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## References

- [1] Abramovich, Y. A. and Aliprantis, C. D., An invitation to operator theory, *Graduate Studies in Mathematics*, Amer. Math. Soc. **50**, Providence, RI, 2002.
- [2] Adams, R. A., *Sobolev Spaces*, Academic Press, New York, 1975.
- [3] Adimurti, A. and Tintarev, K., Hardy inequalities for weighted Dirac operators, *Ann. Mat. Pura Appl.* **189** (2010), 241–251.
- [4] Avkhadiiev, F. G., Hardy type inequalities in higher dimensions with explicit estimate of constants, *Lobachevskii J. Math.* **21** (2006), 3–31.
- [5] Avkhadiiev, F. G. and Laptev, A., Hardy inequalities for non-convex domains. In *Around the Research of Vladimir Maz'ya, I* (F. Laptev, ed.), International Mathematical Series **11**, pp. 1–12, Springer, New York (2010).
- [6] Avkhadiiev, F. G. and Wirths, K.-J., Sharp Hardy type inequalities with Lamb's constants, *Bull. Belg. Math. Soc., Simon Stevin* **18** (2011), 723–736.
- [7] Avkhadiiev, F. G., Families of domains with best possible Hardy constants, *Russian (Iz VUZ)* **57** (2013), 49–52.
- [8] Avkhadiiev, F. G. and Makarov, R. V., Hardy type inequalities on domains with complex complement and uncertainty principle of Heisenberg, *Lobachevskii J. Math.* **40**(9) (2019), 1250–1259.
- [9] Almgren, F. J. and Lieb, E.H., Symmetric decreasing rearrangement is sometimes continuous, *J. Amer. Math. Soc.* **2**(2) (1989), 683–773.
- [10] Alvino, A., Sulla diseguaglianza di Sobolev in spazi di Lorentz, *Boll. Un. Mat. Ital. A* **14** (1977), 148–156.
- [11] Alvino, A., Lions, P.-L. and Trombetti, G., On optimization problems with prescribed rearrangements, *Nonlinear Anal.* **13** (1989), 185–220.
- [12] Ancona, A., On strong barriers and an inequality of Hardy for domains in  $\mathbb{R}^2$ , *J. Lond. Math. Soc.* **33** (1986), 274–290.
- [13] Aronszajn, N., Boundary values of functions with finite Dirichlet integral, *Tech. Report 14*, Univ. of Kansas, 1955, 77–94.
- [14] Balinsky, A., Evans, W. D., Hundertmark, D. and Lewis, R. T., On inequalities of Hardy-Sobolev type, *Banach J. Math. Anal.* **2**(2) (2008), 94–106.
- [15] Balinsky, A., Evans, W. D. and Lewis, R. T., *The Analysis and Geometry of Hardy's Inequality*, Springer, 2015.
- [16] Bañuelos, R. and Wang, G., Sharp inequalities for martingales with applications to the Beurling-Ahlfors and Riesz transformations, *Duke Math. J.* **80** (1995), 575–600.

- [17] Barbatis, G., Filippas, S. and Tertikas, A., A unified approach to improved  $L^p$  Hardy inequalities with best constants, *Trans. Amer. Math. Soc.* **356**(6) (2004), 2169–2196.
- [18] Barbatis, G. and Tertikas, A., On the Hardy constant of non-convex planar domains: the case of a quadrilateral, *J. Funct. Anal.* **266** (2014), 3701–3725.
- [19] Barbatis, G. and Tertikas, A., On the Hardy constant of some non-convex planar domains, in *Geometric Methods in PDEs*. Springer INdAM, vol. 13 (2015), 15–41.
- [20] Benguria, R. D., Frank, R. L. and Loss, M., The sharp constant in the Hardy–Sobolev–Maz’ya inequality in three dimensional upper half-space, *Math. Res. Lett.* **15**(4) (2008), 613–622.
