THE ROTOR: ROTATION OF FRAMES VIA REPRESENTATION OF SYSTEMATIC DIFFERENCES IN TERMS OF SPHERICAL FUNCTIONS

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Abstract. The paper presents a method to derive rotational angles between two reference frames from the systematic differences represented in terms of spherical functions – hereafter referred to as the ROTOR (**ROT**ation by **O**rthogonal **R**epresentation). It is shown that the ROTOR is preferable over the least-squares technique since it (a) takes into account only the harmonics which correspond to rotation, (b) tests them for pure rotation, and (c) discovers the existence of quasi-rotational terms which may smear rotation. Due to these properties the ROTOR yields realistic results even in the case when the observational data contain not only noise but other systematic terms that have nothing to do with rotation. Numerical experiments with the FK5 and three catalogs of radio sources are described.

1. Introduction

The link of frames is an important problem in modern astrometry since there exists a strong demand that at least three types of reference frames (the FK5, VLBI and HIPPARCOS) are to be interconnected. In its general scope this task is very complicated, and a lot of observational programs aiming at measuring of optical and radio positions of stars and radio sources are now in progress (Lindegren and Kovalevsky, 1995). The recent increasing of precision (VLBI, CCD, observations from outer space) poses another problem. The data contain information, and to extract it from observations we must use specific tools – mathematical methods. In this connection one may ask whether the traditional mathematical routines, and among them the classical least-squares technique, are sufficient for treating the high precision observations the modern astrometry is dealing with.

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In a previous paper (Vityazev, 1994, hereafter Paper 1), a new method to derive angles of mutual rotation between two reference frames was proposed. This method does not employ the least-squares technique but implies that the systematic differences in positions of two catalogs are expressed in terms of orthogonal functions. Numerical experiments with the 1535 basic stars of the FK5 showed that ROTOR can be used to obtain the rotation of any reference frame with respect to the FK5. In Paper 2 (Vityazev, 1996) a further study of the ROTOR was made. This time the emphasis was put on the ROTOR's application to the catalogs that are characterized by a low-density distribution of the objects of comparison (such as the catalogs of radio stars and quasars).

The present paper is devoted mainly to practical aspects of the ROTOR's implementation. An improved algorithm which can be applied to any set of points on the celestial sphere is developed. In Section 2, a brief theoretical outline of the ROTOR is made. The modified algorithm is described in Section 3. In the next Section we show why the ROTOR is preferable over the LSM if the data contain non-rotational terms. Numerical experiments demonstrating the execution of the algorithm conclude the paper.

2. Theoretical Outline of the ROTOR

Consider two rectangular systems of coordinates (X, Y, Z) and (X', Y', Z') and connected to them the spherical coordinates (α, δ) and (α', δ'). We suppose that coordinates with primes are obtained by rotating the initial system about the axes X, Y, Z by the angles $\omega_1, \omega_2, \omega_3$, respectively. These angles are regarded as positive if rotation is done in counter-clockwise direction if viewed from the end of the axis. For small rotation angles, the results of the rotation, namely $\Delta\alpha = \alpha - \alpha'$ and $\Delta\delta = \delta - \delta'$ are described by the following basic equations:

$$\Delta\alpha \cos\delta \;=\; \sum_{i=1}^{3} \omega_i \;\phi_i(\alpha, \delta), \tag{1}$$

$$\Delta\delta \;=\; \sum_{i=1}^{2} \omega_i \;\psi_i(\alpha, \delta), \tag{2}$$

where we use two sets of functions

$$\Phi \;:\; \begin{cases} \phi_1(\alpha, \delta) = \sin\delta \;\cos\alpha \\ \phi_2(\alpha, \delta) = \sin\delta \;\sin\alpha \\ \phi_3(\alpha, \delta) = -\cos\delta, \end{cases} \tag{3}$$

$$\Psi \ : \ \begin{cases} \psi_1(\alpha,\delta) = -\sin\alpha \\ \psi_2(\alpha,\delta) = \ \cos\alpha. \end{cases} \tag{4}$$

