6.7 PERIODICITY AND LUMINOSITY OF THE 'PULSAR' MODEL OF QUASARS*

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Abstract. The diagnosis of pulsar-like symptoms in the quasar 3C 345 has renewed interest in differentially rotating supermassive stars (SMS). Quasi-periodic behavior follows from differential rotation. In reviewing early work on this quasar model it is found that SMS models with rotational periods of the order of a few hundred days and with pulsation periods approximately one-quarter as long can supply thermal and magnetic dipole radiation of the order of 10^{48} erg sec^{-1} for somewhat more than 10^6 yr. The surface field required for the magnetic dipole radiation is approximately 10^3 G. The SMS mass taken as typical in the calculations is $3 \times 10^9\ M_{\odot}$.

1. Introduction

This paper will not attempt to explain the Crab pulsar. Under the circumstances it is therefore somewhat unique. On the other hand, it might very well be said that the Crab pulsar explains this paper. Now that others (Morrison, 1969; Cavaliere *et al.*, 1969; Woltjer, 1971) have diagnosed pulsar-like symptoms in quasars there is renewed interest in supermassive stars (SMS) which Hoyle and Fowler (1963a, b) suggested some seven years ago as energy sources for radio galaxies and, after their discovery, for quasars. In later papers Fowler (1964, 1965, 1966a, b, c, d) treated nuclear energy generation in SMS in some detail. In addition, in connection with the binding energy equivalent of the rotational energy, it was especially noted in Fowler (1966c), p. 355 that "Since this energy must be lost during contraction it is another source of the observed energy emissions in quasars." This present paper will attempt to amplify this point in light of current ideas concerning the transformation of gravitational-rotational energy into the radio, infrared, optical, X-ray and cosmic-ray emissions from quasars. The discussion will be limited to the Kelvin-Helmholtz contraction stage of SMS prior to the onset of nuclear burning since it is during this stage that the theoretical periods of rotation and pulsation seem to match those of the observed quasi-periodicities in 3C 345 and other quasars.

In their first discussions of rotating SMS, Hoyle and Fowler noted that angular momentum problems must arise during contraction and that angular momentum and rotational energy would necessarily be transferred to surrounding material. At the same time, in the presence of magnetic fields, electromagnetic processes would generate radiation and relativistic particles. The suggested mechanism for this involved the toroidal winding of the magnetic field between the star and the surrounding material. Following Hoyle *et al.* (1964), Pacini (1967, 1968), and Gold (1968), this idea has

* Supported in part by the Office of Naval Research [Nonr-220(47)].

Davies and Smith (eds.), The Crab Nebula, 364–377. *All Rights Reserved.*

https://doi.org/10.1017/S0074180900007683 Published online by Cambridge University Press

been replaced, in current fashion, by radiation and acceleration involving the rotation of a field as simple as that of a dipole but with retardation effects at the velocity of light circle.

Through the work of Feynman (1963), Iben (1963), Chandrasekhar (1964), and Fowler (1964) it soon came to be realized that non-rotating SMS suffered from a general relativistic instability which led to hydrodynamic collapse early in their evolution. In order to achieve lifetimes for hydrostatic stability of the order of 10^6 yr Durney and Roxburgh (1967) and Fowler (1966b, c, d) investigated rotational effects and found that uniform rotation would stabilize masses of the order of $10^6\ M_\odot$ up to and through hydrogen burning. In addition Fowler found that differential rotation would stabilize masses of the order of $10^9\ M_\odot$ up to and through hydrogen burning. These studies led to estimates for the rotation and pulsation periods of SMS of the order of several tens to a few hundred days. It may prove significant that these periods are of the order of those attributed to the light variations in 3C 345 and other quasars by Kinman *et al.* (1968).

2. The Period of Rotation of SMS

It is usually assumed that angular momentum loss commences when

$$\omega_R^2 R^3 = \alpha^2 GM \tag{1}$$

and maintains this relation thereafter. The notation is standard except that ω_R is the peripheral, equatorial angular velocity in the case of differential rotation and α is of the order of unity or somewhat less and is model dependent. For differential rotation with cylindrical symmetry in an object with polytropic index $n=3$ as discussed by Stoeckly (1965), $\alpha^2=0.456$. For this value Jeans' criterion for instability holds at the center of the SMS. The rotational period can be written as

$$\begin{aligned} P_R &= \frac{2\pi}{\omega_R} = \left(\frac{32\pi^2}{x^3}\right)^{1/2} \frac{GM}{\alpha c^3} \\ &= 1.3 \times 10^{-4} x^{-3/2} \frac{M}{M_\odot} \text{ sec} = 1.5 \times 10^{-9} x^{-3/2} \frac{M}{M_\odot} \text{ day} \end{aligned} \tag{2}$$

where

$$x = \frac{R_s}{R} = \frac{2GM}{Rc^2} \tag{3}$$

is the ratio of the Schwarzschild radius, R_s, to the coordinate radius, R. The variation of $P_R(x)$ is shown in Figure 1. Note that for a given x and α, the period is linear in M. This explains the great difference possible in the rotation rates of neutron stars and SMS even though α is considerably smaller for neutron stars.