- [21] Bennett, C. and Sharpley, R., *Interpolation of Operators*, Academic Press, 1988.
- [22] Bogdan, D. and Dyda, B., The best constant in a fractional Hardy inequality, *Math. Nachr.* **284**(5–6) (2011), 629–638.
- [23] Bourgain, J., Brezis, H. and Mironescu, P., Another look at Sobolev spaces. In *Optimal Control and Partial Differential Equations (In Honor of Professor Alain Bensoussan’s 60th Birthday)* (J. L. Menaldi et al., eds.), pp. 439–455, IOS Press, Amsterdam (2001).
- [24] Bourgain, J., Brezis, H. and Mironescu, P., Limiting embedding theorems for  $W^{s,p}$  when  $s \uparrow 1$  and applications, *J. d’Analyse Math.* **87** (2002), 77–101.
- [25] Brasco, L. and Cinti, E., On fractional Hardy inequalities in convex sets, *Discr. Contin. Dyn. Syst. Ser. A* **18** (2018), 4019–4040.
- [26] Brasco, L. and Franzina, G., On the Hong–Krahn–Szegö inequality for the  $p$ -Laplace operator, *Manuscripta Math.* **142** (2013), 537–557.
- [27] Brasco, L. and Franzina, G., Convexity properties of Dirichlet integrals and Picone-type inequalities, *Kodai Math. J.* **37** (2014), 769–799.
- [28] Brasco, L., Lindgren, E. and Parini, E., The fractional Cheeger problem, *Interfaces and Free Boundaries* **16** (2014), 419–458.
- [29] Brasco, L. and Parini, E., The second eigenvalue of the fractional  $p$ -Laplacian, *Adv. Calc. Var.* **9** (2016), 323–355.
- [30] Brasco, L., Parini, E. and Squassina, M., Stability of variational eigenvalues for the fractonal  $p$ -Laplacian, *Discr. Cont. Dyn. Syst.* **36** (2016), 1813–1845.
- [31] Brasco, L. and Salort, A., A note on homogeneous Sobolev spaces of fractional order, *Annali di Math. Pura ed Appl.* **198** (2019), 1295–1330.
- [32] Brezis, H., *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, New York, 2011.
- [33] Brezis, H., How to recognise constant functions, *Russ. Math. Surv.* **57** (2002), 693–708.
- [34] Brezis, H. and Marcus, M., Hardy’s inequalities revisited, *Ann. Scuola Norm. Sup. Pisa Cl. Sci.* **25**(4) (1997), 217–237.
- [35] Brezis, H., Schaftingen, J. V. and Yung, Po-Lam, Going to Lorentz when fractional Sobolev, Gagliardo and Nirenberg estimates fail, *Calc. Var.* **60**(129) (2021), 1–12.
- [36] Brezis, H., Schaftingen, J. V. and Yung, Po-Lam, A surprising formula for Sobolev norms, *Proc. Natl. Acad. Sci. USA* **118**(8), e2025254118(2021), <https://doi.org/10.1073/pnas.202524118>.
- [37] Browder, F. E., Nonlinear elliptic boundary value problems and the generalised topological degree, *Bull. Amer. Math. Soc.* **76** (1970), 999–1005.

- [38] Buncur, C. and Valdinoci, E., *Nonlocal Diffusion and Applications*, Lecture Notes of the Unione Matematica Italiana, **20**, Springer, Cham: Unione Matematica Italiana, Bologna, 2016.
- [39] Bunt, L. H. N., *Bijdrage tot de Theorie der convexe Puntverzamelingen*. Thesis, University of Groningen, Amsterdam (1934).
- [40] Carbotti, A., Dipierro, S., and Valdinoci, E. *Local Density of Solutions to Fractional Equations*, De Gruyter Studies in Mathematics, **74**, De Gruyter, Berlin, 2019.
- [41] Cassani, D., Ruf, B. and Tarsi, C., Equivalent and attained version of Hardy's inequality in  $\mathbb{R}^n$ , *J. Funct. Anal.* **275** (2018), 3301–3324.
