Advances in Plasma Astrophysics
Proceedings IAU Symposium No. 274, 2010
A. Bonanno, E. de Gouveia Dal Pino, A. G. Kosovichev, eds.

doi:10.1017/S1743921311007010

Kinetic closure conditions for quasi-stationary collisionless axisymmetric magnetoplasmas

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Abstract. A characteristic feature of fluid theories concerns the difficulty of uniquely defining consistent closure conditions for the fluid equations. In fact it is well known that fluid theories cannot generally provide a closed system of equations for the fluid fields. This feature is typical of collisionless plasmas where, in contrast to collisional plasmas, asymptotic closure conditions do not follow as a consequence of an H-theorem This issue is of particular relevance in astrophysics where fluid approaches are usually adopted. On the other hand, it is well known that the determination of the closure conditions is in principle achievable in the context of kinetic theory. In the case of multi-species thermal magnetoplasmas this requires the determination of the species tensor pressure and of the corresponding heat fluxes. In this paper we investigate this problem in the framework of the Vlasov-Maxwell description for collisionless axisymmetric magnetoplasmas arising in astrophysics, with particular reference to accretion discs around compact objects (like black holes and neutron stars). The dynamics of collisionless plasmas in these environments is determined by the simultaneous presence of gravitational and magnetic fields, where the latter may be both externally produced and self-generated by the plasma currents. Our starting point here is the construction of a solution for the stationary distribution function describing slowly-varying gyrokinetic equilibria. The treatment is applicable to non-relativistic axisymmetric systems characterized by temperature anisotropy and differential rotation flows. It is shown that the kinetic formalism allows one to solve the closure problem and to consistently compute the relevant fluid fields *with the inclusion of finite Larmor-radius effects*. The main features of the theory and relevant applications are discussed.

Keywords. kinetic theory, accretion disks, plasmas, magnetic fields

1. Introduction: fluid equations and kinetic closure conditions

In this paper the issue concerned with the determination of kinetic closure conditions for collisionless magnetized axisymmetric accretion disc (AD) plasmas is discussed. The treatment of this problem is a prerequisite of primary importance for getting correct descriptions for the dynamics of collisionless plasmas in terms of suitable fluid equations and the corresponding fluid fields (Cremaschini *et al.* 2008). The result presented here is obtained within the framework of the Vlasov-Maxwell description and concerns, in particular, collisionless magnetoplasmas in astrophysical accretion discs around compact objects (like black holes and neutron stars). In the present context we focus our attention on the specific case of *asymptotic stationary configurations*, in the sense defined in Cremaschini *et al.*, (2010). We also assume that the magnetic field has both poloidal and toroidal components and we restrict our analysis to configurations in which the poloidal field admits locally a family of closed nested magnetic surfaces $\psi = const.$, ψ denoting the poloidal magnetic flux function (Cremaschini *et al.* 2010 and references cited there). The starting point of the work is the construction of a stationary solution to the Vlasov equation for the equilibrium kinetic distribution function (KDF) $\widehat{f_{*s}}$ describing asymptotic (i.e., slowly-varying in time) gyrokinetic equilibria for AD plasmas, as presented in Cremaschini *et al.* 2008 and 2010. This investigation is applicable to non-relativistic axisymmetric systems characterized by temperature anisotropy and flows with differential rotation. By Taylor expanding the KDF in a suitable asymptotic ordering up to a prescribed order of accuracy we wish to compute *analytically* the relevant stationary fluid fields corresponding to collisionless AD plasmas, retaining finite Larmor-radius (FLR) effects in the calculation. It is shown that the kinetic formalism then allows one both to solve consistently the closure problem of the fluid equations and, at the same time, to derive important conclusions about the physical properties of the stationary collisionless AD plasma, as implied by the kinetic analysis (see also Cremaschini *et al.* 2010 for some discussion related to this point). The theory presented is of interest for those studies aimed at investigating stationary AD plasmas from the point of view of fluid MHD theories.

2. Tensor pressure

Concerning the notation adopted, the basic assumptions about the system and the magnetic field configuration, and the details of the construction of the kinetic theory for AD plasmas used here, we refer to the paper by Cremaschini *et al.* 2010. The species number densities and flow velocities have been computed as fluid moments in the same paper. For this reason, in the following we shall focus attention on the calculation of the tensor pressure, which is required as a closure condition in the Euler fluid equation. The total species tensor pressure is defined as $\underline{\underline{\Pi}}_s^{tot} \equiv \int d\mathbf{v} M_s \left(\mathbf{v}-\mathbf{V}_s^{tot}\right)\left(\mathbf{v}-\mathbf{V}_s^{tot}\right)\widehat{f_{*s}}$. Since the AD plasma is collisionless, the KDF is not Maxwellian and we expect to recover some sort of anisotropy in the final form of the pressure tensor. For example, this can be due to the temperature anisotropy, whose origin is related to the conservation of the magnetic moment as an adiabatic invariant. Parallel and perpendicular temperatures are defined with respect to the local direction of the magnetic field. For this reason, it is convenient to introduce the set of orthogonal unitary vectors given by $(\mathbf{b}, \mathbf{e}_1, \mathbf{e}_2)$, where $\mathbf{b} \equiv \frac{\mathbf{B}}{B}$ is the tangent vector to the magnetic field while $\mathbf{e}_1$ and $\mathbf{e}_2$ are two orthogonal vectors in the plane perpendicular to the magnetic field line. Then, from this basis, we can construct the following unitary tensor: $\underline{\underline{\mathbf{I}}} \equiv \mathbf{bb} + \mathbf{e}_1\mathbf{e}_1 + \mathbf{e}_2\mathbf{e}_2$. The tensor pressure has its simplest representation when expressed in terms of the tensor $\underline{\underline{\mathbf{I}}}$ and the vectors $(\mathbf{b}, \mathbf{e}_1, \mathbf{e}_2)$.

