

AN EMPIRICAL $T - \tau$ CURVE FOR THE roAp STAR HR 3831: ATMOSPHERIC STRUCTURE FROM PULSATION AMPLITUDES

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PROBING THE ATMOSPHERE OF A CP2 STAR

The atmospheric structures of magnetic CP2 (Ap) stars are notoriously difficult to model: line blanketing is severe, surface gravities are extremely uncertain, and the surface abundance inhomogeneities lead to different atmospheric properties as a function of position on the star. Seismology of the p -modes of rapidly oscillating Ap (roAp) stars (Kurtz 1990), which vary with periods of a few minutes and amplitudes below 0.01 mag and 1 km/s in light and velocity, has already helped constrain the luminosities – and hence, the $\log g$ values – of some cool CP2 stars (Kurtz 1992, *these proceedings*). We show here that the pulsations of an roAp star can also directly probe the temperature structure of a CP2 atmosphere.

Available multicolour photometry of roAp stars (e.g., Weiss & Schneider 1984) had revealed that their pulsation amplitudes decline more rapidly with increasing wavelength than other known pulsators. Matthews *et al.* (1990) showed that this can be explained by the wavelength dependence of limb darkening and its weighting effect on the integrated amplitude of an $(\ell, m) = (1, 0)$ mode. This dipole mode – which appears to dominate roAp stars; in particular, HR 3831 – has two special characteristics:

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- It produces no changes in the projected area of the stellar disk during the pulsation cycle, so all observed brightness variations can be attributed to flux (i.e., temperature) variations; and
- No matter what the inclination of the mode, limb darkening *increases* the net amplitude measured on the visible disk (except for $i = 90^\circ$, where the net amplitude is zero).

Stronger limb darkening at shorter wavelengths enhances the effective amplitude. By measuring amplitudes at various wavelengths, we can estimate the corresponding limb darkening coefficients and thus, the temperature structure of the atmosphere in a manner akin to that for the Sun.

HR 3831, the most extensively studied of the roAp stars, is the ideal candidate for such an analysis. Its eigenfrequency spectrum appears to be dominated by a dipole mode (Kurtz *et al.* 1992) with a period near 11.8 min; there are empirical constraints on its inclination and magnetic obliquity [$i \geq 38^\circ$; i or $\beta \geq 62^\circ$]; its rotation period is relatively short [~ 2.85 d] so that one can monitor at least one phase of maximum oscillation amplitude and sample different parts of the stellar surface during a short observing run; and it is fairly bright [$V = 6.17$].

OBSERVATIONS

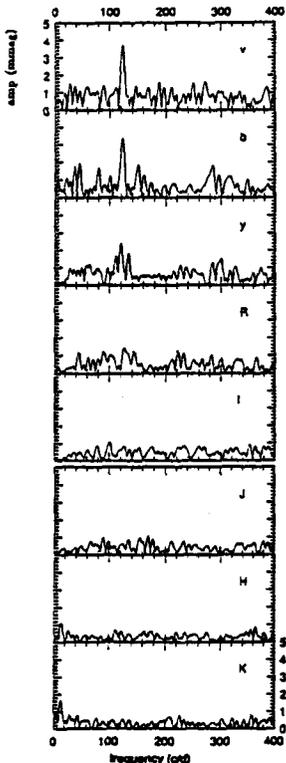


Figure 1.

From a simultaneous run with the CTIO 1.0- and 1.5-m telescopes in November 1991, we obtained high-speed photometry of HR 3831 (sp. F0 Vp) in the optical (Strömgren *vby*, Cousins *RI*) and infrared (*JHK*). Integration times in the optical were ~ 20 sec; in the infrared, ~ 1 min. Figure 1 shows the Fourier amplitude spectra (0–400 c/d) in the 8 bandpasses for the night of 25 Nov. The results for the *v* filter are in the top panel, and wavelength increases downward to the *K* results in the bottom panel. The data span about 3.6 hr or 18 pulsation cycles and were obtained near phase $\phi \simeq 0.4$ of the 2.85-d magnetic period of this star (according to the ephemeris of Kurtz *et al.* 1990); i.e., just before magnetic and pulsation amplitude maximum. The prominent peak visible at shorter wavelengths occurs near 122 d^{-1} ($P = 11.8$ min): the dominant pulsation mode of HR 3831. Note how rapidly the amplitude drops with increasing wavelength, such that only upper limits can be established in the infrared.

TRANSLATING AMPLITUDES INTO ATMOSPHERES

If HR 3831 is pulsating primarily in an $(\ell, m) = (1, 0)$ mode, then one can model its brightness variations as the flux variations of a blackbody whose surface is oscillating in temperature with a dipole pattern. Such models were generated as a function of projected angle α of the pulsation pole, mean temperature T_{eff} and temperature amplitude ΔT_{puls} for all eight observed bandpasses. (Each bandpass was defined by its central wavelength and FWHM.)

