# R CORONAE BOREALIS PULSATIONS

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## ABSTRACT

The Cepheid-like pulsations of some of the R CrB stars should in principle make it possible to determine their masses and hence to place constraints on possible evolution scenarios. We briefly review the evidence for these pulsations and discuss the problem of how these low-mass, hydrogen-deficient carbon stars could have evolved to their present position in the H-R diagram. Linear and nonlinear pulsation calculations are reviewed. It is found that for these large luminosity to mass ratio (L/M) stars a region of pulsation instability extends considerably hotter than for normal high luminosity Cepheids. The envelopes of these models are so nonadiabatic that the identification of modes becomes very difficult since there is frequently no clearly defined nodal structure. For the most extreme L/M cases it is found that the models are unstable in the sense that they appear on the verge of ejecting the outer layers.

## 1. INTRODUCTION

The R Coronae Borealis (R CrB) variables are a relatively small class of stars (approximately 30 are known), but they are of high intrinsic brightness with $M_{bol}$ ranging from about $-4^m$ to $-6^m$ (Warner, 1967). They are characterized by sudden, irregular decreases in luminosity by as much as 9 magnitudes. The recovery to normal brightness may take as long as a few years and is frequently accompanied by fluctuations. In addition to this variation, at least three of these objects are known to undergo more or less periodic changes in brightness similar in nature to Cepheid pulsations. It is this phenomenon that I would like to review.

Before doing so, however, let me say just a few words about the major dimmings which place these objects in the R CrB class. A good review of their general properties is that of Feast (1975). As he points out, the decrease in brightness seems to be due to a veiling of the star by material that lies above its photosphere. The obscuration can be due

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to dust as well as carbon grains. Models ranging from those involving ejection of material by some mechanism (e.g., Alexander et al., 1972, Feast and Glass, 1973) to those which propose semipermanent blotches in orbit around the star (Wing et al., 1972) have been considered. Carbon is known to be overabundant in these stars by factors of ten or more relative to the sun. In fact, the R CrB stars are a subclass of the hydrogen-deficient carbon (HdC) stars with hydrogen underabundant by something like $10^5$. Although based on quite a limited sample, the evidence also indicates that these stars belong to an old disk population.

The three stars which show Cepheid-like pulsations are R CrB itself, RY Sgr and UW Cen. Interestingly, all three have periods close to 40 days. Alexander et al. (1972) have studied the variations of RY Sgr from a decline to minimum in 1967 and a return to normal by 1970. On the rise to maximum and during maximum light, periodic changes in brightness of $\Delta V \cong 0\overset{m}{.}5$ were observed with $\Delta(B-V) \cong 0\overset{m}{.}3$. The period of these variations is approximately 38.6 days. The radial velocity changes with this same period and an amplitude of $\Delta V_R \cong 30$ km/s. It should be kept in mind, however, that the observations indicate that these variations are not as regular as those of the Cepheid variables and this may well be an important clue in understanding them.

The interest in the pulsation properties arises from the fact that the period along with other observed properties will place constraints on the masses of these stars and the pulsations themselves have been suggested as being related to mass ejection and their subsequent decline in brightness (Feast, 1975). A better determination of the mass will be important in trying to understand their past and future evolution.

## 2. EVOLUTIONARY STATE

Most studies assume that a low mass ($\cong 1\ M_\odot$) remnant of a double shell source star is involved in which the object has somehow managed to lose its hydrogen-rich envelope. Schönberner (1979) has investigated asymptotic giant branch evolution with steady mass loss. From the point of view of the HdC stars there are a couple of important results of this work. First, the calculations show that stars with $M \cong 1.4\ M_\odot$ cannot lose their envelopes by ejection since they do not pass through the region of dynamical instability found by Wood (1974), Smith and Rose (1972) and Kutter and Sparks (1974). He points out, however, that this critical mass does depend rather sensitively on the assumed mass loss rates (Wood and Cahn, 1977). The second result is that the flash driven convective shells do not penetrate into the overlying hydrogen-rich layers while the star moves along the Hayashi track. At the moment, then, there is no certain way to produce the HdC stars.