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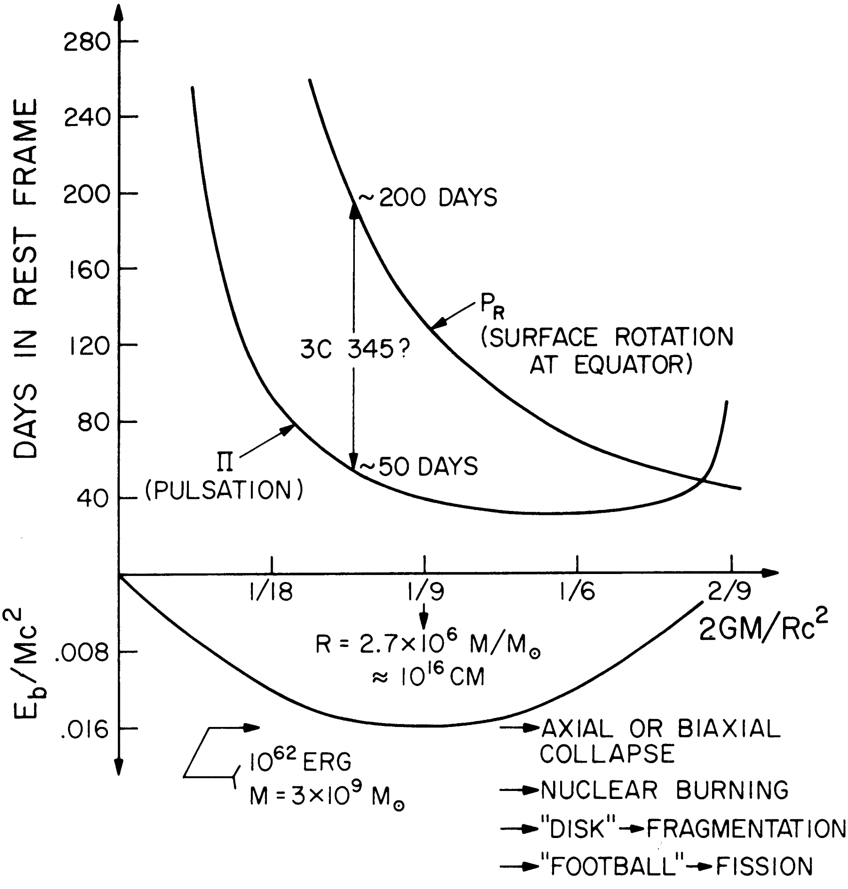

Fig. 1. Rotation period (P_R), pulsation period (Π), and binding energy/rest mass (E_b/Mc^2) as a function of $2GM/Rc^2$ for an SMS with $M = 3 \times 10^9 M_\odot$. Observational data for 3C 345 is fitted at $2GM/Rc^2 = 1/12$ for which $P_R \sim 200$ days, $\Pi \sim 50$ days and $E_b \sim 10^6$ erg.

3. The Binding Energy of SMS

The binding energy of a differentially rotating SMS in hydrostatic equilibrium governed by Equation (1) is given by Fowler (1966c) as

$$\frac{E_b}{Mc^2} = \tfrac{1}{4}K^2\alpha^2 x - \zeta x^2 \tag{4}$$

where ζ is a measure of post-Newtonian general relativistic effects and where KR is an 'effective' radius of gyration for differential rotation. Both ζ and K depend on the polytropic index n. In addition, ζ depends upon the volume average over the star of Γ_4 which is defined by $u/p \equiv (\Gamma_4 - 1)^{-1}$ with u the internal energy per cubic centimeter and p the pressure. I have found that the numerical calculations of Tooper (1966) and of Ipser (1969) can be expressed approximately by

$$\zeta \approx \frac{27}{16(5-n)^2}\left\langle\frac{\Gamma_4 - \frac{4}{3}}{\Gamma_4 - 1}\right\rangle \Rightarrow \left(\frac{9}{8}\right)^2 \quad \text{for} \quad \Gamma_4 = \tfrac{4}{3},\, n = 3 \tag{5}$$

The dimensionless parameter K appears in the expressions for the rotational energy, Ψ, as follows

$$\Psi = \tfrac{1}{2}K^2 M R^2 \omega_R^2 = \tfrac{1}{2}K^2 \alpha^2 G M^2 R^{-1} \tag{6}$$

and

$$\frac{\Psi}{Mc^2} = \tfrac{1}{4}K^2\alpha^2 x\,. \tag{7}$$

It is assumed that radiation pressure greatly exceeds gas pressure in SMS so that the effective ratio of specific heats and all adiabatic coefficients are approximately equal to $\frac{4}{3}$ and the internal structure is thus that of a polytrope of index $n=3$. In this case the Stoeckly model (1965) gives $K^2=2.47$ so that $K^2\alpha^2=\frac{9}{8}$. Note that K^2 is considerably larger than $k^2=0.075$ where kR is the true radius of gyration of a polytrope with index $n=3$. The moment of inertia, I_z, of the SMS about the axis rotation is given by

$$I_z = k^2 M R^2\,. \tag{8}$$

Equation (8) contains k^2 and not K^2 since I_z depends on the mass distribution and not on the angular velocity distribution.

