ON THE FEASIBILITY OF A STAR COORDINATE DETERMINATION IN THE RADIO ASTROMETRY REFERENCE SYSTEM

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At present it is evident that the stability of a celestial coordinates system with an accuracy of at least $0''.01$ per century, with respect to an internal frame can be derived only from VLBI observations of compact extragalactic radio sources. At the same time it is expected that the need for improved stars' coordinates will be strongly felt at least up to the end of this century, because stars are important and more convenient observational objects then very weak extragalactic sources or artificial Earth satellites (AES). One therefore wonders if one can use the high precision and stability of the radio astrometric reference frame to stabilize and improve the reference frame given by star catalogues. The problem of the relationship between these systems has so far been investigated in the form of determining the parameters of their mutual orientation, and/or studying systematic errors of star catalogues (Gubanov, 1978; Gubanov and Kumkova, 1978, 1981). This problem has been attacked by direct photography of several optically identified radio sources or VLBI observations of few radio stars. The present paper shows the feasibility of determining the place of any star of sufficient magnitude with reference to the radio astrometric frame, as given by extragalactic source positions (Gubanov, 1983). This is done by introducing an AES, equipped with radio signal for VLBI observations and a corner reflector for laser ranging, which can be observed by radio methods as well as optically (Gubanov, 1976). This satellite should also be bright enough in the reflected sun's light to be observed in the star field with a precision photoelectric satellite camera.

Consider a three element VLBI complex (A_1, A_2, A_3), which regularly observed an extragalactic source with known coordinates with respect to the radio astrometric reference frame. One such observation on the base of A_1A_2 yields a condition equation of the form:

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$$L_{1,2} = \Delta e_{1,2}\cos\delta\cos h_{1,2} - (\Delta Se)_{1,2}\cos\delta\sin h_{1,2} + \Delta p_{1,2}\sin\delta + \Delta m_{1,2} + \Delta n_{1,2}t, \tag{1}$$

where $L_{1,2} = (c\Delta\tau)_{1,2}$, with c being the velocity of light, $\Delta\tau = \tau - \tau_0$ the difference between the measured (τ) and the precalculated (τ_0) time delay. $\Delta e_{1,2}$ and $\Delta p_{1,2}$ are corrections to the equatorial and polar projections respectively of the base A_1A_2, $\Delta S_{1,2}$ is the correction to the sidereal time on the meridian of the same base, $h_{1,2}$ is the hour angle of the observed source on this meridian; $\Delta m_{1,2}$, $\Delta n_{1,2}$ are corrections for synchronization and relative daily rate, respectively of the local clocks, t is the time reckoned from some arbitrary zero epoch. Analogous equations hold for the other independent base A_1A_3. Two systems of equations (1) can be derived from the observations of a certain group of extragalactic radio sources. Their adjustment (by the method of least squares) will yield two sets of independent estimates for the parameters

$$\Delta e_{1,\nu},\ \Delta p_{1,\nu},\ (\Delta Se)_{1,\nu},\ \Delta m_{1,\nu},\ \Delta n_{1,\nu},\quad (\nu = 2,3). \tag{2}$$

The components of the baselines A_1A_2 and A_1A_3 with respect to the axis of the conventional equatorial coorinate system (x,y,z) are

$$\left.\begin{aligned} \Delta X_{1,\nu} &= \Delta u_1 - \Delta u_\nu = \Delta e_{1,\nu}\cos S_{1,\nu} - (\Delta Se)_{1,\nu}\sin S_{1,\nu},\\ \Delta Y_{1,\nu} &= \Delta v_1 - \Delta v_\nu = \Delta e_{1,\nu}\sin S_{1,\nu} + (\Delta Se)_{1,\nu}\cos S_{1,\nu},\\ \Delta Z_{1,\nu} &= \Delta w_1 - \Delta w_\nu = \Delta p_{1,\nu}, \qquad (\nu = 2,3), \end{aligned}\right\} \tag{3}$$

where (u_i, v_i, w_i) are components of the topocentric vectors

$$\vec{R}_i = \overrightarrow{A_i\,AES}, \qquad i = 1,2,3$$

with respect to the conventional equator system.