There have been several studies which have followed the evolution of stars with a condensed carbon-oxygen core and a helium envelope to see if such objects pass through the region of the Hertzsprung-Russell (H-R) diagram occupied by the R CrB stars. Paczyński (1971) and Trimble and Paczyński (1973) looked at the evolution of helium-rich stars (X=0.0, Z=0.0004 and X=0.0, Z=0.03). They found that it was possible to construct models which evolved from the helium main sequence to the red giant phase for a mass range M=0.8 to 2.0 $M_\odot$. They were not able to explain the observed carbon enrichment of the R CrB stars but did obtain models in the appropriate effective temperature and luminosity range. Other calculations have been carried out by Biermann and Kippenhahn (1971), Dinger (1972) and Uus (1973) with only slightly different results. Schönberner (1977) evolved models from the red giant stage to the white dwarf region of the H-R diagram. These models spent several $10^4$ years crossing the domain of the R CrB stars. The masses turn out to be somewhat smaller than those obtained by Paczyński (1971).

Paczyński (1971) has made the suggestion that instead of the loss of the hydrogen-rich envelope, perhaps at some stage thre was a mixing of essentially all of the star and hence burning of the hydrogen originally in the envelope. Wheeler (1978) extended this idea to point out that such mixing would greatly extend the lifetime of the star and make it possible to have an old disk population object with a mass of 1.5 $M_\odot$ or greater. He discussed this in connection with the possibility that the HdC stars might be progenitors of at least some Type I supernovae. In any event, the evolution picture seems to show that it is possible to evolve stars with helium-rich envelopes into the region of the H-R diagram where the R CrB stars are found, but we are not at all certain how they become helium rich and why there is an enhancement of carbon. Likewise, the mass seems to be constrained by these calculations, but it still may range from 0.7 $M_\odot$ or slightly less to more than 1 $M_\odot$, depending on a number of factors.

## 3. PULSATION CALCULATIONS

Trimble (1972) was the first to carry out a study of the pulsation properties of models which might represent the R CrB stars. She used Christy's (1966) nonlinear code and also presented some unpublished linear, nonadiabatic results due to Gough (1972) and Cogan (1972). Her models assumed a composition of X=0, Z=0.03 with the opacities being a modification of those of Cox and Stewart (1965). The linear results for M=1 $M_\odot$ and L=1.125x$10^4$ $L_\odot$ gave a blue edge for the instability strip at $T_{eff}\cong7850$ K. The growth rates for unstable models were very rapid with an e-folding time for the kinetic energy of one period or less. She assummed that $T_{eff}$ for RY Sgr was 6000 K based on the work of Danziger (1965) and therefore concluded that the period obtained was too long (92 days). It should be noted that if one uses a $T_{eff}\cong7100$ K, as recently determined by Schönberner (1975), the linear nonadiabatic period is close to the observed value of approximately 40 days. Unfortunately, the 1 $M_\odot$ nonlinear model with $T_{eff}$=6000 K turned out to be very unstable and appeared to blow itself apart. A 2 $M_\odot$ model at the same luminosity was better behaved and led to a period (Π) of approximately 48 days. The pulsations were however somewhat irregular with $42\lesssim\Pi\lesssim57$ days. A model with M=2 $M_\odot$ and L=5x$10^3$ $L_\odot$ gave quite reasonable results, but it had a period that was about a factor of two too short (Π≅22 days). The light variations for this model were much larger than observed with $\Delta M_{bol}\cong2^m.5$.

The velocity amplitude (40-50 km/s) and radius variation ($\Delta R/R \cong 0.20$) were, however, much more reasonable. She points out that there is a problem with the L and $T_{eff}$ variations in that the models give too large a variation and the phasing between the two seems to be incorrect. In the models $L_{max}$ and $T_{eff,max}$ occur at the same phase, whereas the observations indicate that $T_{eff,max}$ leads $L_{max}$ in phase by as much as 0.2 of a period. Also $L_{max}$ may be too late, (compared to the observations) with respect to the radial velocity variations by ≅0.25 of a period. She suggests that both of these discrepancies may be due to improper treatment of the outer layers of the star. This could be due in part to a neglect of convection and in part to the use of inadequate opacities (no carbon enrichment was taken into account).

