terminator or limit of the sun's illumination of the earth (at right angles to the line showing the direction of the sun's rays), a' is the point at which the terminator, $a'O'c'$, cuts the elevation of the line of latitude λ.

Referring to the plan, a and a are the points in plan corresponding to a' in elevation ; O is the plan of the polar axis.

It is evident that a place on the circle of latitude λ will receive light while moving along the path represented by the arc ala (plan), and be in darkness while moving along the path represented by the arc ama.

Since these arcs are proportional to the angles they subtend at the centre, we have,

$$\frac{\text{duration of day}}{24 \text{ hours}}=\frac{2\theta}{360^\circ}=\frac{\theta}{180^\circ}.$$

Referring to the figure, it is seen that :

$$a'd' \text{ (elevation)} = bO \text{ (plan)},$$
$$l'd' \text{ (,,)} = Oa \text{ (plan)},$$
$$\angle O'l'd' \text{ (,,)} = \lambda,$$
$$\angle a'O'd' \text{ (,,)} = \delta.$$

Now $$O'd' = R \sin \lambda,$$

and $$l'd' = R \cos \lambda \text{ (where } R \text{ is the radius of the earth).}$$

Again, $$a'd' = O'd' \tan \delta$$
$$= R \sin \lambda \tan \delta.$$

Hence $$bO \text{ (plan)} = R \sin \lambda \tan \delta.$$

Let $\angle aOa$ (plan) $=2\theta$, then $\angle aOb=\theta$,

and $$\cos \theta = \frac{bO}{Oa} = \frac{R \sin \lambda \tan \delta}{R \cos \lambda}.$$
$$= \tan \lambda \tan \delta.$$

Hence $\theta = \cos^{-1}(\tan \lambda \tan \delta)$, which determines θ.

As before, $$\frac{\text{length of day}}{24 \text{ hours}} = \frac{\theta}{180};$$

$$\text{length of day} = \frac{\cos^{-1}(\tan \lambda \tan \delta)}{180} \times 24 \text{ (hrs.)}.$$

The application of the formula to the special cases (i) in which $\delta=0$, (ii) the determination of λ for which $\theta=0$ when $\delta=23\frac{1}{2}^\circ$, (iii) in which $\lambda=0$, (iv) in which $\lambda>66\frac{1}{2}^\circ$ and $\delta=23\frac{1}{2}^\circ$ is interesting.

The formula can be also used for finding the times of sunrise and of sunset.

A. H. Bell.

Sheerness Technical Institute.

Obituary.

Prof. W. H. H. Hudson.

In the late Prof. W. H. H. Hudson, the Association has lost a member of thirty years' standing. He was an earnest and vigorous promoter of many causes in which his ripe judgment, his engaging personality, and profound sympathy will be sorely missed. For some twenty years he was Fellow and Lecturer at St. John's College, Cambridge, and there will be among the members of that society, dating back between the '60's and the '80's, as universal a chorus of appreciation as would be raised by the many pupils with whom he was later associated at King's College, London, and Queen's College, Harley Street. It was

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with peculiar satisfaction that one so closely identified with the movement for the Higher Education of Women found himself able to give the benefit of his enthusiasm and his experience to his fellow governors of Newnham. His work upon the Councils of the London Mathematical Society and of the Mathematical Association, of which he was for many years a distinguished and active member, was highly appreciated. Pupils were constantly passing through his hands to take up their posts as teachers of Mathematics, and he was thus enabled indirectly to sow the good seed which bore fruit eventually in the success of the movement for reform in the teaching of this subject. His own teaching was sound and thorough, and based upon principles which, as a writer in a contemporary has remarked, "received remarkable vindication in the record of his family." His only son showed promise of being one of the most remarkable men of his time, and the three daughters who have survived him are also evidence of the combined effect of nature and nurture upon the members of a highly gifted family. His life was not without its tragedies, but he faced the decrees of Fate with signal fortitude, and in this, as in all else, was a noble example to those who were admitted to the privilege of his friendship.

MATHEMATICAL NOTES.

463. [V. 1. a. δ, ε.] *On Some Arithmetical Conventions.*

A contention that the value of $7-3\times 2$ may be either 1 or 8 recently surprised me, and has led me to look somewhat carefully into the conventions as to the sequence in which arithmetical operations are to be performed.

Many arithmetical books, in their chapter on Fractions, lay down three rules of interpretation. Stated in their baldest form these rules are :

1. Multiplications and divisions must be performed before additions and subtractions. This assigns a meaning to $27\pm 5\times 3$, say $a\pm b\times c$.
2. Multiplications and divisions must be performed in order (from left to right). This assigns a meaning to $a\div b\times c$, viz. $\frac{ac}{b}$.
3. The word 'of' is, however, equivalent to a bracket. According to this,

$$a\div b \text{ of } c \text{ means } \frac{a}{bc}.$$

It will be convenient to state at once the conclusions I have reached, before entering into the arguments on the subject.

These are, that the Rule 1, though not always happily expressed, is a rule of fundamental importance, and is essential to the harmony of arithmetic and algebra ; but that Rules 2 and 3 are of an artificial character, that they are not necessary and that they cannot be defended.

There is little doubt that Rule 1 has suffered from being found in bad company. The case for its separate existence seems to be (apart from mere authority, though Tannery and Workman both adopt it) :

(*a*) Algebraic and arithmetical expressions consist of terms.

(*b*) Apart from brackets + and − separate terms.

(*c*) There is no essential distinction between ab, $a\ b$ and $a\times b$. Each denotes the product of a and b.

If a and b are numbers expressed in figures, the sign × must be used [for may be confused with a decimal point, *Enc. des Sc. Math.* I, 1, i, p. 40]. Contrast $2{\cdot}4\times 3{\cdot}1$, $2{\cdot}4\,.\,3{\cdot}1$ and $2{\cdot}43{\cdot}1$.