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Quantum superalgebras at roots of unity and topological invariants of three-manifolds

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The general method of Reshetikhin and Turaev is followed to develop topological invariants of closed, connected, orientable 3-manifolds from a new class of algebras called pseudo-modular Hopf algebras. Pseudo-modular Hopf algebras are a class of $\mathbb{Z}_2$-graded ribbon Hopf algebras that generalise the concept of a modular Hopf algebra.

The quantum superalgebra $U_q\big(osp(1 \mid 2n)\big)$ over $\mathbb{C}$ is considered with q a primitive N^{th} root of unity for all integers $N \geq 3$. For such a q, a certain left ideal $\mathcal{I}$ of $U_q\big(osp(1 \mid 2n)\big)$ is also a two-sided Hopf ideal, and the quotient algebra $U_q^{(N)}\big(osp(1 \mid 2n)\big) = U_q\big(osp(1 \mid 2n)\big)/\mathcal{I}$ is a $\mathbb{Z}_2$-graded ribbon Hopf algebra.

For all n and all $N \geq 3$, a finite collection of finite dimensional representations of $U_q^{(N)}\big(osp(1 \mid 2n)\big)$ is defined. Each such representation of $U_q^{(N)}(osp(1 \mid 2n))$ is labelled by an integral dominant weight belonging to the truncated dominant Weyl chamber. Properties of these representations are considered: the quantum superdimension of each representation is calculated, each representation is shown to be self-dual, and more importantly, the decomposition of the tensor product of an arbitrary number of such representations is obtained for even N.

It is proved that the quotient algebra $U_q^{(N)}\big(osp(1 \mid 2n)\big)$, together with the set of finite dimensional representations discussed above, form a pseudo-modular Hopf algebra when $N \geq 6$ is twice an odd number.

Using this pseudo-modular Hopf algebra, we construct a topological invariant of 3-manifolds. This invariant is shown to be different to the topological invariants of 3-manifolds arising from quantum $so(2n+1)$ at roots of unity.

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