) *The correspondence* $\alpha: \mathrm{Irr}_i(B) \to \mathrm{Irr}_i(b)$ *given by* $\alpha(\chi) = \chi_N$ *is a bijection for all* $i \geqq 0$.

(15) *The correspondence* $\beta: \mathrm{IBr}(B) \to \mathrm{IBr}(b)$ *given by* $\beta(\phi) = \phi_N$ *is a bijection.*

(16) $J(FD)B \subseteq J(B)$.

(17) $J(FD)B = J(B) = B \,.\, J(FD)$.

(18) $G = N \,.\, N_B^* = N \,.\, N_B$.

(19) *Any simple FG-module in B has D as its vertex, and the FG-module* $(F_D)^G \,.\, e_B$ *is completely reducible (semi-simple), where* $(F_D)^G$ *is the induced FG-module from the trivial FD-module* F_D.

(20) *There exists a projective indecomposable FG-module P in B such that* $D \subseteq \mathrm{Ker}(S)$ *for any composition factor S of P.*

(21) *There exists a projective indecomposable FG-module P in B such that* S_N *is simple for any composition factor S of P, where* S_N *is the restriction of S to FN.*

We use the following notation as well. Let $k_i(B) = |\mathrm{Irr}_i(B)|$ and $l(B) = |\mathrm{IBr}(B)|$, where $|X|$ denotes the number of elements in a set X. If H is a subgroup of G, X_H denotes the restriction of X to FH for an FG-module X and Y^G denotes the induced FG-module from an FH-module Y. For an FG-module X and a positive integer n, nX denotes the direct sum $X \oplus \ldots \oplus X$ (n times). Let U and V be FG-modules. We write $(U, V)^G$ and $[U, V]^G$ for $\mathrm{Hom}_{FG}(U, V)$ and $\dim_F \mathrm{Hom}_{FG}(U, V)$, respectively. For a subgroup H of G, we write $(U, V)_H^G$ for $\mathrm{Tr}_H^G[(U_H, V_H)^H]$, where Tr_H^G is the trace map from $(U_H, V_H)^H$ to $(U, V)^G$ (see [**6**, p. 87]). We write $V \mid U$ if V is (isomorphic to) a direct summand of U. We write $P(U)$ for the projective cover of U. We denote by F_G the trivial FG-module. For a subgroup H and $g \in G$, H^g denotes $g^{-1}Hg$.

It is easy to see that our theorem implies the result of Isaacs and Smith [**9**] since $N_{B_0}^* = O_{p',p}(G)$. Moreover, a number of results now follow from our theorem.

COROLLARY 1. *The results of Isaacs and Smith* [**9**, *Theorems* 2 *and* 4] *and Pahlings* [**17**, *Theorem* 5] *follow from the theorem.*

In [**1**], J. L. Alperin stated a conjecture relating the number $l(B)$ and the number of "weights" in B. For finite groups satisfying the conditions of the theorem here, Alperin's conjecture holds.

COROLLARY 2. *Keep the notation as in the theorem and assume that the theorem holds for B. Then we have the following.*

(i) $k(B) = k(b)$, $k_i(B) = k_i(b)$ *for all* $i \geqq 0$ *and* $l(B) = l(b)$.

(ii) $(|G:N|, p) = 1$, *that is, D is a normal subgroup of a Sylow p-subgroup of G.*

(iii) *D is a Sylow p-subgroup of* N_B^*.

(iv) *Alperin's conjecture for B in* [**1**, *p.* 371] *holds.*

In his paper [**13**], K. Morita proved that $J(FG)$ is principal as a right ideal and a left ideal of FG if and only if G is p-solvable with cyclic Sylow p-subgroups. The next corollary is a kind of a generalization of Morita's result to a block-theoretic form.

COROLLARY 3. *The following conditions are equivalent for a p-block B of G with defect group D.*

(1) *D is cyclic and* $D \subseteq N_B^*$.