The resulting models were then weighted by the following limb-darkening relation:

$$\frac{I_{\nu}(\theta)}{I_{\nu}(0)} = 1 - \beta_{\nu}(1 - \cos\theta).$$

where the values of β_{ν} are free parameters. The weighted amplitude ratios from these models were compared to the observed ratios and solutions to β_{ν} estimated by least-squares fitting.

Since we fit amplitude ratios, we must place some constraint on the fitting procedure to derive values of β . Three fitting approaches were adopted: (1) assuming that $\log\beta = a \log\lambda + b$; a dependence which applies to solar limb darkening over the wavelength range relevant here; (2) fixing β for the v filter at various trial values and allowing β at the other seven wavelengths to vary freely; and (3) fixing β for the K filter at various values and allowing the others to vary.

The derived values of β_{ν} were then used to solve the expression:

$$\frac{I_{\nu}(\theta)}{I_{\nu}(0)} = \int_0^{\infty} \frac{S_{\nu}(\tau_{\nu})}{I_{\nu}(0)} e^{-\tau_{\nu} \sec\theta} \sec\theta \, d\tau_{\nu}$$

where the source function was represented by the expansion

$$S_{\nu}(\tau_{\nu}) = a_0 + a_1\tau_{\nu} + a_2\tau_{\nu}^2 + a_3\tau_{\nu}^3 + a_4\tau_{\nu}^4 + a_5\tau_{\nu}^5$$

and $I_{\nu}(0)$ was sampled over the expected range of luminosity for an F0 V star. The coefficients a_j were obtained by LU inversion of the 6×6 matrix.

Finally, the source function was equated with the Planck function

$$S_{\nu}(\tau_{\nu}) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

to produce the temperature structure as a function of optical depth.

The technique appears relatively insensitive to the choice of mean T_{eff} and ΔT_{puls} , and $I_{\nu}(0)$, within reasonable ranges for CP2 stars. Model fits were performed for a range of mean effective temperatures [$7500 \text{ K} \leq T_{\text{eff}} \leq 8500 \text{ K}$] and tilts of the pulsation pole [$30^\circ \leq \alpha \leq 80^\circ$].

The range of α searched is consistent with limits from the magnetic field variations of HR 3831 and asteroseismology of its p -mode spectrum. (Note that according to the Oblique Pulsator Model (see Kurtz 1990; Matthews 1991) for roAp stars, the pulsation pattern is aligned with the magnetic dipole geometry of the star, so the magnetic and pulsation poles are coincident.) The lower bound on α is set by the phase of the observations in the magnetic cycle of HR 3831; at phase 0.4, the magnetic pole (= pulsation pole) can be no closer to disk centre than $\sim 30^\circ$. The upper bound is less than 90° since at that angle, the observed amplitude of the dipole mode would be zero. In fact, no solutions were possible for $\alpha \geq 65^\circ$, which matches unpublished constraints found by Kurtz for this star.

EVIDENCE FOR A TEMPERATURE INVERSION IN HR 3831

Independent of the mode inclination and fitting procedure adopted, two results are clear from the many trial fits: (1) The pronounced decline in amplitude from the optical to the IR demands a steeper $T - \tau$ gradient than for a grey atmosphere, and (2) A monotonic $T - \tau$ relation cannot satisfactorily account for all the data. No such curves are able to reproduce the observed amplitude ratios at all wavelengths simultaneously. The best compromise for a monotonic solution (Figure 2) leads to an uncomfortably low surface temperature of only 3000 K(!) for an atmosphere with $T_{\text{eff}} = 8000$ K. However, by allowing a temperature inversion in the solution, it is possible to match the observed amplitude ratios, as shown in Figure 3.

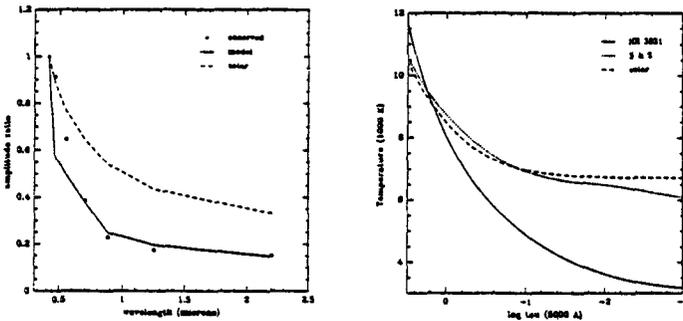


Figure 2. The best fit of a monotonic temperature structure to the amplitude ratios (left panel, solid curve) does not satisfy the data (points) and also yields a $T - \tau$ curve (right panel, solid curve) with an extremely low surface temperature.