If the mean square angular velocity throughout the SMS is defined by

$$\Psi \equiv \tfrac{1}{2}k^2 M R^2 \langle\omega^2\rangle \tag{9}$$

then

$$\frac{\langle\omega^2\rangle^{1/2}}{\omega_R} = \frac{K}{k} = 5.74 \tag{10}$$

whereas the ratio of the central angular velocity to that at the equatorial periphery is given by Fowler (1966c) as

$$\frac{\omega_c}{\omega_R} = 10.9\,. \tag{11}$$

Here and in what follows all numerical values hold for $n=3$.

In the Stoeckly model (1965) it is assumed that a gas cloud with uniform density ($n=0$) and uniform angular velocity equal to ω_R condenses internally under conservation of angular momentum to a polytrope of index n which in the case of interest is equal to 3. The square of the radius of gyration for uniform density is $\frac{2}{5}R^2$ so that the conserved angular momentum, Φ, is given by

$$\Phi = \tfrac{2}{5}MR^2\omega_R \tag{12}$$

and the angular momentum per unit mass by

$$\frac{\Phi}{M} = \frac{2}{5}R^2\omega_R\,. \tag{13}$$

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In the case of angular momentum loss it is assumed that the SMS evolves through a sequence of Stoeckly models governed by Equation (1) so that

$$\Phi = \tfrac{2}{5}\alpha \left(GM^3R\right)^{1/2} \tag{14}$$

and

$$\frac{\Phi}{GM^2/c} = \frac{2\sqrt{2}}{5}\, \alpha x^{-1/2}. \tag{15}$$

In addition, Equations (6) to (12) still hold so that

$$\Psi = \tfrac{5}{4}K^2\Phi\omega_R = 3.09\ \Phi\omega_R \tag{16}$$

$$= \frac{25}{8}\frac{K^2\Phi}{MR^2} = 7.72\,\frac{\Phi^2}{MR^2}. \tag{17}$$

If the mean angular velocity throughout the SMS is defined by

$$\Phi \equiv k^2MR^2 \langle\omega\rangle \tag{18}$$

then

$$\frac{\langle\omega\rangle}{\omega_R} = \frac{2}{5k^2} = 5.33\,. \tag{19}$$

The numerical results for k, K, and ζ given above were calculated assuming sphericity, i.e., no deformation due to the rotation, although Stoeckly (1965) has considered deformation in detail for $n=\tfrac{3}{2}$. In the Stoeckly models with cylindrical symmetry this procedure is exact for k and K. In all equations R should be set equal to the equatorial radius. Changes in ζ will occur with deformation but it is my belief that these will not be large.

4. The Maximum Binding Energy of SMS

Differentiation of Equation (4) indicates that E_b/Mc^2 reaches a maximum value equal to

$$\frac{E_b^{\max}}{Mc^2} = \frac{K^4\alpha^4}{64\zeta} = \frac{1}{64} \tag{20}$$

at

$$x\left(E_b^{\max}\right) = \frac{K^2\alpha^2}{8\zeta} = \frac{1}{9} \tag{21}$$

or $R=9\ R_s$. This is illustrated in Figure 1. Since E_b is the negative total energy of the SMS, it is taken positive along the negative abscissa of the figure. Numerically

$$E_b^{\max} = 3 \times 10^{52}\,(M/M_\odot) \quad \text{erg} \tag{22}$$

Beyond the maximum in E_b or the minimum in total energy hydrostatic equilibrium requires that energy be supplied to the SMS from some source. If nuclear burning has not started or has terminated, the normal losses of energy from the rotating SMS will lead to axial or biaxial collapse at this stage. The eventual onset of nuclear burning may halt the collapse for a period but the ultimate result will be a disk-shaped object unstable to fragmentation or a football-shaped object unstable to fission. Hydrogen burning through the CNO bi-cycle does not occur before the maximum in E_b in SMS with $M > 10^9\ M_\odot$ so that quasi-hydrostatic equilibrium of the Kelvin-Helmholtz contraction stage holds over the interval $0 \leqslant x \leqslant K^2\alpha^2/8\zeta$. At the end of this interval P_R approaches

$$P_R\left(E_b^{\max}\right) = \frac{128\pi\zeta^{3/2}}{K^3\alpha^4}\frac{GM}{c^3} = 4.1 \times 10^{-8}\frac{M}{M_\odot} \text{ day} \tag{23}$$

which is the order of 100 days for M somewhat in excess of $10^9\ M_\odot$.