(2) $J(B) = (x-1)B = B(x-1)$, *where x is an element in D which has the maximal order.*

2. Proof of the theorem. In order to prove our main result, we proceed by a sequence of lemmas. In this section, (n) means the nth condition in the theorem.

LEMMA 1. *If* (1) *holds then any simple FG-module in B has D as its vertex.*

Proof. Easy by [**5**, Corollary 53.8 and Theorem 54.10].

LEMMA 2. *If* (1) *holds then we have the following.*
(i) $\phi_N \in \mathrm{IBr}(b)$ *for any* $\phi \in \mathrm{IBr}(B)$.
(ii) $\chi_N \in \mathrm{Irr}(b)$ *for any* $\chi \in \mathrm{Irr}(B)$.
(iii) *For any* $\bar{\phi} \in \mathrm{IBr}(b)$, *there is some* $\phi \in \mathrm{IBr}(B)$ *with* $\phi_N = \bar{\phi}$.

Proof. These follow from [**11**, Theorem], Lemma 1, the Green correspondence, [**4**, (59.9) Theorem] and Frobenius Reciprocity [**6**, III Theorem 2.5].

LEMMA 3. (1) *implies that D is a Sylow p-subgroup of* N_B^*; *hence* (1) *implies* (18) (*so that* $(|G:N|, p) = 1$).

Proof. Because of the proof of $(1) \Rightarrow (5)$, (7) in [**11**, p. 152], it is enough to show $(|G:N|, p) = 1$. This follows from (18) and [**2**, Proposition (3A)].

LEMMA 4. (1) *implies* (12) *and* (15).

Proof. Easy by Lemmas 2 and 3 and [**11**, Theorem].

LEMMA 5. *If* (1) *holds then* $e_B e_b = e_b e_B = e_B$ *and* $e_B = (1/|G:N|) \sum_{g \in [G \backslash N]} g e_b$, *where* $[G \backslash N]$ *is the set of all representatives of the right cosets of N in G.*

Proof. Fix any $\chi \in \mathrm{Irr}(B)$. By Lemma 3, we can write that $[G \backslash N] = \{g_1, \ldots, g_m\} \subseteq \mathrm{Ker}(\chi)$, where $m = |G:N|$. As usual, let $e_\chi = (\chi(1)/|G|) \sum_{g \in G} \overline{\chi(g)} g$ (see [**8**, p. 274 and (2.12) Theorem]). Let $a = \sum_{i=1}^{m} g_i$. Then

$$e_\chi = (\chi(1)/|G|) \sum_i \Big(\sum_{n \in N} \overline{\chi(n)} n \Big) g_i = (a/m) e_{\chi_N}$$

by Lemma 2(ii) and [**8**, p. 274]. Now, let (R, K, F) be a p-modular system, and let f_B and f_b be the central primitive idempotents of the group algebras RG and RN which correspond to B and b, respectively. Then, we get from [**8**, p. 275, (15.26) Theorem (a) and (15.27) Theorem] and Lemma 4 that

$$f_B = \sum_{\chi \in \mathrm{Irr}(B)} e_\chi = (a/m) \sum_{\psi \in \mathrm{Irr}(b)} e_\psi = (a/m) f_b.$$

LEMMA 6. *If* (1) *holds then* $e_B \bar{S}^G$ *is a simple FG-module in B for any simple FN-module* $\bar{S}$ *in b.*

Proof. By Lemma 2(iii), $S_N \cong \bar{S}$ for some simple FG-module S in B. Then, by Lemma 1 and the Green correspondence, $\bar{S}^G \cong S \oplus X$ for an FG-module X. So it is enough to show $e_B X = 0$. Assume $e_B X \neq 0$. Take any simple submodule T of X in B. Then $0 \neq (T, T)^G \subseteq (T, \bar{S}^G)^G \cong (T_N, \bar{S})^N$. So $T_N \cong \bar{S}$ by Lemma 2(i). Therefore $T \cong S$ since both of them are the Green correspondents of $\bar{S}$. Hence $2 \leqq [S, \bar{S}^G]^G = [S_N, \bar{S}]^N = [\bar{S}, \bar{S}]^N$, a contradiction.