Simultaneously with the extragalactic radio source, a special AES should regulary be observed by the VLBI from the same network (A_1,A_2,A_3). In this case, the observations from the stations A_1 and A_2 as well as A_1 and A_3 should be strictly simultaneous. Moreover, at one of the stations of the network, laser ranging of the same AES should be carried out on clear nights simultaneously with the VLBI observations. If we exclude corrections (Δu_ν, Δv_ν, Δw_ν) with the help of the formulas (3) and parameters (2) from equations of the VLBI observations of the AES (Gubanov et al., 1980), we get a system of three equations with three unknowns together with the equations of the laser ranging of the AES at A_1, say

$$L_i = \frac{u_i}{R_i}\Delta u_1 + \frac{v_i}{R_i}\Delta v_1 + \frac{w_i}{R_i}\Delta w_1 \ , \quad i = 1, 2, 3 \ , \qquad (4)$$

which can be solved because its determinant is generally different from zero.

The solution of the system (4) for each instant at which VLBI observations and laser ranging of one and the same AES are carried out simultaneously yields

$$\vec{R}_1 = (u_1 \, , \, v_1 \, , \, w_1)^T$$

the topocentric position vector of the AES from A_1. In the equations (4) we have:

$$\begin{aligned} L_1 &= c\Delta\tau_1^*/2 \ , \\ L_\nu &= \frac{u_\nu}{R_\nu}\Delta X_{1,\nu} + \frac{v_\nu}{R_\nu}\Delta Y_{1,\nu} + \frac{w_\nu}{R_\nu}\Delta Z_{1,\nu} + \Delta m_{1,\nu} + \\ &+ \Delta n_{1,\nu} t + c\Delta\tau_{1,\nu} + L_1 \ , \qquad \nu = 2, 3 \end{aligned} \qquad (5)$$

where $\Delta\tau_1^*$ is the difference of the measured and precalculated time delay of the laser ranging signal transmitted from A_1.

From formulas (5) one can see that the precision of L_i is determined mainly by the precision of the independent laser ranging and VLBI measurements of the time delays $\Delta\tau_1^*, \Delta\tau_{1,2}, \Delta\tau_{1,3}$, because the other terms included in these formulas are determined more precisely from VLBI observations of the extragalactic radio source group. At present the precision of single measurements by means of laser ranging and VLBI technique may be assumed to be about 10^{-10} sec (Carter, 1980; Silverberg, 1980). The vector $\vec{R}_1$ will give the topocentric equatorial coordinates α_1 and δ_1 of the AES,

$$\alpha_1 = \mathrm{arctg}\,\frac{v_1}{u_1} \, , \quad \delta_1 = \mathrm{arctg}\,\frac{w_1}{\sqrt{u_1^2 + v_1^2}} \qquad (6)$$

and so we have a possibility to determine of corrections $\Delta\alpha_1\cos\delta_1$, $\Delta\delta_1$ with a precision of about 0".01 - 0".001. To determine the position of a star in the same reference frame it is thus necessary to measure relative positions of the star and the moving AES simultaneously with laser ranging and VLBI observations. This requires a fast and accurate photographic satellite camera.

One can talk about an improvement of star positions, entered into modern catalogues, when the accuracy of the measurement is 0"2, while results, to be useful, would have standard errors. Unfortunately, modern photographic satellite cameras provide an accuracy of only 1", and on top of it the derivation of final results is time-consuming. Thus, the necessary satellite camera is unavailable at present. Hovever, recent advances in the microelectronics (the application of charge-coupled devices (CCD-Matrix)) encourage the idea that this problem will be resolved in the nearest future.

Note that there is no need to fill the entire field of the instrument with charge-coupled devices. It is sufficient to have three bands, as is shown in Figure 1, designated by $(\alpha_1 \beta_1)$ $(\alpha_2 \beta_2)$ $(\alpha_3 \beta_3)$. Moreover, there is no need to read electric charges from all the CCD-matrices. It is sufficient to read only two matrices: the central one (A) and that which the target star transits at the moment.