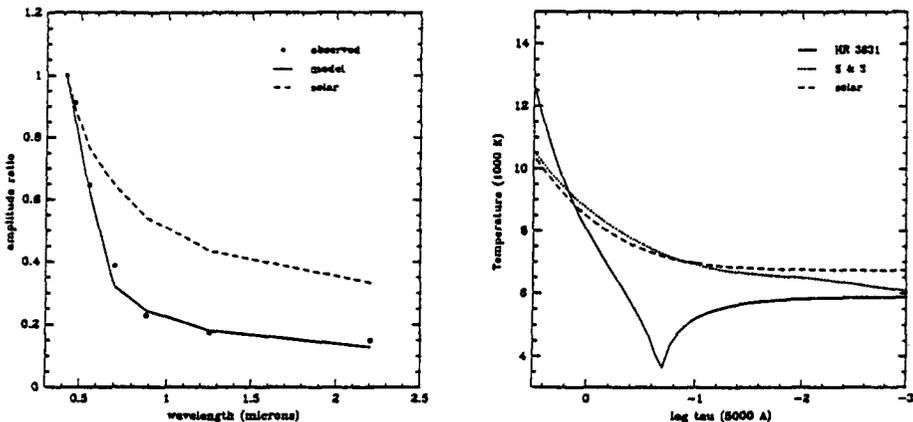


Figure 3. Is there a temperature inversion in the atmosphere of HR 3831? The left panel shows how well the model (solid curve) fits the observed amplitude ratios as a function of wavelength compared to a grey atmosphere (dashed curve). The right panel shows the resulting $T - \tau$ relation (solid curve), compared to the grey case (dashed curve) and Shibahashi and Saio's (1985) model (dotted curve) to account for roAp frequencies observed above the acoustic cutoff of a standard atmosphere.

CONCLUSIONS & CAVEATS

- The observed amplitude ratios (and upper limits) of HR 3831 can only be matched by a **steeper temperature - optical depth relation than for a solar atmosphere**. This is a robust finding, independent of the choice of parameters for the models. It is also in general agreement with the results of Shibahashi & Saio (1985). Although their curve is not so steep (see Fig. 2), it was determined by equating the highest oscillation frequency observed in roAp stars to the acoustic cutoff of their model. Hence, their model should represent a *lower* limit to the $T - \tau$ slope.
- The best solution is obtained by invoking a **temperature inversion near $\log\tau(5000\text{\AA}) \simeq -0.7$** . If real, it could suggest a strong source of opacity near this optical depth. If an artifact of the fitting procedure, it might mean that limb darkening is not the only mechanism weighting the observed pulsation amplitudes. Flux redistribution by photospheric spots is a possibility, particularly if the spots are linked to the magnetic (and pulsation) geometry. However, significant redistribution out to infrared wavelengths of 2.2μ seems unlikely, so it is still difficult to avoid a steep $T - \tau$ curve.
- Note that these results are **valid only for one rotational phase of HR 3831, near magnetic maximum**. The extreme atmospheric properties inferred here may differ from the average over the entire surface of the star.
- **Is a dipole-mode approximation adequate for the pulsations of HR 3831?** Kurtz *et al.* (1992) have argued that the oscillations of HR 3831 are better represented by a “distorted dipole”: a single pulsation mode with $\ell = 0, 1, 2$ and 3 spherical harmonic components dominated by $\ell = 1$. Preliminary tests which include these other components at their relative strengths suggest that this does not change the overall result.
- **Does a single blackbody properly reproduce the unweighted amplitude ratios for pulsation in a real atmosphere, where flux arises from layers of differing temperatures?** Apparently, yes. Models in which the amplitudes are generated from a set of blackbodies spanning temperatures from 4000 – 10,000 K yield un-limb-darkened amplitude ratios similar to those used here. The weighting of the amplitude ratios is dominated by the filtering effect of limb darkening on the pulsation pattern, rather than the fact that fluxes at longer wavelengths arise from cooler atmospheric layers.

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DISCUSSION (Matthews, Wehlau, Rice and Walker)

STEPIEŃ: When Herbert Muthsam and I tried to fit solar metal-content models to the observed energy distribution of Osawa's star (HD 221568), we found that it was necessary to scale the solar metal abundances by a factor of 30. A factor of 10 did not suffice. The $30 \times$ solar model has, of course, a steeper temperature gradient in the outer atmospheric layers than the $10 \times$ solar model.

COWLEY: Does this star have a constellation designation? Has it been studied for abundances? What is the $v \sin i$?

UNIDENTIFIED VOICE: IM Velorum.

KURTZ: The $v \sin i$ is $33 \pm 3 \text{ km s}^{-1}$.

COWLEY: Ugh!

ADELMAN: What is T_{eff} ?

KURTZ: About 8000 K.

WEHLAU: A spectroscopic study of this star would be very interesting indeed. We have applied for observing time several times to study the spectrum and to look for variability which might allow us to map the surface, but were not awarded any time.

BABEL: For an Ap star with T_{eff} around 8000 K, stratification of chemical elements is so large that neither a solar nor $10 \times$ solar model from Kurucz is expected to reproduce very precisely the atmospheric structure. It would be very interesting to investigate if the abundance stratification predicted by the diffusion model could lead to agreement with your steep $T-\tau$ relation.