LEMMA 7 (Okuyama [**15**, Lemma 1]). *Let $B \leftrightarrow e_B$ be a p-block of G with defect group D, and let S be a simple FG-module in B with vertex Q. Assume that $(F_D)^G \cdot e_B$ is a completely reducible FG-module. Then, for any indecomposable direct summand X of S_D, there is a subgroup A of D such that A is conjugate to Q in G and that $X \cong (F_A)^D$.*

Proof. The proof here is completely due to Okuyama (see [**15**, Lemma 1]). Let $Y = (F_D)^G \cdot e_B$, and let $\{S_1, \ldots, S_l\}$ be the set of all non-isomorphic simple FG-modules in B. Since Y is completely reducible, we can write $Y \cong \bigoplus_{i=1}^{l} m_i S_i$ for integers m_i. By Frobenius Reciprocity, $m_i > 0$ for all i. We may assume $S = S_1$, and let $m = m_1$. Since S is Q-projective, it follows from [**6**, II Theorem 3.8] that

$$(Y, S)^G = (Y, S)^G_Q. \qquad (*)$$

Now let R be a subgroup of G such that R does not contain any G-conjugate of Q. Clearly $(Y, S)^G_R \cong m(S, S)^G_R$. Assume that $(Y, S)^G_R \neq 0$. Then $0 \neq (S, S)^G_R$; so that $(S, S)^G = (S, S)^G_R$ since S is simple. This means that S is R-projective (see [**6**, II Theorem 3.8]), a contradiction. Therefore

$$(Y, S)^G_R = 0 \qquad (**)$$

for any subgroup R of G such that R does not contain any G-conjugate of Q. Since $S \mid Y \mid (F_D)^G$, $S \mid (F_Q)^G$ by Mackey Decomposition. Hence, by Mackey Decomposition and Green's theorem [**5**, Corollary 52.5], there is an element $g \in G$ such that $X \cong (F_A)^D$, where $A = Q^g \cap D$.

Thus it is enough to prove that A is conjugate to Q in G. By [**6**, II Lemma 2.5(i)], $[\mathrm{Hom}_F(F_D, S_D)]^G \cong \mathrm{Hom}_F((F_D)^G, S) \cong \mathrm{Hom}_F(Y, S)$ since S is in B. Let $\mathcal{A} = \{A\}$, $\mathcal{B} = \{D \cap A^y \mid y \in G\}$ and $W = \mathrm{Hom}_F(F_D, S_D)$. Then we have from [**6**, II Lemma 3.5] that

$$(Y, S)^G/(Y, S)^G_A \cong (F_D, S_D)^D \Big/ \Big[\sum_{B \in \mathcal{B}} (F_D, S_D)^D_B \Big].$$

Now, assume that A does not contain any G-conjugate of Q. Then $(Y, S)^G_A = 0$ by $(**)$; so that $(F_D, S_D)^D_B = 0$ for all $B \in \mathcal{B}$ from the above since $(Y, S)^G \cong (F_D, S_D)^D$ by Frobenius Reciprocity. Since $A \in \mathcal{B}$, $(F_D, S_D)^D_A = 0$. On the other hand, we have that $(F_D, X)^D \neq 0$ by Frobenius Reciprocity and $X \mid S_D$ and that X is A-projective. Hence $(F_D, S_D)^D_A \neq 0$ from [**6**, II Lemma 3.13], which is a contradiction.

Therefore A contains a G-conjugate of Q; so that $|Q| \leqq |A| = |Q^g \cap D| \leqq |Q^g| = |Q|$. This implies that A is conjugate to Q in G. This completes the proof of the lemma.