- [42] Chowdhury, I., Csaló, G., Roy, P. and Sk, F., Study of fractional Poincaré inequalities on unbounded domains, *Discr. Contin. Dyn. Syst.* **41**(6) (2021), 2993–3020.
- [43] Chen, B.-Y., Hardy-type inequalities and principal frequency of the  $p$ -Laplacian, arXiv: 2007.06782v1.
- [44] Chen, Z.-Q. and Song, R., Hardy inequality for censored stable processes, *Tohoku Math. J.* **55** (2003), 439–450.
- [45] Cianchi, A., Quantitative Sobolev and Hardy inequalities, and related symmetrization principles. In *Sobolev Spaces in Mathematics I* (V. Maz'ya, ed.), International Mathematical Series, pp. 87–116, Springer, New York (2009).
- [46] Cuesta, M., Minimax theorems on  $C^1$  manifolds via Ekeland variational principle, *Abstr. Appl. Anal.* **13** (2003), 757–768.
- [47] Davies, E. B., Some norm bounds and quadratic form inequalities for Schrödinger operators (II), *J. Operator Theory* **12** (1984), 177–196.
- [48] Davies, E. B., The Hardy constant, *Quart. J. Math. Oxford* (2) **46** (1994), 417–431.
- [49] Davies, E. B. and Hinz, A., Explicit constants for Rellich inequalities in  $L^p(\Omega)$ , *Math. Zeit.* **227**(3) (1998), 511–523.
- [50] Di Castro, A., Kuusi, T. and Palatucci, G., Local behavior of fractional  $p$ -minimisers, *Ann. Inst. H. Poincaré Anal. Non Linéaire* **33**(5) (2015), 1279–1299.
- [51] Drábek, P. and Milota, J., *Methods of Nonlinear Analysis: Applications to Differential Equations*, Birkhäuser, Basel, 2007.
- [52] Drelichman, I. and Durán, R. G., Improved Poincaré inequalities in fractional Sobolev spaces, *Ann. Acad. Sci. Fennicae Math.* **43** (2018), 885–903.
- [53] Dubinskii, Yu. A., A Hardy-type inequality and its applications, *Proc. Steklov Inst. Math.* **269** (2010), 106–126.
- [54] Dunford, N. and Schwartz, J. T., *Linear Operators I*, Interscience, New York and London, 1958.
- [55] Dyda, B., A fractional order Hardy inequality, *Illinois J. Math.* **48** (2004), 575–588.
- [56] Dyda, B., Fractional Hardy inequality with remainder term, *Coll. Math.* **122** (1) (2011), 59–67.
- [57] Dyda, B. and Frank, R. L., Fractional Hardy–Sobolev–Maz'ya inequality for domains, *Studia Math.* **208** (2012), 151–166.
- [58] Dyda, B., Ihnatsyeva, L. and Vähäkangas, A. V., On improved fractional Sobolev–Poincaré inequality, *Ark. Mat.* **54** (2016), 437–454.
- [59] Dyda, B., Lehrbäck, J. and Vähäkangas, A. V., Fractional Hardy–Sobolev type inequalities for half spaces and John domains, *Proc. Amer. Math. Soc.* **140**(8) (2018), 3393–3402.

- [60] Edmunds, D. E. and Evans, W. D., *Hardy Operators, Function Spaces and Embeddings*, Springer Monographs in Mathematics, Springer, Berlin/Heidelberg/New York, 2004.
- [61] Edmunds, D. E. and Evans, W. D., *Representations of Linear Operators between Banach Spaces*, Birkhäuser, Basel, 2013.
- [62] Edmunds, D. E., Hurri-Syrjänen, R. and Vähäkangas, A. V., Fractional Hardy-type inequalities in domains with uniformly fat complement, *Proc. Amer. Math. Soc.* **142**(3) (2014), 897–907.