The analysis shows that the most convenient way observing is a continuous satellite tracking. The AES image will then be practically stationary in the field of the instrument and will be displayed on the central CCD-matrix A. Star images will migrate in the direction given by topocentric projection of the AES orbit on the celestial sphere, and this will be known with sufficient accuracy. Since the positions of the target stars are approximately known, one can predict which CCD-matrix the star will be imaged on.

The limiting magnitude of stars and the AES depends on the AES velocity, and hence on the altitude of its orbit as well as the sensitivity of charge-coupled devices which can be very high, counting on advances in technology especially thermal noise suppression. The radius of AES orbit should be about 12.5 thousand kilometers, its inclination and equator density about 75° (Gubanov et al., 1976) so that the orbit projection would cover the entire celestial sphere for 5 years.

Consider the AES passing a group of 3 stars, two of which cross the field of the instrument at the edges and the third in the center. Their paths indicated by dashed lines in Figure 1 and the points at which their positions are measured are provided with subscripts j,k= 1, 2, 3 (j is the index of position in the field of the instrument and k is the star index). Laboratory measurements will bring the coordinate systems of all the CCD-matrices into good agreement, so that one may consider the common ortho-

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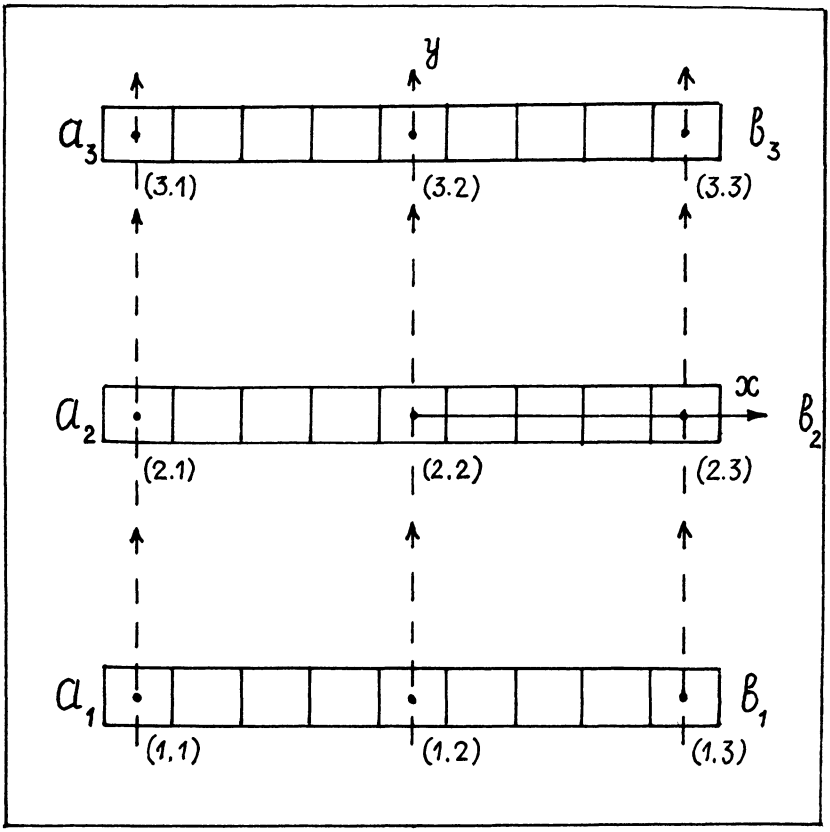

Fig. 1

gonal system of measured coordinates (x, y) with the origin at O (Figure 1) to coincide with the tangential point of the focal plane of the instrument. These measurements should insure the equality of the scale of the measurement in Ox and Oy. If we precalculate differential corrections for refraction, aberration, plate tilt and others, including the terms of the highest order, the relationship between the tangential (X_{jk}, Y_{jk}) and mesured (x_{jk}, y_{jk}) coordinates of the k-th star at the instant t_j will be an expansion, rotation and shift of origin, thus require only four parameters for its description, namely m, the scale, β is the angle between the axis (x, y) and (X, Y), as well as c and d, the coordinates of the origin of the (x, y) system.