Proof of Theorem. In our previous paper [**11**], we have already proved that the conditions (1)–(7) are all equivalent. Now, (9)$\Rightarrow$(8) is trivial. Since $D \subseteq N_b^*$, (8)$\Rightarrow$(1) is clear. We get (1)$\Rightarrow$(12) and (1)$\Rightarrow$(15) by Lemma 4. (1)$\Rightarrow$(18) holds from Lemma 3. (12)$\Rightarrow$(2), (15)$\Rightarrow$(7), (15)$\Rightarrow$(9), (13)$\Rightarrow$(4), (18)$\Rightarrow$(7), (14)$\Rightarrow$(13) and (17)$\Rightarrow$(16) are all trivial. (1)$\Leftrightarrow$(16) is obtained from [**12**, Lemma 1] since D is a p-group. (10)$\Rightarrow$(11) is easy.

Assume (11). Then $J(FD)e_B = J(FD)e_b e_B \subseteq J(FD)bB = J(b)B = J(B)$ since $J(FD)b = J(b)$ by [**10**, 3.3 Proposition]. Hence, we get (1) by [**12**, Lemma 1].

Assume (12). Then (1) holds from the above; so that $(|G:N|, p) = 1$ by Lemma 3. Hence we get (14).

Assume (19). Take any simple FG-module S in B, and let $S_D = \oplus X_i$, where each X_i is an indecomposable FD-module. Then, since $(F_D)^G \cdot e_B$ is completely reducible, we have from Lemma 7 that $X_i \cong (F_{A_i})^D$ for each i, where A_i is a subgroup of D and is conjugate to the vertex of S. So, by (19), $X_i \cong F_D$ for all i, which means $D \subseteq \mathrm{Ker}(S)$. This implies (1).

Assume (1). The latter part of (11) holds from Lemma 5. Thus, $J(B)b = J(FG)e_Be_bFN = J(FG)e_BFN = J(FG)e_B = J(B)$. Similarly, $b \cdot J(B) = J(B)$. By [**12**, Lemma 1], $D \subseteq N_B^*$ if and only if $J(FD)e_B \subseteq J(B)$. Hence, by [**10**, 3.3, Proposition], $J(b)B = J(FD)bB = J(FD)e_BbFG \subseteq J(B)bFG \subseteq J(B)$; so that $J(b)B \subseteq b \cdot J(B)$. Similarly, $B \cdot J(b) \subseteq J(B)b$. Now, the following argument is due to Motose and Ninomiya [**14**, Theorem 1]. Let $\{g_1, \ldots, g_m\}$ be a set of representatives of the right cosets of N in G such that $g_1 = 1$. Then $FG = \bigoplus_{i=1}^{m} g_iFN$. Take any $x \in J(B)$; then we can write $x = \sum_{i=1}^{m} g_iy_i$ for elements $y_i \in FN$. Let $\tilde{S}$ be any simple FN-module in b, and take any $\tilde{s} \in \tilde{S}$. Then $\tilde{S}^G = \bigoplus_{i=1}^{m} (g_i \otimes_{FN} \tilde{S})$ and $\sum g_i \otimes (y_i\tilde{s}) = (\sum g_iy_i) \otimes \tilde{s} = x \otimes \tilde{s} = x(1 \otimes \tilde{s}) \in x\tilde{S}^G \subseteq J(B)e_B\tilde{S}^G$. Hence $\sum g_i \otimes (y_i\tilde{s}) = 0$ since $e_B\tilde{S}^G$ is simple by Lemma 6. Thus $g_i \otimes y_i\tilde{s} = 0$ for all i. This means $y_ie_b\tilde{S} = e_by_i\tilde{S} = 0$ for all i and for all simple FN-modules $\tilde{S}$ in b. Hence $y_ie_b \in J(b)$ for all i; so that $xe_b = e_Bxe_b = \sum e_Bg_i(y_ie_b) \in e_BFG \cdot J(b) = B \cdot J(b)$. Thus we have $J(B)e_b \subseteq B \cdot J(b)$; so that $J(B)b \subseteq B \cdot J(b)$. Since the condition (1) is symmetric, we similarly obtain $b \cdot J(B) \subseteq J(b)B$. These imply (11).