When we are sure that the observational quantities $\Delta\alpha$ and $\Delta\delta$ comprise nothing else but the rotational terms and noise (normally distributed random variables with zero mean), the least-squares routine is probably the best one to derive the rotational angles ω_1, ω_2, ω_3. At present, least-squares determinations of the rotational angles is common practice. Nevertheless, long experience of catalog comparison (Brosche, 1966; Schwan, 1977; Bien *et al.*, 1978; etc.) shows us that the differences $\Delta\alpha$ and $\Delta\delta$ have, as a rule, a much more complicated structure than that given by the right-hand sides of equations (1) and (2). Following Brosche (1966), we will use the following representations of the differences $\Delta\alpha\cos\delta$ and $\Delta\delta$

$$\Delta\alpha\cos\delta \ = \ \sum_{j=0}^{n} C_j \ K_j(\alpha,\delta) \ + \ \epsilon, \tag{5}$$

$$\Delta\delta \ = \ \sum_{j=0}^{n} C_j{}' \ K_j(\alpha,\delta) \ + \ \epsilon', \tag{6}$$

where C_j and $C_j{}'$ are the coefficients of expansion of the systematic differences in terms of the spherical functions

$$K_j(\alpha,\delta) \ = \ \begin{cases} P_{n0}(\delta) & k=0, \ l=1 \\ P_{nk}(\delta) \ \sin k\alpha & k\neq 0, \ l=0 \\ P_{nk}(\delta) \ \cos k\alpha & k\neq 0, \ l=1 \end{cases} \tag{7}$$

and $P_{n0}(\delta)$, $P_{nk}(\delta)$ are the Legendre polynomials. The indices j and n,k,l are linked by

$$j \ = \ n^2 \ + \ 2k \ + \ l \ - \ 1. \tag{8}$$

Returning to our problem, one may note that if the left-hand sides of equations (1)–(2) and (5)–(6) are obtained from comparing two astrometric catalogs, then both pairs of equations may be regarded as models of systematic differences. Models (1) and (2) are physical since they were derived from the consideration of relative rotation of two coordinate systems. At the same time, these models are incomplete, because the real differences may comprise non-rotational effects. Models (5) and (6) are complete since they are based on a complete orthogonal system of functions, but they are not physical, for, in general, one cannot say what physics stands behind every term of equations (1) and (2). But, if we know (or suppose) the nature of the systematic differences $\Delta\alpha$ and $\Delta\delta$, the physics of the coefficients

C_j and C'_j can be clarified without any problem. Thus, in the case of rigid rotation one has

$$C_j = \sum_{i=1}^{3} \omega_i \frac{(\phi_i, K_j)}{(K_j, K_j)}, \tag{9}$$

$$C'_j = \sum_{i=1}^{2} \omega_i \frac{(\psi_i, K_j)}{(K_j, K_j)}, \tag{10}$$

where the scalar product is defined as

$$(p, q) = \int_0^{2\pi} d\alpha \int_{-\pi/2}^{\pi/2} p(\alpha, \delta) q(\alpha, \delta) \cos\delta \, d\delta. \tag{11}$$

TABLE 1. Spherical functions which represent rotation in right ascension.

n	0	1	2	3	4	5	6	...
k = 1, l = 1		-	$\frac{k_2}{4}\omega_1$	-	$\frac{k_4}{4}\omega_1$	-	$\frac{k_6}{4}\omega_1$	...
k = 1, l = 0		-	$\frac{k_2}{4}\omega_2$	-	$\frac{k_4}{4}\omega_2$	-	$\frac{k_6}{4}\omega_2$	...
k = 0, l = 1	$-\frac{l_0}{2}\omega_3$	-	$-\frac{l_2}{2}\omega_3$	-	$-\frac{l_4}{2}\omega_3$	-	$-\frac{l_6}{2}\omega_3$	...

TABLE 2. Spherical functions which represent rotation in declination.

n	0	1	2	3	4	5	6	...
k = 1, l = 1		$-\frac{m_1}{4}\omega_2$	-	$-\frac{m_3}{4}\omega_2$	-	$-\frac{m_5}{4}\omega_2$	-	...
k = 1, l = 0		$\frac{m_1}{4}\omega_1$	-	$\frac{m_3}{4}\omega_1$	-	$\frac{m_5}{4}\omega_1$	-	...
k = 0, l = 1	-	-	-	-	-	-	-	...

