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# GROUPS–ST ANDREWS 1985

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The conference Groups–St Andrews 1985 was held at the University of St Andrews from 27 July to 10 August 1985. The conference received financial support from the Edinburgh Mathematical Society, the London Mathematical Society and the British Council. There were 366 participants from 43 countries registered for the conference. Although the conference did not specialize in a particular area of group theory, a glance at Mathematical Reviews shows that the work of the participants is mainly under classifications 20D, 20E and 20F. In part this is because the conference followed an earlier conference [**6**] which was primarily based on topics falling under 20F.

The five main speakers at Groups–St Andrews 1985 have contributed survey articles to the separate volume [**31**] which also contains other survey and research articles. Research articles submitted by participants to the Edinburgh Mathematical Society form the contents of this volume (Part 1 Volume 30, of the Proceedings of the Edinburgh Mathematical Society). The range of topics covered in both the volume [**31**] and this volume are representative of the activity of the conference in both the formal and informal sessions.

It is of interest to consider the effect of a large specialist conference on the area of mathematics it covers. For example the influence of the major finite group theory conference [**10**] in 1979 and of Groups–St Andrews 1981 is easily detected by explicit references to papers in the Conference Proceedings, by papers written by joint authors who collaborated because of the conference and in papers motivated by ideas which were raised and discussed at the conference. Monographs such as [**33**] and [**37**] also exert much influence as do books such as the "Essays for Philip Hall" [**16**] which contains well-written survey articles.

For the rest of this article we single out several of the areas of group theory which, it seemed to us, the conference considered important in the current development of the subject. We wish to thank the many conference participants whose ideas have helped us with this article.

The classification of finite simple groups in 1980 clearly ended a major phase in the development of group theory. This work is now being made more accessible by Gorenstein in a series of books [**13**], [**14**] and [**15**] where he presents all the ideas necessary for the classification and also gives an overview of the inter-relations between the various parts of the jigsaw. Studying the ramifications of the classification heralds a new phase of group theory which is well under way. In order to apply the classification

*This paper is an introduction to the Proceedings of Groups–St Andrews 1985.

theorem to prove other important theorems about general finite groups requires a detailed knowledge of the structure of finite simple groups such as the subgroup structure and character tables. Attempts to discover these details and so to understand the sporadic simple groups (in particular the Monster [**8**]) and relate them to other areas of mathematics such as number theory and geometry have led to important, and as yet not well understood, results; see Norton's article in [**2**]. A major catalogue of properties of the "interesting" finite simple groups is given by Conway *et al.* in the $\mathbb{ATLAS}$ [**9**]. It was in the final stages of preparation at the time of the conference and several talks described $\mathbb{ATLAS}$ related work, see for example Wilson's article on maximal subgroups in [**31**].

A more minor, yet nonetheless important, classification theorem more recently obtained is the classification of the simple periodic linear groups. The theorem showing that an infinite simple periodic linear group is a Chevalley group or a twisted Chevalley group over a locally finite field was obtained independently by a number of authors, see for example [**4**], [**21**] and [**35**]. Important work on the classification of the irreducible complex representations of the groups of rational points of reductive groups over finite fields has been done by Lusztig and others, see [**27**] and [**28**]. The Deligne–Lusztig theory of irreducible characters of Chevalley groups constructed using the theory of $l$-adic cohomology is made accessible to a wider audience by Carter's book [**7**].

Tits' local characterization theorems for buildings [**36**] have had important implications for finite groups and finite geometries. The paper [**36**] introduces the concept of a chamber system and the results apply to the geometries of some of the sporadic groups showing that their universal covers are affine buildings. Work on buildings and on the geometries of finite groups is described in [**32**]. Papers on amalgams of finite groups and finite geometries appear in [**1**] and these topics are also discussed in Tits' article in [**31**].

Many important recent ideas are described in the articles presented to Groups–St Andrews 1985. For instance the variety of representation techniques developed by Culler and Shalen [**11**] have had considerable influence in different areas. The set of all representations $R(G)$ of a finitely generated group $G$ in $SL(2,\mathbb{C})$ is thought of as a complex algebraic variety. Now if $G=\langle x_1,x_2,\dots,x_n\rangle$ then $r\in R(G)$ corresponds to the point $(r(x_1),\ r(x_2),\dots,r(x_n))\in SL(2,\mathbb{C})^n\subseteq\mathbb{C}^{4n}$. These points of $\mathbb{C}^{4n}$ satisfy algebraic equations coming from the defining relations of $G$. Some of the developments arising from this work are described in Baumslag's article in [**31**], see also [**3**].

