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Transformation properties of certain partial differential equations, solutions and integrability

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In this thesis we discuss particular properties pertaining to partial differential equations (pde's). The existence of conservation laws, constants of motion, a Lax pair, pseudopotentials and the Painlevé property are studied and connections among these are investigated. It is shown that if a pde can be derived from a Lagrangian density then it may be possible to determine its energy levels from the corresponding Hamiltonian density and solitary wave solution. Some pde's are solved by standard techniques such as Hirota's method and the group theoretical approach which leads to similarity solutions. Considerable attention is given to the Painlevé property and several Painlevé tests have been defined. The concept of integrability is critically analysed with respect to particular properties an equation possesses.

From partial use of a Painlevé test we have introduced methods which obtain travelling wave solutions to most pde's and systems of pde's with constant coefficients.

Also from partial use of a Painlevé analysis a technique is introduced to solve a Korteweg-de Vries type equation with variable coefficients and non-uniformities. This method leads to a constraint on the coefficient functions and if this constraint is satisfied then the pde possesses the Painlevé property. Other types of KdV equations are obtained by reduction and solved by this method.

The Korteweg-de Vries-Burgers' equation is extensively discussed with respect to integrability. A survey of exact solutions is given which shows that there is essentially only one exact solution known at this stage.

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