Assume (1). Then (11) and (16) hold by the above. So (16) and [**10**, 3.3. Proposition] imply $J(B) = J(b)B = J(FD)bB \subseteq J(FD)B \subseteq J(B)$; so that $J(B) = J(FD)B$. Similarly, $J(B) = B \cdot J(FD)$. So we get (17).

Assume (1). By Lemma 1, the first-half part of (19) holds. The following argument is due to Motose and Ninomiya [**14**, Theorem 1]. From the above, (17) holds, so that $J(B) = B \cdot J(FD)$. Hence

$$\begin{aligned} J(B)((F_D)^G \cdot e_B) &= J(B)(e_B \cdot (F_D)^G) = J(B)(F_D)^G = J(B)(FG \otimes_{FD} F_D) \\ &= J(B)FG \otimes_{FD} F_D = B \cdot J(FD) \otimes_{FD} F_D = B \otimes_{FD} J(FD)F_D = 0, \end{aligned}$$

which implies that $(F_D)^G \cdot e_B$ is completely reducible. Hence we obtain (19).

Assume (1). Then σ is an F-algebra-homomorphism and $\sigma(e_b) = e_B$ by Lemma 5. Suppose $y \in b$ and $\sigma(y) = 0$. By Lemma 5, $0 = ye_B = e_By = (\sum g_i/m)e_by = \sum (g_i/m)y \in \bigoplus_i (g_i \otimes b) = FG \otimes_{FN} b \subseteq FG$, where $\{g_i\}$ and m are the same as in the proof of Lemma 5. Hence $y = 0$; so that σ is a monomorphism. Now $J(B) = J(b)B$ since (5) holds from the above. Thus, by Lemma 2(i), $P(S)_N \cong P(S_N)$ for any simple FG-module S in B. Thus $\dim_F B = \dim_F b$ from Lemma 4; so that σ is an isomorphism, which means that we have (10).

$(1) \Rightarrow (20)$ is clear.

Assume (20). Let L be the intersection of the kernels of all composition factors of P. Then $L = N_B^*$ by [**18**, 3.2. Proposition (b) and 2.3. Definition] (see the results of H. Pahlings [**16**, Proposition 1] and [**17**, Theorem 5 and Lemma in p. 245]). Hence $D \subseteq N_B^*$ by (20), which means that (1) holds.

Therefore, we have proved that (1)–(20) are all equivalent.

$(7) \Rightarrow (21)$ is trivial.

Assume (21). Then, for any composition factor S of P, we have $D \subseteq \mathrm{Ker}(S)$ by (21) and [**5**, Theorem 53.9(i)] since D is normal in N. Hence, we obtain (20). This completes the proof of the theorem.

3. Proofs of the corollaries.

Proof of Corollary 1. By [**2**, Proposition (3D)], $N^*_{B_0} = O_{p',p}(G)$. So the theorem, [**5**, Lemma 59.6] and [**7**, Theorem 2.1] imply the corollary.

Proof of Corollary 2. (i) is clear by the theorem. (ii) and (iii) are proved in Lemma 3. So it is enough to claim (iv). Take any weight (Q, T) for G in B. Then, by [**1**, Lemma 1] and [**4**, (59.9) Theorem], the Green correspondent gT of T with respect to $(G, Q, N_G(Q))$ is a simple FG-module in B since $(F_D)^G \cdot e_B$ is completely reducible by (19) in the theorem. Thus Q is conjugate to D in G by (19) in the theorem since the Green correspondence preserves vertices. So we may assume $Q = D$. On the other hand, by [**1**, p. 372], $l(b)$ equals the number of weights for G in B of the form $(D, \tilde{S})$. Therefore we obtain (iv) since $l(B) = l(b)$ by (i).