Evaluating the integrals corresponding to various values of the indices n, k, l, we find that the rotational angles ω_1, ω_2, ω_3 are proportional to specific coefficients C_j, C'_j (Tables 1 and 2). The factors of proportionality are defined over the quantities:

$$k_n = R_{n1} \int_{-\pi/2}^{\pi/2} P_{n1}(\delta) \sin\delta \cos\delta \, d\delta, \tag{12}$$

$$l_n = R_{n0} \int_{-\pi/2}^{\pi/2} P_{n0}(\delta) \cos^2 \delta \, d\delta, \tag{13}$$

$$m_n = R_{n1} \int_{-\pi/2}^{\pi/2} P_{n1}(\delta) \cos \delta \, d\delta, \tag{14}$$

where

$$R_{nk} = \sqrt{2n+1} \begin{cases} \sqrt{2\frac{(n-k)!}{(n+k)!}} & k \neq 0 \\ 1 & k = 0. \end{cases} \tag{15}$$

From this follows that if for a given set of points we could calculate the coefficients $\bar{C}_j$, $\bar{C}'_j$ which correspond to unit rotational angles ($\omega_1 = \omega_2 = \omega_3 = 1'$), then the angles of rotation would be evaluated without any difficulties from systematic differences under consideration.

3. The ROTOR's Implementation

For a practical realization of the ROTOR one should have an appropriate procedure to derive the coefficients C_j, C'_j from individual differences $\Delta\alpha \cos\delta$ and $\Delta\delta$ available for stars or radio sources common to the two catalogs of comparison. It is likely, that the method proposed by Brosche (1966) is the best to maintain the ROTOR. For the sake of reference we will call this technique the ORM (Orthogonal Representation Method). For a given set of individual differences $\Delta\alpha \cos\delta$ and $\Delta\delta$ and for a chosen significance level the ORM derives the coefficients C_j, C'_j that yield the systematic parts in Eqs. (5)–(6) together with their root mean square errors σ, σ'. It should be noted that the rms errors characterize the level of noise in the data and that they are independent of j due to normalization of spherical functions. Now, we are in a position to describe the practical algorithm of the ROTOR.

Given are the points $\{\alpha_i, \delta_i\}$, $i = 1, 2, ..., N$ at which the individual differencies $\Delta\alpha_i \cos(\delta_i)$ and $\Delta\delta_i$ are known. The algorithm consists of the following steps.

1. Using the ORM calculate the coefficients $C_j \pm \sigma$ and $C'_j \pm \sigma'$ to represent the systematic differences by spherical harmonics.

2. Calculate at given points the artificial individual differences

$$\Delta\alpha_i \cos(\delta_i) = \phi_1(\alpha_i, \delta_i), \tag{16}$$

$$\Delta\delta_i = \psi_1(\alpha_i, \delta_i), \qquad i = 1, 2, ..., N, \tag{17}$$

which correspond to mutual rotation of frames around X axis ($\omega_1 = 1'$).

https://doi.org/10.1017/S0252921100046960 Published online by Cambridge University Press

3. With the help of the ORM represent the artificial individual differences in terms of spherical harmonics. Find those j's for which the coefficients of expansion $\bar{C}_j$, $\bar{C}'_j$ have non-zero values.

4. For all of the j's specified in step 3, find the estimates of rotational angle ω_1

$$(\omega_1)_j = \frac{C_j}{\bar{C}_j} \pm \frac{\sigma}{\bar{C}_j}; \quad \text{from} \quad \Delta\alpha\cos\delta; \tag{18}$$

$$(\omega_1)_j = \frac{C_j{}'}{\bar{C}'_j} \pm \frac{\sigma'}{\bar{C}'_j} \quad \text{from} \quad \Delta\delta. \tag{19}$$

5. Test the data for rotation. The main idea of testing is: if all significant values ω_1 are similar (formalization is given below) then one can conclude that initial differences do contain the component due to rotation around X axis. For the final result one should adopt the value ω_1 obtained from the lowest value of j. If the values ω_1 turn out to be discordant, there is no pure rotation in the data.

6. To derive the rotational angles ω_2 and ω_3 the steps 2 – 5 must be repeated with the artificial differences calculated from Eqs. (16–17) with ϕ_2, ϕ_3, ψ_2 and ϕ_3 substituted accordingly.

The formal tests which are used at step 5 follow from Eqs. (18–19). Consider the quantities:

$$T_1(n,m) = \frac{C_{n11}}{C_{m11}}\frac{\bar{C}_{m11}}{\bar{C}_{n11}}; \quad T_2(n,m) = \frac{C_{n10}}{C_{m10}}\frac{\bar{C}_{m10}}{\bar{C}_{n10}};$$
$$n,m = 2,4,6,\ \dots,\ n \neq m; \tag{20}$$

$$T_3(n,m) = \frac{C_{n01}}{C_{m01}}\frac{\bar{C}_{m01}}{\bar{C}_{n01}};$$
$$n,m = 0,2,4,\ \dots,\ n \neq m; \tag{21}$$

$$T'_1(n,m) = \frac{C'_{n10}}{C'_{m10}}\frac{\bar{C}'_{m10}}{\bar{C}'_{n10}}; \quad T'_2(n,m) = \frac{C'_{n11}}{C'_{m11}}\frac{\bar{C}'_{m11}}{\bar{C}'_{n11}}$$
$$n,m = 1,3,5,\ \dots,\ n \neq m. \tag{22}$$

