Le Chapitre 6 par contre l'est infiniment plus. La topologie des réseaux en termes de bord et de cobord, séduira les esprits en mal d'usage de théories générales, ainsi que les applications aux lois de Kirchoff et à un problème de transport. Le Chapitre 7 final, dans les limites de ses quelques pages, introduit quelques concepts relatifs à la topologie à trois dimensions, qui reçoit tant d'attention dans la recherche actuellement. Sans doute aucune trace de celle-ci ici, mais les quelques commentaires sur l'orientabilité et les espaces de configuration sont remarquables par leur clarté et leur simplicité.

En définitive, un livre à recommander comme introduction à une partie de la topologie particulièrement attrayante. Il faut féliciter l'auteur d'avoir prouvé que ceci peut se faire dès les premières années de college, et le remercier d'avoir tracé le chemin si fermement, ce que soulignent les nombreux exercices et illustrations qui accompagnent chaque chapitre. Une courte mais efficace bibliographie termine l'ensemble.

Quelques rares fautes typographiques ont été relevées. Quelques figures ne sont pas très claires (Exemple: fig. 1.30 p.31), ou sont non convaincantes (fig. 5.1 p.134). Mais ce sont là vétilles auxquelles le lecteur ou l'instructeur remédiera facilement.

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<u>Analytic functions of several complex variables</u>, by R.C. Gunning and Hugo Rossi, Prentice-Hall, 1965. \$12.95.

This book has already established itself, in the short time it has been out, as a standard reference book for complex analysis. One of the primary reasons for this is that it is the only book available which directs the fundamental problems of global analysis on complex spaces. The subject of several complex variables has traveled a very evolutionary path. The first period of development included contributions by Cousin, Poincaré, E.E. Levi, Hartogs and others, which was summarized in the classic text of 1934 by Behnke and Thullen. Starting from there Oka turned out a remarkable series of nine papers between 1934 and 1952 solving several of the outstanding problems in the subject, notably the solution of Cousin's problem on a domain of holomorphy, and the solution to the Levi problem, giving a geometric characterization of domains of holomorphy. Henri Cartan made many important contributions to the subject during this period and one of the most noticeable innovations was the use of sheaf theory and the notion of coherent analytic sheaves introduced in Cartan's Seminare ENS of 1950-51. This gave a generality which carried far beyond their original applicability. At about this time as generalizations of Riemann surfaces and complex manifolds there appeared various definitions of complex spaces, due to Serre and Behnke-Stein, as well as others. Grauert and Remmert unified this theory in the 1950's, proving the equivalence of the various definitions, and making important contributions to the theory of coherent analytic sheaves on complex spaces. It is this development which is

presented in a modern framework by the book under review. An important new tool in several complex variables, the solution to the $\bar{\partial}$ -Neumann problem on complex manifolds, due to J. J. Kohn, arose at the same time that this book was in preparation, and so is not included. On complex manifolds, this does present an alternative approach to the subject and to some of the fundamental theorems. This point of view has been taken by Hormander in a more recent book on several complex variables. However, the book by Gunning and Rossi is the only book which treats these problems on complex spaces, for which the differential forms and partial differential equation methods do not yet work. Basically the sheaf theory and proofs originally due to Oka go back to the Weierstrass point of view and power series, whereas the book by Hormander represents the latest step in a long line of developments coming from the Riemann point of view and the Dirichlet principle.

We shall now try to give a survey of some of the topics covered. Chapter I is one of the most elementary and yet, at the same time, one of the deepest chapters in the book. It contains some basic facts which are prerequisite for the rest of the book such as the definitions of analytic objects in several complex variables (complex manifolds, holomorphic maps, etc.) and those facts which carry over easily from function theory in one variable. Two sections are devoted to proving Oka's theorems on the solution to Cousin's (Mittag-Leffler's) problem

for certain domains in C^n , (i.e. given locally, principal parts of a meromorphic function, find a global meromorphic function with these principal parts), and to the theory of polynomial convexity using the original proof of Oka recast into the language of differential forms. There are then two sections of independent interest, the first of which deals with the problem of simultaneous analytic continuation, a phenomenon discovered by Hartogs' occurring in C^n for n > 1. Here

the authors give a complete solution for domains spread over $\ \boldsymbol{C}^n$

(unramified coverings of a subdomain of C^n to the envelope of holomorphy

problem, i.e. finding a maximal domain over C^n to which all holomorphic functions in a domain D continue and proving that it is holomorphically convex. The second section deals with important applications of several complex variables to the theory of Banach algebras.

In Chapter II is presented the classical theory of the local ring of convergent power series at a point and the Weierstrass preparation theorem. In Chapter III analytic subvarieties of C^n are discussed from a local point of view. The Nullstellensatz is proved using the Weierstrass preparation theorem to bridge the gap between polynomials and power series. Then the local parametrization for analytic subvarieties is proved, giving a representation of a local analytic subvariety (the common zeros of a finite number of holomorphic functions) as a ramified (analytic) cover of some lower dimensional subspace.