- [63] Edmunds, D. E. and Evans, W. D., The Rellich inequality, *Rev. Mat. Complut.* **29**(3) (2016), 511–530.
- [64] Edmunds, D. E. and Evans, W. D., *Spectral Theory and Differential Operators*, 2nd edition, Oxford University Press, Oxford, 2018.
- [65] Edmunds, D. E. and Evans, W. D., *Elliptic Differential Operators and Spectral Analysis*, Springer Nature, Switzerland AG, 2018.
- [66] Edmunds, D. E., Evans, W. D. and Lewis, R. T., Fractional inequalities of Rellich type, *Pure Appl. Funct. Anal.*, to appear.
- [67] Edmunds, D. E., Gogatishvili, A. and Nekvinda, A., Almost compact and compact embeddings of variable exponent spaces, *Studia Math.*, to appear.
- [68] Edmunds, D. E., Lang, J., Non-compact embeddings of Sobolev spaces, to appear.
- [69] Edmunds, D. E., Lang, J. and Mihula, Z., Measure of noncompactness of Sobolev embeddings on strip-like domains, *J. Approx. Theory* **269** (2021), 105608.
- [70] Edmunds, D. E. and Triebel, H., *Function Spaces, Entropy Numbers, Differential Operators*, Cambridge University Press, Cambridge, 1996.
- [71] Ekeland, I., On the variational principle, *J. Math. Anal. Appl.* **47** (1974), 324–353.
- [72] Evans, W. D. and Harris, D. J., Sobolev embeddings for generalized ridged domains, *Proc. Lond. Math. Soc.* **54** (1987), 141–175.
- [73] Evans, W. D. and Lewis, R. T., On the Rellich inequality with magnetic potentials, *Math. Zeit.* **251** (2005), 267–284.
- [74] Evans, W. D. and Lewis, R. T., Hardy and Rellich inequalities with remainders, *J. Math. Inequalities* **1**(4) (2007), 473–490.
- [75] Evans, W. D. and Schmidt, K. M., A discrete Hardy–Laptev–Weidl inequality and associated Schrödinger-type operators, *Rev. Mat. Complut.* **22**(1) (2009), 75–90.
- [76] Fernández-Martínez, P., Manzano, A. and Pustyl'nik, E., Absolutely continuous embeddings of rearrangement-invariant spaces, *Mediterr. J. Math.* **7** (2010), 539–552.
- [77] Filippas, S., Maz'ya, V., and Tertikas, A., Critical Hardy-Sobolev inequalities, *J. Math. Pures. Appl.* **87** (2007), 37–56.
- [78] Frank, R. L., Lieb, E. and Seiringer, R., Hardy–Littlewood–Thirring inequalities for fractional Schrödinger operators, *J. Amer. Math. Soc.* **21**(4) (2008), 925–950.
- [79] Frank, R. L., Eigenvalue bounds for Laplacians and Schrödinger operators: a review, *arXiv:1603.09736v2* 4 Nov. (2017), 1–24.
- [80] Frank, R. L., and Larsen, S., Two consequences of Davies' Hardy inequality, *Funktsional Anal. i Prilozhen* **55** (2021), 118–121.
- [81] Frank, R. L. and Loss, M., Hardy–Sobolev–Maz'ya inequalities for arbitrary domains, *J. Math. Pures Appl.* **97** (2012), 39–54.

- [82] Frank, R. L. and Seiringer, R., Non-linear ground state representations and sharp Hardy inequalities, *J. Functional Anal.* **255** (2008), 3407–3430.
- [83] Frank, R. L. and Seiringer, R., Sharp fractional Hardy inequalities in half-spaces. In *Around the Research of Vladimir Maz'ya, I* (F. Laptev, ed.), International Mathematical Series **11**, pp. 161–167, Springer, New York (2010).