If the noiseless data contain mutual rigid rotation of frames then all the values of rotational angles should coincide, and all the quantities defined by Eqs. (20–22) should be equal to unity. When noise is present, the coefficients C_j and C'_j are random quantities. Consequently, the magnitudes $(\omega_i)_j$, $i = 1,2,3$ are random values too. Now, the values $T_i(n,m)$ and $T'_i(n,m)$ instead of being equal to unity should satisfy the following inequalities:

(1) for angles $\omega_1, \omega_2, \omega_3$ from $\Delta\alpha\cos\delta$

$$1 - \sigma_i \le T_i(n,m) \le 1 + \sigma_i, \tag{23}$$

where

$$\sigma_1 = \sigma\, T_1(n,m)\sqrt{C_{n11}^{-2} + C_{m11}^{-2}}\,,$$

$$\sigma_2 = \sigma\, T_2(n,m)\sqrt{C_{n10}^{-2} + C_{m10}^{-2}}\,,$$

$$n,m = 2,4,6,\,\dots\,,\quad n \ne m; \tag{24}$$

$$\sigma_3 = \sigma\, T_3(n,m)\sqrt{C_{n01}^{-2} + C_{m01}^{-2}}\,,$$

$$n,m = 0,2,4,\,\dots\,,\quad n \ne m; \tag{25}$$

(2) for angles ω_1, ω_2 from $\Delta\delta$

$$1 - \sigma_i' \le T_i'(n,m) \le 1 + \sigma_i', \tag{26}$$

where

$$\sigma_1' = \sigma'\, T_1'(n,m)\sqrt{(C_{n10}')^{-2} + (C_{m10}')^{-2}}\,,$$

$$\sigma_2' = \sigma'\, T_2'(n,m)\sqrt{(C_{n11}')^{-2} + (C_{m11}')^{-2}}\,,$$

$$n,m = 1,3,5,\,\dots\,,\quad n \ne m. \tag{27}$$

4. The LSM in the Presence of Systematic Noise

It is valid to say that the LSM is the best technique to derive the angles of rotation from systematic differences provided they consist of nothing else but rotational terms defined by Eqs. (1–2) plus stochastic noise (normally distributed random values with zero mean). This is not true when the differences contain some systematic terms beyond the model of rotation. To clarify the situation consider following theorems which have been proved (in more general form) in Paper 1.

Theorem 1. If the systematic differences $\Delta\alpha\cos\delta$ are represented in terms of spherical functions

$$\Delta\alpha\cos\delta = \sum_{nkl} C_{nkl}\; K_{nkl}(\alpha,\delta) \tag{28}$$

with arbitrary coefficients C_{nkl}, then the LSM solution of Eq. (1) looks as follows:

$$\omega_1 = \frac{3}{2}\sum_{\nu=1}^{\infty} C_{2\nu,1,1}\; k_{2\nu}, \quad \omega_2 = \frac{3}{2}\sum_{\nu=1}^{\infty} C_{2\nu,1,0}\; k_{2\nu}, \tag{29}$$

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$$\omega_3 = -\frac{3}{4}\sum_{\nu=1}^{\infty} C_{2\nu,0,1}\; l_{2\nu}. \tag{30}$$

Theorem 2. If the systematic differences $\Delta\delta$ are represented in terms of spherical functions with arbitrary coefficients C'_{nkl}, then the LSM solution of Eq. (2) looks as follows:

$$\omega_1 = -\frac{1}{2}\sum_{\nu=1}^{\infty} C'_{2\nu-1,1,0}\; m_{2\nu-1}, \quad \omega_2 = \frac{1}{2}\sum_{\nu=1}^{\infty} C'_{2\nu-1,1,1}\; m_{2\nu-1}. \tag{31}$$