Wood (1976) conducted a search for envelope excitation in high luminosity helium stars. He was primarly interested in higher $T_{eff}$ and shorter periods than those being discussed here, but his results do extend over a broad range of $T_{eff}$ and are therefore quite relevant. His linear results are quite interesting. Models were constructed for $M=0.6\ M_\odot$ and $M=1.0\ M_\odot$ with $L=5x10^3\ L_\odot$ for both masses and $L=10^4\ L_\odot$ for the $M=1.0\ M_\odot$ case. The composition was assumed to be $X=0.0$, $Z=0.02$ and the opacities were due to Cox and Stewart (1970). He did include convection in the mixing length approximation in both his linear and non-linear models. His linear, non-adiabatic models varied in $T_{eff}$ from 5000 K to 15,000 K. Figure 1 gives the linear results for models with $L=10^4\ L_\odot$ and $M=1.0\ M_\odot$. On the left is plotted the growth rate $\omega_R$ (in units of $yr^{-1}$) as a function of log $T_{eff}$ while on the right log P (where P is the period in days) is plotted as a function of log $T_{eff}$. This latter plot shows a rather peculiar behavior of the nonadiabatic (solid lines) modes. The dashed lines are the adiabatic fundamental and overtone modes, which at low log $T_{eff}$ agree rather well with the nonadiabatic cases. As one goes to higher $T_{eff}$ the nonadiabatic periods decrease faster than the adiabatic ones. In the case of the fundamental mode the nonadiabatic period then approaches, and for log $T_{eff} \gtrsim 4.0$ becomes larger than the adiabatic period. For the overtones, the corresponding nonadiabatic period decreases until at higher $T_{eff}$ it is close to the adiabatic period for the next higher overtone. These transition cases are identified e.g., by $P_{12}$ (first overtone approaching second overtone at higher $T_{eff}$). There is also a mode which at high $T_{eff}$ seems to be associated with the first overtone but at lower $T_{eff}$ appears to cross the (1,2) curve. Wood refers to this as $P_1$. If we look at the stability of the various modes as displayed in part (a) of Figure 1, we note that the fundamental mode is unstable at low $T_{eff}$ with the FBE at log $T_{eff} \cong 3.88$. The $P_{12}$ mode has a red edge at log $T_{eff} \cong 3.92$ and a blue edge at log $T_{eff} \cong 4.02$. The $P_1$ mode is seen to be stable at all log $T_{eff}$. The $P_{23}$ mode is unstable between $T_{eff} \cong 4.0$ and 4.1 so that there is a large range of $T_{eff}$ for which unstable models with a variety of periods are found.

For models with smaller L/M, Wood found that both the nonadiabatic fundamental and first overtone were identifiable with their adiabatic counterparts over the whole range of log $T_{eff}$ investigated, while the

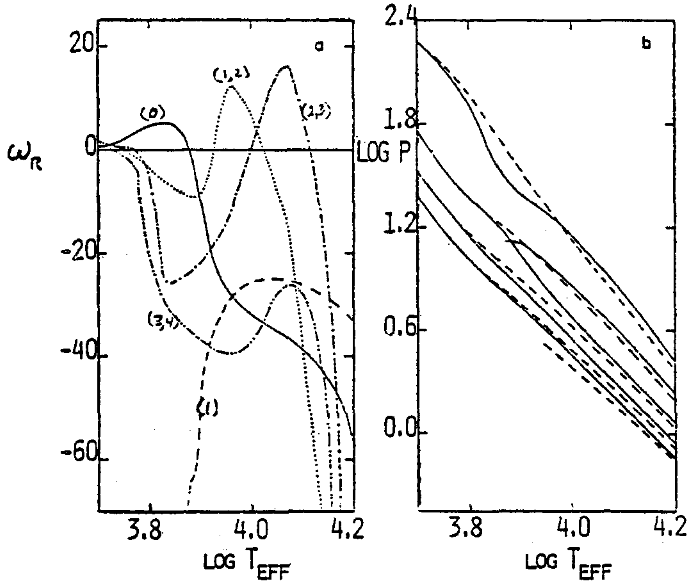

Figure 1. (a) Real part $\omega_R(yr^{-1})$ of the linear nonadiabatic eigenvalue plotted against log $T_{eff}$ for models with $L=10^4$ $L_\odot$, $M=M_\odot$. The labels on each curve refer to the appropriate modes as discussed in the text. (b) Log P (where P is the period in days) versus Log $T_{eff}$. The dashed curves are for the linear adiabatic case and the solid curves for the linear nonadiabatic. The top curve is for the fundamental mode and the lower curves for the overtones. The model parameters are the same as those in (a). This figure is from Wood (1976).

transition cases occured for higher modes.