A recent result of Stöhr [**34**] proves the following: Let $G(n)$ be the free centre-by-metabelian group of rank $n$. Then Aut $G(2)$ and Aut $G(3)$ are infinitely generated but Aut $G(n)$, $n>3$, is finitely generated. (Here Aut $G(n)$ means the automorphism group of $G$.) This theorem is important as it presents a situation which the articles by Bachmuth and Mochizuki in [**31**] indicate may be typical in the following sense: If $F(n)$ is the free group of rank $n$ in a variety which is not defined by a power law (only commutator laws) then does there exist $N$ such that Aut $F(n)$ is finitely generated if and only if $n<N$?

At the time of Groups–St Andrews 1981 it appeared that applying computing to solve group theory problems was a specialist area. However at Groups–St Andrews 1985 it was clear that the computer has become a standard tool for a large number of group theorists. Although several different group theory systems have been implemented on computers to handle specific problems the most general, and widely used, system is

CAYLEY; see Cannon's article (and some other articles) in [**2**]. Neumann's article in [**31**] is an example of how group theorists are now thinking in terms of algorithms and their implementation.

The theory of $p$-groups has over recent years received impetus from the use of computers. However much interesting new work discussed at Groups–St Andrews 1985 was not so motivated. Firstly there is important work of Roggenkamp and Scott on the isomorphism problem for integral group rings of $p$-groups, see [**31**]. Secondly several clever constructions of examples of "badly behaved" infinite $p$-groups contribute significantly to Burnside problems. Gupta and Sidki have developed a technique to construct infinite $p$-groups by extending groups by tree automorphisms [**18**], [**19**] and [**20**]. This same technique is used by Gupta [**17**] to construct, in an extremely simple way, 2-generator infinite recursively presented not finitely presentable $p$-groups. Another spectacular but very difficult result in this area is Ol'sanskiĭ's proof [**29**] that for any sufficiently large prime $p$, there exists an infinite group in which every proper subgroup has order $p$. This paper is one of a series in which Ol'sanskiĭ uses powerful geometric methods representing relations in groups by diagrams in a similar way to small cancellation theory. A later paper in this series is [**30**] where Ol'sanskiĭ shows the existence of a non-abelian variety of groups, defined by a single law with two entries, in which all finite groups are abelian.

The work of Leedham-Green *et al.* shows an increasing evidence for the interaction between $p$-groups and other areas of mathematics such as profinite groups, Lie groups and computing. A group of order $p^n$ and class $c$ is said to have *coclass* $n-c$. Leedham-Green and Newman [**25**] proposed a programme to classify $p$-groups based on the idea of the coclass. Considerable progress has been made in verifying the conjectures in this paper; see for example [**24**]. Major progress has also been made in the classification of the $p$-groups of maximal class, that is coclass 1, in a series of papers, see for example [**23**]. The work of Lubotzky and others makes ideas on $p$-adic Lie groups useful in abstract group theory [**26**]. Finitely generated linear groups over fields of characteristic 0 are characterized in terms of dense embeddings in analytic $p$-adic groups, see Lubotzky's article in [**31**].

Not only in the area of $p$-groups have clever constructions of examples made important new contributions. In the theory of equations over groups an important open problem is the Kervaire conjecture, namely that any system of equations over a group $G$ can be solved in some overgroup of $G$. Recent work by Stallings, Gersten and Howie has attacked the Kervaire conjecture via various stronger conjectures. All these stronger conjectures are shown to be false by a recently constructed counterexample of Gersten [**12**].

Often papers are as important for the ideas introduced as for the results actually proved. One such important new concept is that of a constrained module introduced by Kropholler in [**22**]. If $k$ is a field and $G$ is a group, a $kG$-module $M$ is called *constrained* if, for each $g \in G$, $M$ is locally finite dimensional as a $k\langle g\rangle$-module. Kropholler uses this concept in his proof that a finitely generated soluble group is minimax if and only if it has no section of type $C_p \, wr \, C_\infty$ for any prime $p$. Constrained modules also appear in the work of Brookes [**5**] on finitely generated soluble groups.

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