Proof of Corollary 3. Assume (1). Then we easily get (2) by (17) of the theorem since $J(FD) = (x-1)FD$, where $D = \langle x \rangle$. Conversely, assume (2). Since $J(B)$ is a principal left and right ideal, B is a serial (Nakayama) algebra by the result of Morita [**13**, Theorem 1]; so that D is cyclic. Then, by (2) and [**12**, Lemma 1], $D \subseteq N^*_B$.

4. A remark. It is clear that p-blocks which satisfy the conditions in the theorem are not nilpotent blocks in general (see [**3**] for nilpotent blocks). For example, let B_0 be the principal p-block of a finite p-solvable group of p-length one which is not a p-nilpotent group. Then B_0 is not nilpotent but satisfies the conditions in the theorem from Corollary 1.

On the other hand, let $p = 3$ and $G = \mathrm{SL}(2, 3)$. Then G has three 3-blocks, say, the principal block, the non-principal block B of full defect and the block of defect zero. Then B is a nilpotent block since G is 3-nilpotent (see [**3**, p. 118]). Now B has the unique irreducible 3-modular representation L of degree 2 which is canonically given by SL(2, 3). Hence the defect group of B is not contained in $\mathrm{Ker}(L)$, and $\mathrm{Ker}(L) = N^*_B$. This means that nilpotent blocks do not satisfy the conditions in the theorem in general.

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REFERENCES

1. J. L. Alperin, Weights for finite groups, in *The Arcata Conference on Representations of Finite Groups,* Proc. Sympos. Pure Math. **47** (Part 1) (1987), 369–379.

2. R. Brauer, Some applications of the theory of blocks of characters of finite groups IV, *J. Algebra* **17** (1971), 489–521.

3. M. Broué and L. Puig, A Frobenius theorem for blocks, *Invent. Math.* **56** (1980), 117–128.

4. C. W. Curtis and I. Reiner, *Methods of representation theory vol.* II (Wiley-Interscience, 1987).

5. L. Dornhoff, *Group representation theory* (Dekker, 1972).

6. W. Feit, *The representation theory of finite groups* (North-Holland, 1982).

7. W. Hamernik and G. O. Michler, On vertices of simple modules in p-solvable groups, *Mitt. Math. Sem. Giessen* **121** (1976), 147–162.

8. I. M. Isaacs, *Character theory of finite groups* (Academic Press, 1976).

9. I. M. Isaacs and S. D. Smith, A note on groups of p-length 1, *J. Algebra* **38** (1976), 531–535.

10. R. Knörr, Blocks, vertices and normal subgroups, *Math. Z.* **148** (1976), 53–60.

11. S. Koshitani, On the kernels of representations of finite groups, *Glasgow Math. J.* **22** (1981), 151–154.

12. G. O. Michler, The kernel of a block of a group algebra, *Proc. Amer. Math. Soc.* **37** (1973), 47–49.

13. K. Morita, On group rings over a modular field which possess radicals expressible as principal ideals, *Sci. Rep. Tokyo Bunrika Daigaku. Sect. A* **4** (1951), 177–194.

14. K. Motose and Y. Ninomiya, On the subgroups H of a group G such that $J(KH)KG \supset J(KG)$, *Math. J. Okayama Univ.* **17** (1975), 171–176.

15. T. Okuyama, p-radical groups are p-solvable, *Osaka J. Math.* **23** (1986), 467–469.

16. H. Pahlings, Groups with faithful blocks, *Proc. Amer. Math. Soc.* **51** (1975), 37–40.

17. H. Pahlings, Normal p-complements and irreducible characters, *Math. Z.* **154** (1977), 243–246.

18. W. Willems, On the projectives of a group algebra, *Math. Z.* **171** (1980), 163–174.

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