Chapter IV introduces sheaf theory and develops the theory of sheaves of modules necessary to study analytic sheaves on subdomains

of C^n , which are \mathcal{P} -modules where \mathcal{P} is the sheaf of germs of

holomorphic functions in C^n . This theory is carried over to subvarieties

of a subdomain of $C^{''}$. Using that as a local model, the theory of analytic spaces is introduced in Chapter V as a ringed space, that is, a topological space with a structure sheaf on it, which is locally isomorphic

to a subvariety of a domain in C^n with its corresponding structure sheaf, introduced in Chapter IV. In this chapter the elementary theory of holomorphic functions on an analytic space is presented, followed by Remmert's proper mapping theorem. This theorem asserts that the image of an analytic space under a proper holomorphic map is again an analytic space. This is used to prove the Remmert-Stein theorem concerning the closure of a subvariety V defined in a domain minus a subvariety W. This is used in turn to prove Chow's theorem that an analytic subvariety of projective space is algebraic.

Chapter VI presents the basic theory of Čech cohomology for paracompact spaces with coefficients in a sheaf of Abelian groups, and the fundamental long exact sequence in cohomology associated to a short exact sequence of sheaves is presented. Resolutions of sheaves and the "abstract de Rham theorem" are used to give an alternative representation for cohomology groups in terms of differential forms when there is a manifold structure present. At the end of this chapter an important lemma due to Cartan concerning holomorphic matrices is proved. From this, the authors use the cohomology theory and syzygy theory (resolutions by free sheaves) to piece together local information in order to obtain a weak form of Cartan's famous theorems A and B

for a polydomain in C^n .

In Chapter VII Stein spaces are introduced. These are the natural generalizations of domains of holomorphy to abstract complex spaces. They are also a good generalization to several complex variables of open Riemann surfaces, and the natural object upon which to do function theory since they are axiomatically endowed with an ample supply of global non-constant holomorphic functions. It is then shown that a Stein space X can be exhausted by analytic polyhedra, relatively compact domains defined $\{x \in X: |f_1(x)| < 1, \ldots, |f_r(x)| < 1\}$, where the f_1 are holomorphic functions in X. These are a generalization of a polydomain and one proves the fundamental theorems of Cartan for Stein spaces by proving them first for these special domains and using a limiting process. Special analytic polyhedra, defined by n functions, where $n = \dim X$, are then introduced and used to give Bishop's proof that a Stein manifold X of

dim n can be properly and holomorphically embedded in C^{2n+1} . Other uses of special analytic polyhedra in relation to the problem of simultaneous analytic continuation and extension of subvarieties are developed here. In Chapter VIII the concept of Frechet sheaf is introduced in order to topologize the cohomology groups of a space with coefficients in such a sheaf as a topological vector space. The fundamental theorems A and B of Cartan are enunciated and proved, using the topological machinery developed in order to carry out the limiting process

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discussed earlier. Then there are various standard applications of these theorems to Cousin problems and divisors on a complex space. A final section discusses locally free sheaves and holomorphic vector bundles, and proves a tubular neighbourhood theory for complex submanifolds of C^n .

The last chapter deals with the important geometric concept of pseudo convexity. Grauert's solution to the Levi problem for strongly pseudoconvex subdomains of a complex manifold is presented here along with a complete discussion of the relation between pseudoconvexity, holomorphic convexity and domains of holomorphy for domains spread over C^{n} .

This book is an important reference tool for people working in the general area of several complex variables. The beginner will no doubt experience difficulty in trying to master this book, as it demands a fair amount of mathematical maturity and background at times. This is due in part to the mixture of analysis, algebra, geometry, and topology which go into the subject, and the book draws freely on fundamental results in these various fields. In addition to this some proofs don't seem as clear as they could be. There are also mistakes in cross referencing and one section in particular, Lemma 20 through the end of Section B in Chapter III, uses definitions and notations introduced much later in the book. Some theorems are incorrect as stated and the reader must read carefully at times to be sure he understands just what is true. However, the authors have assimilated a lot of mathematics here in one book, and it's clear that it was not an easy task. A second edition of this book with some careful revision and editing in certain places would enhance the quality of this book, and increase its usefulness. Nevertheless, the reviewer is very happy that such a book was written, and it does do justice to a survey of an exciting area of mathematical development.

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Introduction to Cybernetics, by V.M. Glushkov. Academic Press, 1966. Originally published as "Vvedeniye v Kibernetiku" Ukr. Acad. of Sci. Kiev, 1964. 322 pages. \$11.75.

This book is a unified treatment, at a beginning graduate level, of several topics related to cybernetics. The theories of algorithms and Boolean functions are developed and applied to the theory and construction of automata, self organizing systems, and perceptrons. The use of Algol to program these for a digital computer is presented. The book closes with a presentation of examples of what has been accomplished with "machine proof of theorems" and an introduction to some of the problems remaining.

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