- [84] Franzina, G. and Palatucci, G., Fractional  $p$ -eigenvalues, *Riv. Mat. Univ. Parma* **5**(2) (2014), 315–328.
- [85] Fremlin, D. H., Skeletons and central sets, *Proc. Lond. Math. Soc.* **74** (1997), 701–720.
- [86] Gagliardo, E., Proprietà di alcune classi di funzioni in più variabili, *Ricerche Mat.* **7** (1958), 102–137.
- [87] Gesztesy, F., Pang, M. H. and Stanfill, J., Bessel-type operators and a refinement of Hardy's inequality, arXiv:2102.00106v36 March 2021.
- [88] Giaquinta, M., *Multiple Integrals in the Calculus of Variations and Nonlinear Elliptic Systems*, Annals of Mathematics Studies **105**, Princeton University Press, Princeton, 1983.
- [89] Gilbarg, D. and Trudinger, N. S., *Elliptic Partial Differential Equations of Second Order*, Springer-Verlag, Berlin-Heidelberg-New York, 1977.
- [90] Grisvard, P., *Elliptic Problems in Nonsmooth Domains*, Monographs and Studies in Mathematics **24**, Pitman, Boston, 1985.
- [91] Guzu, D., Kapitanski, L. and Laptev, A., Hardy inequalities for discrete magnetic Dirichlet forms, *Pure Appl. Funct. Anal.*, to appear.
- [92] Hadwiger, H., *Vorlesungen über Inhalt, Oberfläche und Isoperimetrie*, Springer, Berlin/Göttingen/Heidelberg, 1957.
- [93] Hajłasz, P., Pointwise Hardy inequalities, *Proc. Amer. Math. Soc.* **127**(2) 1999, 417–423.
- [94] Hardy, G. H., An inequality between integrals, *Messenger Math.* **54** (1925), 150–156.
- [95] Haroske, D. D. and Triebel, H., *Distributions, Sobolev Spaces, Elliptic Equations*, European Mathematical Society, Zürich, 2008.
- [96] Herbst, I., Spectral theory of the operator  $(p^2 + m^2)^{1/2} - ze^2/r$ , *Comm. Math. Phys.* **53**(3) (1977), 285–294.
- [97] Hewitt, E. and Stromberg, K., *Real and Abstract Analysis*, Springer, New York, 1965.
- [98] Hoffmann-Ostenhof, M., Hoffmann-Ostenhof, T. and Laptev, T., A geometrical version of Hardy's inequality, *J. Funct. Anal.* **189** (2002), 539–548.
- [99] Hurri-Syränen, R. and Vähäkangas, A. V., On fractional Poincaré inequalities, *J. Anal. Math.* **120** (2013), 85–104.
- [100] Hurri-Syränen, R. and Vähäkangas, A. V., Fractional Sobolev-Poincaré and fractional Hardy inequalities in unbounded John domains, *Mathematika* **61**(2) (2015), 385–401.
- [101] Hurri-Syränen, R., Weighted fractional Poincaré type inequalities, *Colloq. Math.* **157** (2019).
- [102] Iannizzotto, A., Mosconi, S. and Squassina, M., Global Hölder regularity for the fractional  $p$ -Laplacian, *Rev. Mat. Iberoam.* **32** (2016), 1353–1392.
- [103] Iannizzotto, A. and Squassina, M., Weyl-type laws for fractional  $p$ -eigenvalue problems, *Asymptotic analysis* **88** (2014), 233–245.

- [104] Itoh, J.-I. and Tanaka, M., The Lipschitz continuity of the Lipschitz function to the cut locus, *Trans. Amer. Math. Soc.* **353**(1) (2001), 21–40.
- [105] Iwaniec, T. and Martin, G., Riesz transforms and related singular integrals, *J. Reine Angew. Math.* **473** (1996), 25–57.
- [106] Janson, S., Taibleson, M. and Weiss, G., Elementary characterizations of the Morrey-Campanato spaces. In: *Harmonic Analysis* (Cortona 1992), Lecture Notes in Math., vol 1992, pp. 101–114, Springer, Berlin (1983).