Although I do not feel that the cause of this strange behavior is clearly understood, Wood has pointed out that in the case of the $P_1$ mode, the radial eigenfunction resumbles a first overtone at higher $T_{eff}$ and as one goes to lower $T_{eff}$ the node disappears. He attributes this change to the appearance of a strong running wave component in the exterior part of the envelope. This suggests that care must be taken in the choice of the outer boundary conditions.

Wood (1976) also carried out some nonlinear calculations and although they were for shorter periods than those of interest here, they should be briefly mentioned. A model with $L=10^4$ $L_\odot$, $M=1$ $M_\odot$ and log $T_{eff}=3.78$ was found to be quite unstable with no repeatable light curve. For higher $T_{eff}$ (Log $T_{eff}=4.0$ and Log $T_{eff}=4.08$), he was able to obtain models in higher modes, $P_{12}$ and $P_{23}$ with periods of 5 days and 2 days respectively. The light amplitudes were approximately $1^m$ and the velocity amplitudes were of the order ±20 km/s.

King et al. (1980) have recently attempted to extend the study of these stars using newer opacity and equation of state data which include the overabundance of carbon. The primary concern was to see if we could obtain a better estimate of the masses of R CrB stars from pulsation theory. Linear, nonadiabatic models were constructed with M=0.8 $M_\odot$ to 3.0 $M_\odot$ and L=$3\times10^3$ $L_\odot$ to $2\times10^4$ $L_\odot$. Opacity and equation of state data were from the Huebner et al. (1977) opacity library for a mixture of Y=0.90 and $Z_C$=0.10 (referred to as HE9C1). At the time the calculations were made we had opacities only for T≳12,000 K and therefore we used the Stellingwerf (1975) opacity fit for lower T. As will be indicated later, this defect is probably more important than previously thought. The basic result of the linear calculations was that a large L/M was needed to obtain a 40 day period at the approximate $T_{eff}$ for these stars (7100±600 K for RY Sgr as determined by Schönberner, 1975). For the longest period unstable mode the blue edge occurs at higher $T_{eff}$ for larger values of L/M. This, of course, was also found by Wood (1976). The paper following this includes a figure showing this result for our calculations. The difficulty is in identifying the modes that are present since they frequently do not seem to correspond in any direct way to the adiabatic fundamental and higher overtones.

A sequence of models at L=$1.13\times10^4$ $L_\odot$ was studied using the nonlinear theory. Table 1 indicates the masses considered. $T_{eff}$ was fixed at 6300 K. This value, slightly outside the lower range indicated for RY Sgr, was chosen to assure that the models at higher masses would be pulsationally unstable. The 1.2 $M_\odot$ and 1.4 $M_\odot$ models were unstable in the sense that the pulsations grew rapidly and appeared on the verge of ejecting mass. The growth of the pulsations up to the time that the accelerations became too large to follow was quite regular, unlike that reported by Trimble (1972) and Wood (1976). Figure 2 shows the growth of the radius variations for the 1.4 $M_\odot$ model. A maximum radial velocity of ≅50 km/s was reached while the escape velocity for this model is ≅80 km/s. For the higher masses stable limit cycles were reached. The pulsations were quite repeatable from cycle to cycle. The magnitudes of the velocity and radius variations, as indicated in Table 1, are in reasonable agreement with observations, but again as found by other investigators the light variation is considerably larger than observed.

Table 1. Nonlinear Models at $L/L_\odot=1.13\times10^4$ and $T_{eff}$=6300 K.

| Mass ($M/M_\odot$) | Π (days) | $\Delta M_{bol}$ | $\Delta R/R$ | $\Delta V_R$ (km/s) |
|---|---|---|---|---|
| 1.2 | 44 | --- | --- | --- |
| 1.4 | 43 | --- | --- | --- |
| 1.6 | 39 | $2^m.0$ | 0.17 | 27 |
| 2.0 | 36 | $2^m.2$ | 0.23 | 40 |