It is interesting to note that the coefficients C_{nkl} and C'_{nkl} which are proportional to the angles $\omega_1, \omega_2, \omega_3$ (see Tables 1 and 2) are essentially the same that enter Eqs. (29–31) to define the parameters of rotation in the LSM solutions. Thus, we see that mutual rotation of frames is portrayed only by particular subsets of spherical functions. For the sake of reference we define these subsets as

$$E = \begin{cases} n = 2,4,6,...; \; k = 1; \; l = 1; \\ n = 2,4,6,...; \; k = 1; \; l = 0; \\ n = 0,2,4,...; \; k = 0; \; l = 1, \end{cases} \tag{32}$$

$$E' = \begin{cases} n = 1,3,5,...; \; k = 1; \; l = 1; \\ n = 1,3,5,...; \; k = 1; \; l = 0. \end{cases} \tag{33}$$

So, to detect rotation in $\Delta\alpha\cos\delta$ and $\Delta\delta$ one may use only the E and E' subsets of spherical functions. Keeping this in mind, we can point out two essential faults of the LSM when the systematic differences contain non-rotational components.

(a) The statistical significance of the LSM solution is diminished by the non-rotational components. We will show this for α-solution (situation with the δ-solution is treated analogously). Suppose that

$$\Delta\alpha\cos\delta = \sum_{j\in E} C_j K_j(\alpha,\delta) + \sum_{j\in G} C_j K_j(\alpha,\delta), \tag{34}$$

where summation over E produces rigid rotation and G is a subset of spherical harmonics defined as "the set of all except E". In the least-squares procedure the root mean square errors of rotational angles are defined by equation

$$\sigma(\omega_i) = W_i\sqrt{\frac{\sum_i \epsilon_i^2}{N-3}}, \quad i = 1,2,3, \tag{35}$$

where W_i are the weights of unknowns. In this formula due to Eq. (34)

$$\epsilon_i = \Delta\alpha_i \ \cos(\delta_i) - \sum_{k=1}^{3} \omega_k \ \phi_k(\alpha_i, \delta_i) = \sum_{j \in G} C_j K_j(\alpha_i, \delta_i). \qquad (36)$$

In the classical form of the least-squares procedure the root mean square errors describe the level of random part in observed quantities. In our case, as it is seen from Eqs. (34–36), even in absence of stochastic noise the values $\sigma(\omega_i)$ may happen to become very large due to the presence of non-rotational terms (summation over G). We see that the LSM does not discriminate between stochastic noise and any other components which are beyond the rotation. In this sense, the non-rotational terms that are represented by the G subset of spherical harmonincs may be regarded as "systematic noise".

(b) The LSM may yield a solution which would be adopted as a rotation even if the systematic differences contain no rotational components. Indeed, the orthogonal functions belonging to E may represent either rotational terms (R-terms), given by Eqs. (1–2), or other functions, which are not orthogonal to the functions from E. Henceforth, we will call such functions the quasi-rotational terms (the Q-terms).

Let us suppose now, that the systematic differences are composed of the Q-terms and of nothing else (in Eq. (34) we set $C_j = 0$ for all $j \in G$). In this case the coefficients C_j, C'_j with $j \in E$ will not be equal to zero, and after summation according to Eqs. (29–31), they will produce non-zero values of rotational parameters with $\sigma(\omega_i) = 0$, $i = 1, 2, 3$ due to Eq. (36). Since the summation in Eq. (36) is necessarily finite, it may happen that the residuals of the corresponding series give but small ϵ_i. In this case, the formal errors of rotational parameters may be found too small to reject the spurious solution.

Thus, we see that in the presense of systematic noise the LSM yields too high rms errors of the rotational angles and, what is more dangerous, the LSM does not discriminate between rotational and quasi-rotational terms. Consequently, one never knows whether the LSM solution reflects rotation or something else, for in the LSM technique the model is taken for granted. In contrast to that, the ROTOR gives realistic rms errors of the rotational angles, since it takes into account only the rms errors of the coefficients C_j and C'_j belonging to the E and E' subsets of spherical harmonics. The most valuable feature of the ROTOR is its ability to test the compatibility of data to the rotational model. In this way the ROTOR is protected from confusing the rotational components with the non-rotational. Here, we clearly see the difference between the LSM and the ROTOR. In the presence of the R – and Q – terms the LSM fits the functions of the $\Phi-$ and $\Psi-$ bases to *all*

https://doi.org/10.1017/S0252921100046960 Published online by Cambridge University Press

the components of which the systematic differences are comprized, and this may lead to a fictitious solution. The numerical example demonstrating this property of the ROTOR is given in Paper 1.

5. The ROTOR in Practice

To show how the ROTOR works in practice, we used the following sequence of catalogs: the FK5-Basic (Fricke *et al.*, 1988); the catalogs of radio stars (Svidunovich, 1990), containing 205 stars; the JPL catalog of 104 radio sources (Melbourne *et al.*, 1983); the catalog IERS-s which is a sample of 50 sources from the general IERS catalog (IERS Annual Report for 1988). The IERS-s was taken as an extreme for which the ROTOR is ineffective due to the poor distribution of radio sources over the sky.