- [107] Karadzhov, G. E., Milman, M. and Xiao, J., Limits of higher-order Besov spaces and sharp reiteration theorems, *J. Functional Anal.* **221** (2005), 323–339.
- [108] Kinnunen, J., Lehrbäck, J. and Vähäkangas, A. V., *Limits of Maximal Function Methods for Sobolev Spaces*, AMS Math. Surveys, American Math. Soc., Providence, RI, 2021.
- [109] Kolyada, V. and Lerner, A., On limiting embeddings of Besov spaces, *Studia Math.* **171**(1) (2005), 1–13.
- [110] Kufner, Maligranda, L. and Persson, L.-E., *The Hardy Inequality: About its History and Some Related Results*, Vydavatelský servis, Pilsen, 2007.
- [111] Kuusi, T., Mingione, G. and Sire, Y., Nonlocal equations with measure data, *Comm. Math. Phys.* **337**(3) (2015), 1317–1368.
- [112] Kwaśnicki, M., Ten equivalent definitions of the fractional Laplace operator, *Fract. Calc. Appl. Anal.* **20** (2017), 51–57.
- [113] Lamberti, P. D. and Pinchover, Y.,  $L^p$  Hardy inequality on  $C^{1,\gamma}$  domains, *Annali Scuola Normale Superiore Pisa* **19** (2019), 1135–1159.
- [114] Landau, E., A note on a theorem concerning positive terms, *J. Lond. Math. Soc.* **1** (1926), 38–39.
- [115] Laptev, A. and Sobolev, A. V., Hardy inequalities for simply connected planar domains, *Amer. Math. Soc. Transl. Ser. 2* **225** (2008), 133–140.
- [116] Laptev, A. and Weidl, T., Hardy inequalities for magnetic Dirichlet forms, *Oper. Theory: Adv. Appl.* **108** (1999), 299–305.
- [117] Lang, J. and Musil, V., Strict  $s$ -numbers of non-compact Sobolev embeddings into continuous functions, *Constr. Approx.* **50**(2) (2019), 271–291.
- [118] Lang, J. and Nekvinda, A., Embeddings between Lorentz sequence spaces are strictly singular, arXiv:2104.00471v1 (April 2021).
- [119] Lefèvre, P. and Rodríguez-Piazza, L., Finitely strictly singular operators in harmonic analysis and function theory, *Adv. Math.* **255** (2014), 119–152.
- [120] Lehrbäck, J., Pointwise Hardy inequalities and uniformly fat sets, *Proc. Amer. Math. Soc.* **136**(6) (2008), 2193–2200.
- [121] Lehrbäck, J., Weighted Hardy inequalities and the size of the boundary, *Manuscripta Math.* **127** (2008), 249–273.
- [122] Lewis, J. L., Uniformly fat sets, *Trans. Amer. Math. Soc.* **308** (1988), 177–196.
- [123] Lewis, R. T., Singular elliptic operators of second order with purely discrete spectra, *Trans. Amer. Math. Soc.* **271** (1982), 653–666.
- [124] Lewis, R. T., Li, J. and Li, Y., A geometric characterization of a sharp Hardy inequality, *J. Funct. Anal.* **262**(7) (2012), 3159–3185.
- [125] Lieb, E. H., On the lowest eigenvalue of the Laplacian for the intersection of two domains, *Invent. Math.* **74**(3) (1983), 441–448.
- [126] Lieb, E. H. and Loss, M., *Analysis*, 2nd edition, Amer. Math. Soc. Graduate Studies in Math. **14**, 2001.

- [127] Lindgren, E. and Lindqvist, P., Fractional eigenvalues, *Calc. Var. Partial Differential Equations* **49** (2014), 795–826.
- [128] Lindqvist, P., Notes on the  $p$ -Laplace equation, Report. University of Jyväskylä Department of Mathematics and Statistics, 2006.