5. The Period of Pulsation of SMS

The fundamental mode of radial pulsation of a star has a period which depends critically on its physical properties, in particular, on Chandrasekhar's adiabatic coefficient, $\Gamma_1 \equiv -\mathrm{d}\ln p/\mathrm{d}\ln V$. As noted previously the main pressure support in a SMS is due to radiation and $\Gamma_1 \approx \frac{4}{3}$. In this case Fowler (1966c) has shown that

$$\sigma^2 = \frac{4E_b}{3I_z} = \frac{Mc^2}{3I_z}\left[K^2\alpha^2 x - 4\zeta x^2\right] \tag{24}$$

Thus

$$\left(\frac{\sigma}{\omega_R}\right)^2 = \left(\frac{P_R}{\Pi}\right)^2 = \frac{2}{3}\left(\frac{K}{k}\right)^2\left[1 - 4\left(\frac{\zeta}{K^2\alpha^2}\right)x\right] \tag{25}$$

where σ is the pulsational angular frequency and Π is the period. Equation (25) leads to the numerical results

$$\begin{aligned}\left(\frac{P_R}{\Pi}\right)^2 &= \frac{2}{3}\left(\frac{K}{k}\right)^2\left[1 - 4.5x\right] \\ &= \left(\frac{2}{3} \to \frac{1}{3}\right)\left(\frac{K}{k}\right)^2 = 22 \to 11 \quad \text{for} \quad 0 \leqslant x \leqslant \tfrac{1}{9}\end{aligned} \tag{26}$$

The variation of Π with x is illustrated in Figure 1. Note that the radial pulsations are stable until σ^2 becomes negative at $x = K^2\alpha^2/4\zeta = 2x\left(E_b^{\max}\right) = \frac{2}{9}$. However, as noted previously, energy must be supplied if collapse is to be avoided when $E_b^{\max}$ is reached. The ratio $(P_R/\Pi) = (\frac{2}{3})^{1/2}\ (K/\kappa)$ for $x = 0$ varies considerably with polytropic index. For $n = 2$, 3, 3.5 and 4, $P_R/\Pi = 2.2$, 4.7, 8.0 and 15 respectively at $x = 0$.

6. Discussion of the Periods and Binding Energy

Over the Kelvin-Helmholtz contraction stage Equation (26) indicates that $P_R/\Pi \approx 4$. This is illustrated in Figure 2. This ratio is of some interest in connection with the observations on 3C 345 where Kinman *et al.* (1968) attribute a rest frame period of

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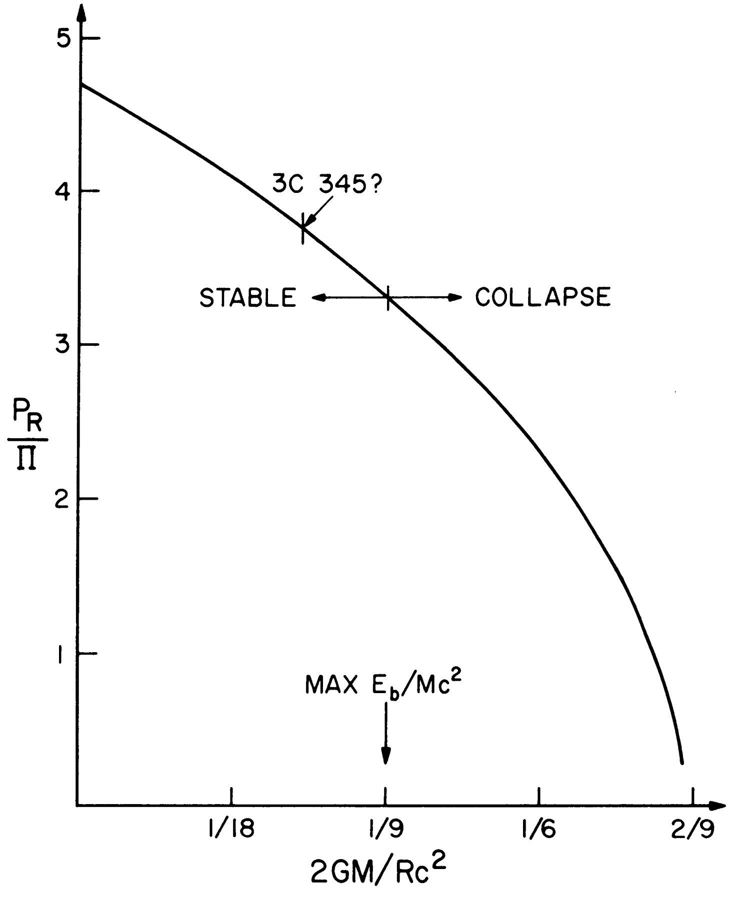

Fig. 2. The ratio of rotation period to pulsation period for SMS as a function of $2GM/Rc^2$.

200 days to rotation and find some additional evidence for a second period of ~ 50 days. As indicated in Figures 1 and 2 a fair fit to these observations is obtained for $x = \frac{1}{12}$ and $M = 3 \times 10^9\ M_\odot$.

