The function defined by the integral :

$$w=\int_0^z \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}} \quad (0<k<1)$$

is now considered in this circle :

$$w=f(z').$$

Obviously $f(z')$ is analytic within the circle and continuous on the boundary, and it is further shown in all detail that the circumference goes over in a one-to-one manner and continuously into the perimeter of a rectangle in the w-plane. Thus all the conditions of Darboux's Theorem in its restricted form are satisfied for the circle and the rectangle. Hence the interior and circumference of the circle go over into the interior and perimeter of the rectangle in the desired manner. It remains merely to transform back from the circle to the half-plane. This completes the proof.

R.C. now says that the same reasoning would show that the function

$$w=\int_0^{z^3} \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}$$

carries the upper half-plane over into a rectangle, since this function also transforms the axis of reals in the z-plane in a one-to-one manner and continuously into the axis of reals in the w-plane. True, but the other condition, namely, that this function of z be analytic in the upper half of the z-plane, is not fulfilled ; for at the point $z=w$, where $w=e^{2\pi i/3}$, this latter function has a branch point. Thus the example which is cited to confound my proof fails to fulfil the conditions of the theorem.

Very truly yours,

William Fogg Osgood.

Harvard University,
2nd February, 1938.

THE ASSES' BRIDGE.

To the Editor of the *Mathematical Gazette*.

Sir,—When one recalls early youth it brings thoughts of a Society for the Improvement of Mathematical Teaching. One might begin from the Abacus as operated by a governess ; or from the multiplication table which, when one thinks of it, is a far more complex table of double entry than most of such tables in the modern advanced mathematic, yet somehow we all, whether clever or dull, had to conquer it. But the famous Asses' Bridge has special claims. One remembers that in improved Euclids it was simply abolished by the device of turning over the isosceles triangle in space like a pancake so as to cover itself upside-down. Yet that was hardly respectful to the great Greek originals : and indeed it shocked the purists. But why was it forbidden? Euclid has now disappeared, gone out of sight like the other texts, mostly more concise, once provided for

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us by the great mathematicians of the past, and perhaps nobody outside the profession knows what has taken their place (except possibly the technical schools which are effective practically). But one apparently obtains the following account from one of the amended Euclids that formed an intermediate stage in the transition. Prop. V, the famous Asses' Bridge, depended vitally on Prop. IV, which asserted that when two triangles have two pairs of sides equal in length each to each and the angles contained by them equal they can be moved, presumably by sliding, into exact superposition. But the imperfectly informed is tempted to ask the question : Can they? or may it not be necessary to turn one of them over? which is just the forbidden operation in space that is foreign to self-contained congruence. And this, if it really be so, vitiates the Asses' Bridge which essentially hangs on to it, so that Euclid himself cannot evade consideration of this idea of turning over in the outside space. In fact, may it be that an angle ACB is essentially a different one from BCA in that it is affected by a different sign? Here one seems to approach the domain of trigonometry. Again, if two triangles have all three corresponding pairs of sides of the same lengths, each to each, they need not be capable of sliding in their plane into superposition : to that end the signs of the angles of the triangle must be all the same, say positive : perhaps trigonometry escapes this ambiguity. The upshot is apparently that for consistent doctrine a fundamental direction of positive angular rotation in the plane of the geometry must be specified, after the manner of a corkscrew.* And, indeed, when one thinks of it, has Euclid with all his rigour attained to any proof that a consistent frame of uniform geometry subsists at all, one which is the same from whatsoever standpoint or origin it is surveyed? Or does he merely have to assume its existence, and develop the consequences, the number π and all the rest of them, secure so long as he discovers no internal contradiction. Perhaps Pascal, who is said to have re-discovered Euclid for himself, entertained such ideas as these. Here our learner if enterprising can hardly keep himself

* This principle of rotational quality has been dealt with by the chemists, in their own way, ever since the early days of Pasteur, and doubtless they will derive still further fruit from it. It applies to geometry on a spherical sheet still more emphatically : a spherical triangle cannot possibly slide, nor even be turned over, into coincidence with its opposite or polar triangle. Illustrations by extensions of this kind into cognate domains would possibly make the discussions on the philosophy of elementary geometry less difficult to follow. In its own domain Hilbert's tract is perhaps still the best guide for the specialist explorer of foundations. In the days before the Great War the Board of Education published under German international influence a series of volumes of essays on elementary mathematical education which are doubtless still to be found. Nowadays centralisations are illustrated by the different English and Scotch geometries, referred to in the recent extensive publications of the *Gazette* which came to hand after this letter was forwarded. Half a century ago there was more freedom for local centres of education, and general education was pursued at the Universities. One notices the recent remarks of an experienced immigrant on the present isolation (in the public schools and universities) of English mathematical education from physical science, which is in so marked contrast with one's memories.

away, in the course of his trigonometry, from Riemann's modern plane, unusual and fruitful and thus mysterious, consisting of several interconnected sheets—unless he takes refuge from originality in the complications of algebraic analysis provided for him in the texts. And what of consistent measures of time in view of astronomical light-ranging with Bradley's finite speed of light? The Improvement and Vivification of Mathematical Teaching surely sets an urgent problem!

Another phase of this subject is set by a conundrum once propounded to the writer from a ladies' school, where it had made a great sensation, as indeed it did when expounded to him for advice. The argument proved irrefragably a geometrical result that was in common sense quite wrong: so what was to become of the faculty of human reasoning? The key to the paradox proved to be that though the reasoning employed was right, the diagram on which it was based was wrong. If it is drawn so that an essential point P of it lies on one side of an essential line AB, then all is well; but if by bad drawing it is put on the other side, then everything may be upset. Euclid had an adequate notion that in his own simple domain his diagrams should be verified: but what of more complex cases like the one that so intelligently disturbed the ladies' rational atmosphere?

With the suitable hesitations the writer must present himself as

DIDASCULUS.

GEOMETRY REPORT. GEOMETRY IN SCOTLAND.

To the Editor of the *Mathematical Gazette.*

DEAR SIR,—May I draw your readers' attention to what I think is an important mistake (so far as Scotland is concerned) in the excellent new Geometry Report.

On p. 183, in the Appendix on "Geometry in Scotland" it is stated that, for the Scottish Leaving Certificate Examination, "This geometrical strictness makes it necessary to prove *Euclid* VI. 1, as in *Euclid* . . ." and there is a footnote explaining that this involves using the definition of proportion which Euclid used. But the compiler of the Appendix seems not to have noticed an asterisk in "Education (Scotland) Note as to Mathematics", on which the Appendix is based; that asterisk seems to refer to a footnote which says that the fact (*Euclid*, VI. 1) should be thoroughly known, but formal proof will not be required in examinations.

May I also point out that the page headings give no help to the reader trying to look up anything in the Report. Anyone reading the Report does not wish to be reminded of its title on nearly every page. I would suggest giving on the left-hand page the title of the section, and on the right-hand page subtitles (as given on pp. v, vi of the Report). Could not this be done when the Report is reprinted?

Yours truly,

A. W. SIDDONS.