- [129] Li, Y., and Nirenberg, L., The distance to the boundary, Finsler geometry and the singular set of viscosity solutions of some Hamilton-Jacobi equations, *Comm. Pure Appl. Math.* **18**(1) (2005), 85–146.
- [130] Loss, M. and Sloane, C., Hardy inequalities for fractional inequalities on general domains, *J. Functional Anal.* **259** (2010), 1369–1379.
- [131] Mantegazza, C. and Mennici, A. C., Hamilton-Jacobi equations and distance functions on Riemannian manifolds, *Appl. Math. Optim.* **47** (2003), 1–25.
- [132] Marcus, M., Mizel, V., and Pinchover, Y., On the best constant in Hardy's inequality in  $\mathbb{R}^n$ , *Trans. Amer. Math. Soc.* **350**(8) (1998), 3237–3255.
- [133] Matskewitch, T. and Sobolevskii, P., The best possible constant in generalized Hardy's inequality for convex domains in  $\mathbb{R}^n$ , *Nonlinear Anal. Theory Methods Appl.* **28**(9) (1997), 1601–1610.
- [134] Maz'ya, V. and Shaposhnikova, T., On the Bourgain, Brezis and Mironescu theorem concerning limiting embeddings of fractional Sobolev spaces, *J. Functional Anal.* **195** (2002), 230–238; corrig. *ibid.* **201** (2003), 298–300.
- [135] Maz'ya, V., *Sobolev Spaces*, Springer, Berlin, Heidelberg, 1985.
- [136] Milman, M., Notes on limits of Sobolev spaces and the continuity of interpolation scales, *Trans. Amer. Math. Soc.* **357**(9) (2005), 3425–3442.
- [137] Mosconi, S. and Squassina, M., Recent progresses in the theory of nonlinear nonlocal problems, *Bruno Pini Math. Analysis Sem.* **7** (2016), 147–164.
- [138] Moser, J., On Harnak's theorem for elliptic differential equations, *Comm. Pure Appl. Math.* **16** (1961), 577–591.
- [139] Motzkin, T. S., Sur quelques propriétés caractéristiques des ensembles convex, *Atti Real. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. Serie VI* **21** (1935), 562–567.
- [140] Naibullin, R., Hardy and Rellich type inequalities with remainders, to appear.
- [141] Nečas, J., *Les méthodes directes en théorie des équations elliptiques*, Masson, Paris, 1967.
- [142] Nezza, E. di, Palatucci, G. and Valdinoci, E., Hitchhiker's guide to the fractional Sobolev spaces, *Bull. Sci. Math.* **136**(5) (2012), 521–573.
- [143] Owen, M. P., The Hardy–Rellich inequality for polyharmonic operators, *Proc. Roy. Soc. Edin.* **129 A** (1999), 825–839.
- [144] Perera, K., Agarwal, R. P. and O'Regan, D., *Morse theoretic aspects of  $p$ -Laplacian type operators*, Mathematical surveys and monographs, Amer. Math. Soc., Providence, RI, 2010.
- [145] Peetre, J., Espaces d'interpolation et théorème de Soboleff, *Ann. Inst. Fourier* **16** (1966), 279–317.
- [146] Pick, L., Kufner, A., John, O. and Fučík, S., *Function Spaces*, Vol. 1 (2nd revised and extended edition), De Gruyter, Berlin/Boston, 2013.
- [147] Pietsch, A., *History of Banach Spaces and Linear Operators*, Birkhäuser, Boston, 2007.
- [148] Pinchover, Y. and Goel, D., On weighted  $L^p$ -Hardy inequalities on domains in  $\mathbb{R}^n$ , arXiv: 2012.12860v2.



- [149] Prats, M. and Saksman, E., A  $T(1)$  theorem for fractional Sobolev spaces on domains, *J. Geom. Anal.* **27**(3) (2017), 2490–2518.