It is important to note that the ratio P_R/Π is relatively independent of the uncertain parameter α. Furthermore the result $P_R/\Pi \approx 4$ for differential rotation is quite different than the customary results for uniform rotation $(K = k)$ without the general relativistic correction, namely $P_R/\Pi = (\frac{2}{3})^{1/2} = 0.82$. If the pulsar search-light model is applied literally to quasars then it must be recalled that, for differential rotation, P_R holds only for 'hot spots' on or near the equator. The rotational periods at the poles are of the order $P_R/10$ on the Stoeckly model. Differential rotation implies shear and thus centers of activity are not expected to survive more than a few complete rotations. This may be relevant to the 'quasi-periodic' behavior of 3C 345 and other quasars.

In the models under discussion $\alpha^2 = 0.456$ on the basis of the Jeans' criterion at the center of the star, namely $\omega_c^2 = \frac{4}{3}\pi G\varrho_c$, and the equipotential and equidensity contours at the center of the star are flat. Using the Roche approximation I have been able to show that the equatorial radius is slightly less than six times the polar radius. Thus the SMS is considerably deformed and it is of interest to consider lower values for α^2 for which the spherical approximation is more accurate. If, for example, α^2 is decreased by $\sqrt{10}$ to 0.144 then the ratio of the equatorial to polar radii for the contours at the center is 1.2 and at the surface 2.5. However, it will be noted from Equation (23) that M must be decreased by a factor of 10 to maintain the same period as before the change in α^2. Moreover, under these circumstances $E_b^{\max}/Mc^2$ from Equation (20) is decreased by 10 and $E_b^{\max}$ by 100. This means, for example, a reduction in mean luminosity by a factor of 10 over a period shortened by a factor of 10. In this connection, however, it is possible to invoke magneto-turbulence to compensate for the reduction in α^2 and to restore the original values for P_R and $E_b^{\max}$. This possibility has been treated by Bardeen and Anand (1966).

Uniform rotation leads to a considerable reduction in $E_b^{\max}$ since Equations (20) and (21) are replaced by

$$\frac{E_b^{\max}}{Mc^2} = \frac{k^4\alpha^4}{64\zeta} = \frac{1}{14\,400} \tag{27}$$

and *(uniform rotation)*

$$x(E_b^{\max}) = \frac{k^2\alpha^2}{8\zeta} = \frac{1}{135} \tag{28}$$

In the numerical evaluation, $\alpha^2 = 1$, which corresponds to the Jean's criterion at the surface and to an equatorial radius approximately $\frac{3}{2}$ times the polar radius. Thus $E_b^{\max}$ is reduced by more than a factor of 200. Even so, $E_b^{\max} = 1.3 \times 10^{50}$ $(M/M_\odot)$ which is not to be sneezed at.

In any case it is of crucial importance to monitor quasars continuously to ascertain whether regular or quasi-regular periods are characteristic of these objects. The differentially rotating SMS model has readily calculable periods as indicated here. If future observations ultimately establish such periods then considerable support will be given to the SMS model for quasars. It must also be remembered that SMS may exhibit relaxation oscillations (Fowler, 1966a) and sporadic flare phenomena.

7. The Luminosity of SMS

Hoyle and Fowler (1963a) showed that the thermal luminosity of a stable SMS is proportional to the mass according to the approximate relation

$$l_{th} \equiv \frac{L_{th}}{Mc^2} = 10^{-16}\ \text{sec}^{-1} \tag{29}$$

or

$$L_{th} = 2 \times 10^{38} \left(\frac{M}{M_\odot}\right) \text{erg sec}^{-1} \tag{30}$$

Equation (30) leads to $L_{th} = 6 \times 10^{47}$ erg sec^{-1} for $M = 3 \times 10^9\ M_\odot$ as suggested for 3C 345 above. When combined with equation (22) this leads to a Kelvin-Helmholtz quasi-static contraction time of 1.5×10^{14} sec or 5×10^6 yr if no other type of radiation occurs. This interval is increased by 10^6 yr if hydrogen burning occurs during the stage of stable pulsations before $2x(E_b^{\max})$ is reached.

In accordance with current fashion it is now appropriate to add to L_{th} the radiative magnetic dipole luminosity on the assumption that the SMS has a surface magnetic field with component, B, normal to the axis of rotation. According to Deutsch (1955) this luminosity is given by

$$L_{md} = \frac{2\varepsilon}{3c^3} B^2 R^6 \omega_R^4 \tag{31}$$

where the factor $\varepsilon \equiv \langle\omega^4\rangle/\omega_R^4 \approx K^4/k^4 \approx 10^3$ has been introduced in recognition of the differential rotation which has cylindrical symmetry in the Stoeckly model (Stoeckly, 1965) and thus extends to the surface. There is considerable ambiguity in this procedure and even more in the nature of the magnetic field which can cut across the differentially rotating cylindrical shells. There is the difficult problem, too, associated with the survival of differential rotation in company with the magnetic field as well as with internal convection and turbulence. Notwithstanding these difficulties considerable insight can be gained into the behavior of the SMS under the energy and angular momentum loss implied by Equation (31). It will be found in what follows that the field B, necessary to yield $L_{md} \sim L_{th}$, is only a few thousand gauss and that the magnetic field energy is thus quite small compared to the rotational energy.

