ON 3-MANIFOLDS WITH SUFFICIENTLY LARGE DECOMPOSITIONS*: CORRIGENDUM

WOLFGANG HEIL

I would like to thank G. A. Swarup for pointing out that our proof in the above paper does not yield the theorem as stated, but rather gives the following.

THEOREM. Let M, N be compact, orientable, boundary irreducible 3-manifolds. Suppose that ∂M , ∂N contain no 2-spheres. Suppose that M has sufficiently large decomposition,

$$M \approx M_1 \# M_2 \# \ldots \# M_n.$$

If there exists an isomorphism $\psi : \pi_1(M) \to \pi_1(N)$ which respects the peripheral structure, then N has a decomposition $N \approx N_1 \# \dots \# N_n \# H$, where $N_i \approx M_i$ and H is a homotopy 3-sphere.

The original theorem follows from this only if we assume in addition that (at least) n - 1 of the summands of M admit orientation reversing homeomorphisms (or if n - 1 summands of M are non-orientable and satisfy the conditions of Remark 1).

In general the original theorem is false: Let F_1 , F_2 be irreducible, closed, orientable, and sufficiently large Seifert fiber spaces such that $\pi_1(F_1) \not\cong \pi_1(F_2)$ and F_1 , F_2 do not admit orientation reversing homeomorphisms. (F_i exists, e.g., $F_i = (0, 0; p | b_i; \alpha_{i1}, \beta_{i1}; \ldots; \alpha_{ir_i}, \beta_{ir_i})$ with $p \ge 2, b_i \ne -r_i - b_i$. Then (see [2]) F_i admits no orientation reversing fiber preserving homeomorphisms and therefore, by Walhausen's result [1], no orientation reversing homeomorphisms.) Let M and N be the two possible connected sums of F_1 and F_2 . Then $\pi_1(M) \cong \pi_1(N)$, but $M \ne N$.

References

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Florida State University, Tallahassee, Florida

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