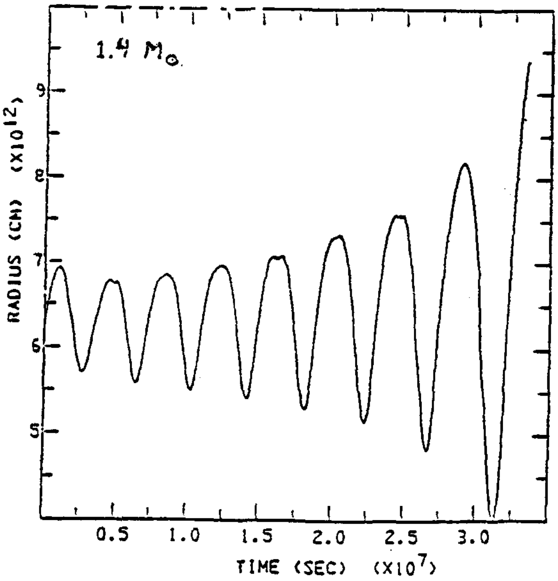

Figure 2. Radius variation as a function of time for a model with $L=1.13\times10^{4}$ $L_{\odot}$, $T_{eff}=6300$ K, $M=1.4$ $M_{\odot}$. This figure is from King et al. (1980).

More recent calculations with the HE9C1 opacities extended to T<12,000 K indicate a greater sensitivity to the details of the outer structure of the star than was first believed. The strange behavior of the non-adiabatic modes found by Wood (1976) was also encountered in this study and was found to depend in a complicated way on the stellar parameters.

## 4. SUMMARY AND CONCLUSIONS

It has been shown by a number of investigators that it is possible to evolve low-mass stars through that region of the H-R diagram where the R CrB and other HdC stars are found. It has not yet been possible, however, to calculate in any detail the process which leads to the helium-rich envelope and to the overabundance of carbon.

Pulsation calculations have shown that the very nonadiabatic envelopes of these large L/M stars lead to complications in understanding the possible radial modes. Pulsational instability is possible over a fairly wide range of $T_{eff}$ with periods appropriate to the R CrB stars and also to the hotter helium stars. The details of the pulsation calculations do not agree very well with the observations at this point and the indication is that they may be rather sensitive to the envelope structure near the surface. More care is probably required with regard to the choice of opacities and surface boundary conditions. Unfortunately, because of these dificulties, it has not been possible to refine the masses for these stars. We still do not know if they are closer to

1 $M_\odot$ or 2 $M_\odot$. In order to discuss their subsequent evolution and also to understand how they might have arrived at their present location in the H-R diagram, it is important to resolve this uncertainty.

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## DISCUSSION

CAHN: There are several things that I want to say. First, Icko Iben has been disturbed by the appearance of carbon stars in the Magellanic Cloud. He has a paper in press in which he talks about carbon stars of lower mass than we previously thought about. This may have some bearing on these objects. Second, does the number of stars you observe have any relation to the rate that stars should be leaving the peak of the asymptotic giant branch?

KING: We should think of these stars as a subset of the hydrogen-deficient carbon stars. We see them as R CrB stars, but their evolution is probably going to give you a helium star later on as they evolve to the blue. Warner, I believe, has looked into this and finds that the number is not unreasonable in terms of the number of helium-rich white dwarfs that you get. So there is some consistency.

CAHN: Did you do any nonlinear calculations for less massive stars? You were getting into trouble at 1.4 $M_\odot$ and you would expect even lower mass stars to be even less stable.

KING: Yes, the less massive they are the more problems you have. $1M_\odot$ is bad enough.

STARRFIELD: Did you go to higher effective temperature and did you use any even more carbon enriched mixture, like half helium and half carbon?

KING: No, we haven't gone beyond ten percent carbon, which seems consistent with the observations, and we haven't done any nonlinear calculations that we are satisfied with, beyond 6300K. We would like to do models closer to 7000K. The observed pulsating R CrB stars do not seem to have effective temperatures much higher than this.

SMEYERS: You showed a diagram in which you plotted log P versus log $T_{eff}$ and your strange mode extended to high values of $T_{eff}$. Do you have the feeling that at the high effective temperatures the non-adiabatic terms are more important than at the low effective temperatures?

KING: You might get that for the strange mode, however, you recall that for the fundamental of Peter Wood, the adiabatic periods differ more at intermediate temperatures. There is a kink in the curve and then the period approaches the adiabatic value at both lower and higher effective temperatures.

SMEYERS: I discussed this point earlier with John Cox. When you go from the adiabatic approximation to the full nonadiabatic problem, you increase the order of the system with respect to time. This may make it possible to get the additional mode that you are seeing.

J. COX: I think that the nonadiabatic effects are more important at the lower temperatures but I am not sure.

KING: I think that we have to worry about the boundary condition that we are using at the surface. At the moment, we use a standing wave boundary condition. Peter Wood found evidence that there are running waves present as the strange mode becomes more important. This could be a serious problem.