It might have been better to have used an expression due to Goldreich and Julian (1969) for L_{md} in Equation (31). In their expression the $\frac{2}{3}$ in Equation (31) becomes $\frac{1}{8}$ but, more important, B is the strength at the poles of a magnetic field parallel to the axis of rotation. When there is no winding of the field in differential rotation in Equation (35) to come, change the numerical coefficient from 24 to 128.

On the assumption of the conservation of magnetic flux it is possible to write

$$F_0 = B_0 R_0^2 = BR^2 \tag{32}$$

where F_0 is a constant measure of the flux and the subscript designates some fiducial time. Then (1), (3), (31) and (32) yield

$$L_{md} = \frac{2\varepsilon\alpha^4 F_0^2 G^2 M^2}{3c^3 R^4} \tag{33}$$

and

$$l_{md} \equiv \frac{L_{md}}{Mc^2} = \frac{x^4}{\tau} \tag{34}$$

where τ, which has the dimensions of time, is given by

$$\tau = \frac{24 G^2 M^3}{\varepsilon c^3 \alpha^4 F_0^2} = \frac{24 k^4 G^2 M^3}{c^3 K^4 \alpha^4 F_0^2} \tag{35}$$

The total luminosity must be equated to the time rate of decrease of the total energy of the system and thus to the rate of increase of the binding energy according to the equation

$$\frac{\mathrm{d}E_b}{\mathrm{d}t} = L_{\text{tot}} = L_{th} + L_{md}\,. \tag{36}$$

Although E_b must contain the post-Newtonian term which sets a limit on the energy which can be extracted from the SMS, namely $E_b^{\max}$ from Equation (20), yet the gravitational and rotational red-shift corrections to L_{tot} can be neglected. The correction factor is $\sim(1-x)$ which varies from unity down to $\frac{8}{9}$.

Substitution of Equations (4), (29) and (34) into Equation (36) leads to the following convenient equation

$$\frac{dt}{dx} = -\frac{R}{x}\frac{dt}{dR} = \frac{K^2\alpha^2(1-8\zeta x/K^2\alpha^2)}{4(l_{th}+x^4/\tau)} \tag{37}$$

where it will be recalled that l_{th} is a constant $=10^{-16}\ \sec^{-1}$. For arbitrary reasons x rather than R has been used in the right-hand side of Equation (37). This equation can be integrated to yield

$$t = \frac{K^2\alpha^2 X}{4l_{th}}\left[\frac{1}{4\sqrt{2}}\ln\frac{x^2+xX\sqrt{2}+X^2}{x^2-xX\sqrt{2}+X^2} + \frac{1}{2\sqrt{2}}\tan^{-1}\frac{xX\sqrt{2}}{X^2-x^2} - \frac{4\zeta X}{K^2\alpha^2}\tan^{-1}\frac{x^2}{X^2}\right] \tag{38}$$

where X is the value of x at which $l_{md}=l_{th}$ so that from equation (34)

$$X^4 = \tau l_{th}. \tag{39}$$

In Equation (38) it has been taken that $t=0$ at $x=0$ or $R=\infty$. Simple expressions are obtained for t in two interesting cases as follows:

Case I, $l_{th} \gg l_{md}$

$$t = \frac{K^2\alpha^2 x}{4l_{th}}\left(1-\frac{4\zeta}{K^2\alpha^2}x\right) = 3\times10^{16}x(1-4.5x)\ \sec \tag{40}$$

where it is taken that $t=0$ at $x=0$ or $R=\infty$. The time finally reached at $x=x(E_b^{\max})=K^2\alpha^2/8\zeta$ is then

$$t_{th} = \frac{K^4\alpha^4}{64\zeta l_{th}} = \frac{E_b^{\max}}{L_{th}} = 1.5\times10^{14}\ \sec = 5\times10^6\ \text{years} \tag{41}$$

as found previously and as expected since L_{th} has been taken constant.

Case II, $l_{md} \gg l_{th}$

$$1-\frac{t}{T} = \left(\frac{x_0}{x}\right)^3\left(1-\frac{12\zeta x}{K^2\alpha^2}\right) = \left(\frac{x_0}{x}\right)^3(1-13.5\,x) \tag{42}$$

$$= \left(\frac{R}{R_0}\right)^3\left(1-\frac{27GM}{Rc^2}\right) = \left(\frac{\omega_{R_0}}{\omega_R}\right)^2(1-13.5x) = \left(\frac{B_0}{B}\right)^{3/2}(1-13.5x) \tag{43}$$

where, in this case, $x=x_0$ at $t=0$ neglecting the relativistic term and

$$T = \frac{K^2\alpha^2\tau}{12x_0^3} = \frac{3}{32}\frac{\tau}{x_0^3} \tag{44}$$