In all the numerical runs the systematic differences $\Delta\alpha\cos\delta$ and $\Delta\delta$ have been calculated for each star or radio source in such a way that systematic differences were taken to be "rotation plus systematic noise":

$$\Delta\alpha\cos\delta = \sum_{i=1}^{3}\omega_i\ \phi_i(\alpha,\delta) + \sum_{j\in G} C_j K_j(\alpha,\delta), \tag{37}$$

$$\Delta\delta = \sum_{i=1}^{2}\omega_i\ \psi_i(\alpha,\delta) + \sum_{j\in G'} C'_j K_j(\alpha,\delta), \tag{38}$$

where G, G' denote the sets of empty cells in Tables 1–2. The input differences have been calculated with $\omega_i = 0''\!.5$, $i = 1,2,3$ and $C_j = C'_j = 3'$ for $k = 0,1$; $n = 0,1,...6$.

The rotational parameters derived with the LSM and with the ROTOR are shown in Tables 3 and 4.

We see that the LSM did not reconstruct the angles correctly, whereas for all the catalogs (except the IERS-s) the ROTOR gave strict solutions with very small rms error. As was said above, this is one of the beneficial properties of the ROTOR since it recognizes rotation and rejects the systematic noise (the non-zero values of rms error are explained by the remnants of rotation in the highest harmonics).

One of the purposes of our study was to find the catalog for which the ROTOR loses its efficiency. Table 4 shows that this catalog is the IERS-s. Thus, we can say that the situation when the ROTOR can not be used is defined by sets of approximately 50 points or less with uneven distribution in a comparatively narrow zone of declination.

TABLE 3. Rotational angles obtained with the LSM for various catalogs. Model: "rotation plus systematic noise". Columns 2–4 from Eq. (1), columns 5–6 from Eq. (2). Unit: 1 arcsec.

Catalog	ω_1	ω_2	ω_3	ω_1	ω_2
FK5	0.622 ± 0.505	0.632 ± 0.506	0.453 ± 0.213	0.561 ± 0.291	0.584 ± 0.293
RS	-1.43 ± 1.80	-2.96 ± 1.72	0.803 ± 0.914	1.566 ± 1.006	0.961 ± 1.148
JPL	4.90 ± 1.67	5.78 ± 1.51	-0.14 ± 0.60	-0.94 ± 0.92	1.23 ± 0.90
IERS-s	22.9 ± 4.8	-0.93 ± 4.02	-2.82 ± 1.72	-1.11 ± 2.42	5.85 ± 2.49

TABLE 4. Rotational angles obtained with the ROTOR for various catalogs. Model: "rotation plus systematic noise". Columns 2–4 from Eq. (1), columns 5–6 from Eq. (2). Unit: 1 arcsec.

Catalog	ω_1	ω_2	ω_3	ω_1	ω_2
FK5	0.498 ± 0.002	0.501 ± 0.002	0.500 ± 0.001	0.501 ± 0.001	0.501 ± 0.001
RS	0.501 ± 0.008	0.501 ± 0.002	0.506 ± 0.003	0.502 ± 0.003	0.501 ± 0.004
JPL	0.496 ± 0.010	0.511 ± 0.008	0.501 ± 0.008	0.503 ± 0.005	0.501 ± 0.002
IERS-s	0.808 ± 0.184	0.558 ± 0.101	0.426 ± 0.195	0.156 ± 0.285	1.422 ± 0.342

6. Conclusions

The success of the ROTOR's application depends on the possibility of representing the systematic differences by at least several low order spherical functions. To achieve this, the catalogs of comparison must have a sufficient number of objects in common properly distributed over the celestial sphere.

Unfortunately, catalogs of radio sources contain much less objects than catalogs of stars. And this is an obstacle not only in the task of deriving the mutual orientation of two reference frames based on radio sources, but in the general problem of their comparison. Nevertheless, the numerical experiments described in this paper and in Papers 1 and 2 give evidence that for various models of systematic differences and for a wide range of catalogs the ROTOR is preferable to the least-squares technique with respect to deriving the mutual orientation of two reference frames.

Acknowledgements. This research was supported by a grant from the American Astronomical Society. The author appreciates the assistance of A. Tsvetkov who wrote a computer program for numerical experiments.

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