- [150] Ponce, A., A new approach to Sobolev spaces and connections to  $\Gamma$ -convergence, *Calc. Var. Partial Diff. Eqs.* **19** (2004), 229–255.
- [151] Qui, H. and Xiang, M., Existence of solutions for fractional  $p$ -Laplacian problems via Leray-Schauder nonlinear alternative, *Boundary value problems* (2016), doi 10.1186/s13661-016-0593-8.
- [152] Rafeiro, H., Samko, N. and Samko, S., Morrey-Campanato spaces: an overview, *Operator Theory, Advances and Applications* **228** (2013), 293–323.
- [153] Rellich, F., Halbeschränkte Differential operatoren höherer Ordnung. In *Proceedings of the International Congress of Mathematicians 1954*, vol. III, pp. 243–250, Noordhoff, Groningen (1956).
- [154] Robinson, D. W., Hardy and Rellich inequalities on the complement of convex sets, *J. Austr. Math. Soc.* **108**(1) (2020), 98–119.
- [155] Robinson, D. W., The weighted Hardy constant, arXiv:2103.07.848v1 14 Mar. 2021.
- [156] Samko, S., Best constant in the weighted Hardy inequality: the spacial and spherical version, *Fractional Calculus and Applied Analysis* **8**(1) (2005), 39–52.
- [157] Schilling, R. L., *Measure, Integrals and Martingales*, Cambridge University Press, Cambridge, 2005.
- [158] Schilling, R. and Kühn, F., *Counterexamples in Measure and Integration*, Cambridge University Press, Cambridge, 2021.
- [159] Slavíková, L., Almost compact embeddings, *Math. Nachr.* **285** (2012), 1500–1516.
- [160] Sloane, C. A., A fractional Hardy–Sobolev–Maz’ya inequality in the half-space, *Proc. Amer. Math. Soc.* **139** (2011), 4003–4016.
- [161] Slobodeckij, L. N., Generalised Sobolev spaces and their applications to boundary value problems of partial differential equations (Russian), *Leningrad Gos. Ped. Inst. Učep. Zap.* **197** (1958), 54–112.
- [162] Sobolevskii, P. E., Hardy’s inequality for the Stokes problem, *Nonlinear Analysis, Methods & Applications* **30**(1) (1997), 129–145.
- [163] Solomyak, M. Z., A remark on the Hardy inequalities, *Integr. Equ. Oper. Theory* **19** (1994), 120–124.
- [164] Stromberg, K. R., *An Introduction to Classical Real Analysis*, Wadsworth, Belmont, 1981.
- [165] Talenti, G., An inequality between  $u^*$  and  $|\nabla u|^*$ , *Inst. Series Num. Math.*, **103** (1992), 175–182.
- [166] Thomas, J. C., *Some problems associated with sum and integral inequalities*, Ph.D. thesis, Cardiff University, Wales (2007).
- [167] Tidblom, J., A geometrical version of Hardy’s inequality for  $W_0^{1,p}(\Omega)$ , *Proc. Amer. Math. Soc.* **132**(8) (2004), 2265–2271.
- [168] Tidblom, J., A Hardy inequality in the half-space, Research report in mathematics 3, Department of Mathematics, University of Stockholm, 2004.
- [169] Triebel, H., *Theory of Function Spaces*, Birkhäuser, Basel, 1983.
- [170] Triebel, H., *Interpolation Theory, Function Spaces, Differential Operators* (2nd edition), Barth, Heidelberg, 1995.



- [171] Triebel, H., *The Structure of Functions*, Birkhäuser, Basel, 2001.
- [172] Yafaev, D., Sharp constants in the Hardy–Rellich inequalities, *J. Functional Anal.* **168**(1) (1999), 121–144.
- [173] Zhou, Y., Fractional Sobolev extension and imbedding, *Trans. Amer. Math. Soc.* **367**(2) (2015), 959–979.