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The time elapsed between $x=K^2\alpha^2/12\zeta$ and $x(E_b^{max})=K^2\alpha^2/8\zeta$ or between $l_{md}=(\frac{2}{3})^4\, l_{md}^{max} \approx 0.2 l_{md}^{max}$ and l_{md}^{max} is

$$\Delta t_{md} = \frac{64\zeta^3}{3K^4\alpha^4}\,\tau = \frac{512\zeta^3 k^4 G^2 M^3}{c^3 K^8 \alpha^8 F_0^2} = 3.65\,\frac{G^2M^3}{c^3F_0^2} \tag{45}$$

Equations (38), (40) and (43) indicate that R decreases with time while x, ω_R, and B increase with time. The rotational energy then increases with time according to Equations (6) or (7) while the angular momentum decreases according to Equations (14) or (15). This is in striking contrast to 'rigid' neutron stars for which R remains constant as ω_R, Ψ and Φ decrease. Neutron stars spin down under angular momentum loss while SMS contract and spin up. In the latter case the rotational energy increases but the binding energy also increases at the expense of gravitational energy. In the Newtonian approximation for an SMS in units of GM^2/R the various energies can be expressed for $K^2\alpha^2=\frac{9}{8}$ as

$$\Omega:E_b:\Psi:H = \tfrac{3}{2}:\tfrac{9}{16}:\tfrac{9}{16}:\tfrac{1}{4} \tag{46}$$

where Ω is the gravitational binding energy and H is the internal heat energy. From energy conservation $E_b=\Omega-H-\Psi$ and from the virial theorem $\Omega=H+2\Psi$ so that $E_b=\Psi$ in the Newtonian approximation. In the 3C 345 example $R\approx10^{16}$ cm and $GM^2/R\approx2\times10^{62}$ erg so $\Omega=3\times10^{62}$ erg, $E_b=\Psi\approx10^{62}$ erg, and $H\approx5\times10^{61}$ erg. Note that $\Psi/\Omega=\frac{3}{8}$ but when deformation is taken into account Ψ remains unchanged while Ω increases by $\sim6^{1/3}$ so $\Psi/\Omega\Rightarrow0.2$. General relativistic effects reduce the binding energy at maximum ($x=\frac{1}{9}$) by a factor of two to $E_b^{max}=\frac{1}{2}\Psi$.

In order to indicate as simply as possible the behavior of SMS with both thermal and magnetic dipole luminosity, Figure 3 has been drawn for the case where $L_{md}=L_{th}$ at $X=\frac{1}{12}$ with $M=3\times10^9\,M_\odot$ as in the calculations for 3C 345 above. From Equation (39) this is equivalent to setting

$$\tau = \frac{(3K^2\alpha^2/32\zeta)^4}{l_{th}} = \frac{1}{12^4 l_{th}} = 5\times10^{11}\ \text{sec} = 2\times10^4\ \text{years} \tag{47}$$

Thus

$$\Delta t_{md} = 2\left(\frac{3}{32}\right)^3 \frac{K^4\alpha^4}{\zeta l_{th}} = 5\times10^5\ \text{years} \tag{48}$$

It then follows that

$$L_{tot} = 6\times10^{47}\,[1+(12x)^4]\quad \text{erg sec}^{-1} \\ \Rightarrow 2.5\times10^{48}\ \text{erg sec}^{-1}\quad \text{at}\quad x=\tfrac{1}{9} \tag{49}$$

and

$$t = 1.32\times10^6\left[\ln\frac{(12x)^2+12\sqrt{2x}+1}{(12x)^2-12\sqrt{2x}+1} + 2\tan^{-1}\frac{12\sqrt{2x}}{1-(12x)^2} - \frac{3\sqrt{2}}{2}\tan^{-1}(12x^2)\right]\text{years} \Rightarrow 4.4\times10^6\ \text{years}\quad \text{at}\quad x=\tfrac{1}{9} \tag{50}$$