Assuming AES image to be the origin of the tangential coordinates (X, Y), we have, trasforming to equatorial coordinates of the stars (α_k, δ_k) and the satellite (α_j, δ_j)

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$$\left.\begin{aligned} \Delta m\, p_{jk} - (\Delta\beta m)_j\, q_{jk} - (\Delta\alpha\cos\delta)_k A_{jk} + \Delta\delta_k B_{jk} + M_{jk} = 0, \\ \Delta m\, q_{jk} + (\Delta\beta m)_j\, p_{jk} - (\Delta\alpha\cos\delta)_k C_{jk} - \Delta\delta_k D_{jk} + N_{jk} = 0, \end{aligned}\right\} \quad (7)$$

where Δm is the scale correction, $\Delta\beta$ the correction to the angle between the two coordinates systems, and $\Delta\alpha_k, \Delta\delta_k$, are the corrections to the stars' coordinates.

Various symbols in Eqs. (7) have the following meaning:

$$p_{jk} = \xi_{jk}\cos\beta_j - \eta_{jk}\sin\beta_j \;, \quad q_{jk} = \xi_{jk}\sin\beta_j + \eta_{jk}\cos\beta_j \;;$$

$$M_{jk} = m\Delta\xi_{jk}\cos\beta_j - m\Delta\eta_{jk}\sin\beta_j \;, \quad N_{jk} = m\Delta\xi_{jk}\sin\beta_j + m\Delta\eta_{jk}\cos\beta_j \;;$$

$$A_{jk} = [\cos\delta_k\cos\delta_j + \sin\delta_k\sin\delta_j\cos(\alpha_k - \alpha_j)]/E_{jk}^2 \;,$$

$$B_{jk} = \sin\delta_j\sin(\alpha_k - \alpha_j)/E_{jk}^2 \;,$$

$$C_{jk} = \sin\delta_k\sin(\alpha_k - \alpha_j)/E_{jk}^2 \;,$$

$$D_{jk} = \cos(\alpha_k - \alpha_j)/E_{jk}^2 \;,$$

$$E_{jk} = \sin\delta_k\sin\delta_j + \cos\delta_k\cos\delta_j\cos(\alpha_k - \alpha_j) \;.$$

In these equations,

$$\xi_{jk} = x_{jk} - x_j \;, \quad \eta_{jk} = y_{jk} - y_j$$

are differences between the stars' measured coordinates and the AES at the instant t_j, and

$$\Delta\xi_{jk} = \xi_{jk} - (\xi_{jk})_0 \;, \quad \Delta\eta_{jk} = \eta_{jk} - (\eta_{jk})_0$$

are the differences between their measured and precalculated values.

The most difficult task is to provide for the stability of the correction $\Delta\beta_j$, although this is needed only for that very short time intervals, during which the additional condition equation system (7) is valid. This interval is 3 minutes for a field of 5° x 5° and an AES altitude of 6000 km. The task is evidently simplest to solve when the instrument is mounted at the equator.

Table 1 gives the weight coefficients of the unknowns in system (7), derived for the group of three stars in Figure 1 and solved by least squares, with $\Delta\beta_j = \Delta\beta =$ const.

Table 1.

β	Δm	$\Delta\beta m$	$(\Delta\alpha\cos\delta)_1$	$\Delta\delta_1$	$(\Delta\alpha\cos\delta)_2$	$\Delta\delta_2$	$(\Delta\alpha\cos\delta)_3$	$\Delta\delta_3$
0°	6	6	2	2	3	3	2	2
45°	6	6	2	2	3	3	2	2

The data of the Table show that the results are independent of the angle β , both coordinates of each star are determind with eqal accuracy. The latter decreases toward the edge but not significantly.

So, we can conclude that at present there exists the feasibility of a rather accurate determination of stella: coordinates and coordinates of other optical objects with respect to an internal frame of reference which itself is assumed to be realized by positions of extragalactic radio sources. Assuming that this reference frame was determined by essentially new methods of ground based observations, independent of the methods of classical astrometry, the only device, needed for the implementation of the proposals in this paper, is a very precise and fast operating satellit photoelectric camera.

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