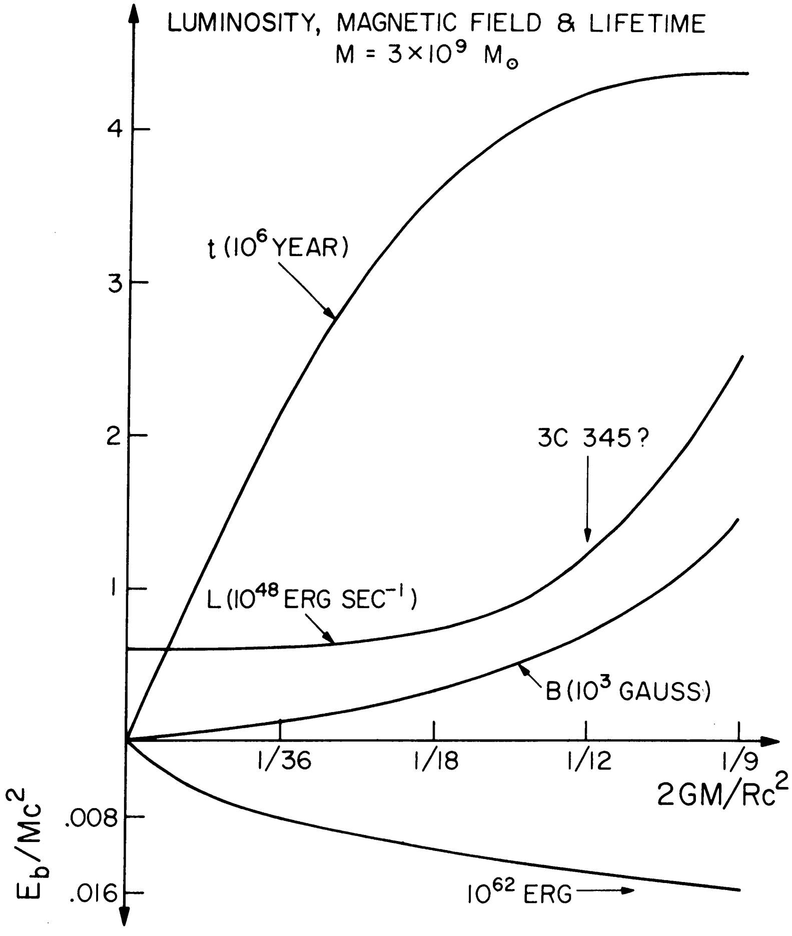

Fig. 3. Luminosity, magnetic field and lifetime of a SMS with $M = 3 \times 10^9\ M_\odot$ and $L_{md}/Mc^2 = L_{th}/Mc^2 = 10^{-16}\ \text{sec}^{-1}$ at $2GM/Rc^2 = 1/12$.

as illustrated in Figure 3. The necessary magnetic field can be calculated from Equations (35), (32) and (47) with the result

$$B^2 = \frac{3}{2}\left(\frac{32}{3}\right)^4 \frac{\zeta^4 k^4 c^5}{K^{12}\alpha^{12}G^2 M} x^4 l_{th} \tag{51}$$

and

$$B = 6.2 \times 10^9 \left(\frac{M_\odot}{M}\right)^{1/2} x^2$$
$$\Rightarrow 1400\ \text{G at}\ x = \tfrac{1}{9} \quad \text{for} \quad M = 3 \times 10^9\ M_\odot \tag{52}$$

as illustrated in Figure 3. A simple calculation shows that the magnetic field energy, $B^2R^3/6$, if B extends throughout the SMS, is equal to $\sim 3 \times 10^{53}$ erg which is small compared to the rotational energy $\Psi \sim 10^{62}$ erg. This justifies the neglect of the magnetic field in all considerations except the magnetic dipole luminosity.

In summary it has been found that the SMS model for quasars with rotational periods of the order of a few hundred days and with radial pulsation periods approximately one-quarter as long can supply thermal and magnetic dipole radiation of the order of 10^{48} erg sec^{-1} for somewhat more than 10^6 yr. The surface field required for the magnetic dipole radiation is approximately 10^3 G. The mass of the SMS taken as typical in these calculations is $3 \times 10^9 \, M_\odot$. Approximately $\frac{1}{64}$ of the rest mass energy or $\sim 10^{62}$ ergs is released during the Kelvin-Helmholtz contraction stage which is stabilized by differential rotation. Bardeen and Wagoner (1969) have shown that considerably more of the rest mass energy (up to 40%) can be released in the collapse to a disk shaped structure but this happy circumstance has not been treated in this paper. The periods during this collapse stage will be considerably shorter than discussed in this paper. However, the results of Bardeen and Wagoner do justify in some measure the neglect in this paper of higher order rotational and gravitational terms. Salpeter and Wagoner (1970) have recently circulated a preprint in which the relationship between supermassive disks and stars is discussed in some detail.

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Discussion

J. P. Ostriker: What is the ratio of kinetic to gravitational energy? If this ratio is more than $\frac{1}{4}$ and previous calculations may be used as a guide, then the model will be unstable to a non-axisymmetric $(m = 2)$ perturbations – that is fission will occur.

W. A. Fowler: The ratio is approximately $\frac{3}{8}$ from calculations based on spherical symmetry. Deformation does not change the rotational energy but the gravitational energy roughly doubles so a better estimate for the ratio is 0.2.

J. A. Roberts: Would the field of 1000 G exist in the region producing the radio synchrotron emission, since this should produce easily detectable circular polarization.

W. A. Fowler: That might be the case but Dr. Cavaliere can give the answer.

A. G. Cavaliere: May I add a comment on this. The radio-emission phenomena in QSS (van der Laan, Kellermann and Pauliny-Toth 'clouds') probably originate in lumped masses ejected beyond the 'critical surface' with radius $r \approx c/\Omega$ existing around these massive rotating cores as well as around rotating neutron stars. There the general magnetic field is much lower than the surface value ($B \sim 1/r^2$ or $1/r^3$ up to the critical surface, and $\sim 1/r$ from there on). The internal field of the clouds, from the cited model, is also smaller (< 1 G) when they become transparent.

https://doi.org/10.1017/S0074180900007683 Published online by Cambridge University Press