By Taylor expanding the equilibrium KDF (Cremaschini *et al.* 2010), the species pressure tensor $\underline{\underline{\Pi}}_s^{tot}$ for magnetized plasmas can be written as follows: $\underline{\underline{\Pi}}_s^{tot} \simeq \underline{\underline{\Pi}}_s + \Delta\underline{\underline{\Pi}}_s$, where $\underline{\underline{\Pi}}_s$ is the leading-order term (with respect to all of the expansion parameters), while $\Delta\underline{\underline{\Pi}}_s$ represents the first-order (i.e., $O(\varepsilon)$) correction term. In particular, $\underline{\underline{\Pi}}_s$ is given by

$$\underline{\underline{\Pi}}_s \equiv T_{\perp s} n_s \underline{\underline{\mathbf{I}}} + n_s \left[T_{\parallel s} - T_{\perp s}\right]\mathbf{bb}, \tag{2.1}$$

which is expressed in terms of the parallel and perpendicular temperatures. On the other hand, $\Delta\underline{\underline{\Pi}}_s$ can be written as

$$\Delta\underline{\underline{\Pi}}_s \equiv \Delta\Pi_s^1 \underline{\underline{\mathbf{I}}} + \Delta\Pi_s^2 \mathbf{bb} + \Delta\underline{\underline{\Pi}}_s^3, \tag{2.2}$$

in which $\Delta\Pi_s^1$ and $\Delta\Pi_s^2$ are the diagonal first-order anisotropic corrections to the pressure tensor, while $\Delta\underline{\underline{\Pi}}_s^3$ contains all of the non-diagonal contributions. More precisely, $\Delta\Pi_s^1$ is given by

$$\begin{aligned}\Delta\Pi_s^1 \equiv\ & \gamma_1 \Omega_s R T_{\perp s} n_s - \gamma_2 \frac{2\Omega_s R T_{\perp s}^2 n_s}{B} + \gamma_3 \frac{\Omega_s R T_{\perp s}^2 n_s}{M_s}\left(5 + \frac{T_{\parallel s}}{\Delta_{T_s}}\right) \\ & + \gamma_3 \left(\Omega_s R\right)^3 T_{\perp s} n_s + \gamma_3 \frac{2\Omega_s I^2 T_{\parallel s} T_{\perp s} n_s}{R B^2 M_s} + \gamma_3 \frac{8\Omega_s R T_{\perp s}^2 n_s}{M_s}\left[\frac{3}{4} - \frac{I^2}{R^2 B^2}\right],\end{aligned} \tag{2.3}$$

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while $\Delta\Pi_s^2$ is defined as

$$\begin{aligned}\Delta\Pi_s^2 \equiv\ & \gamma_1 \Omega_s R n_s \left[T_{\|s} - T_{\perp s}\right] - \gamma_2 \frac{\Omega_s R T_{\|s} T_{\perp s} n_s}{B} \\ & + \gamma_3 \frac{\Omega_s R n_s T_{\|s} T_{\perp s}}{M_s} \left(5 + 3\frac{T_{\|s}}{\Delta_{T_s}}\right) + + \gamma_3 \left(\Omega_s R\right)^3 n_s \left[T_{\|s} - T_{\perp s}\right] + \\ & + \gamma_3 \frac{2\Omega_s R n_s T_{\|s} T_{\perp s}}{M_s} - \gamma_3 \frac{8\Omega_s R T_{\perp s}^2 n_s}{M_s} \left[\frac{3}{4}\frac{I^2}{R^2 B^2} + 1\right] + \gamma_2 \frac{2\Omega_s R T_{\perp s}^2 n_s}{B} + \\ & - \gamma_3 \frac{\Omega_s R T_{\perp s}^2 n_s}{M_s}\left(5 + \frac{T_{\|s}}{\Delta_{T_s}}\right) + \gamma_3 \frac{6\Omega_s I^2 n_s T_{\|s}^2}{R B^2 M_s} - \gamma_3 \frac{4\Omega_s I^2 T_{\|s} T_{\perp s} n_s}{R B^2 M_s}. \end{aligned} \tag{2.4}$$

Finally, $\Delta\underline{\underline{\Pi}}_s^3$ is symmetric and is given by

$$\begin{aligned}\Delta\underline{\underline{\Pi}}_s^3 \equiv\ & \gamma_3 \frac{16\Omega_s R T_{\perp s}^2 n_s}{M_s} \left(\mathbf{e}_1 \mathbf{e}_2 \colon \mathbf{e}_\varphi \mathbf{e}_\varphi\right) \left[\mathbf{e}_1 \mathbf{e}_2 + \mathbf{e}_2 \mathbf{e}_1\right] + \\ & + \gamma_3 \frac{4\Omega_s I T_{\|s} T_{\perp s} n_s}{B M_s} \left(\left(\mathbf{e}_2 \cdot \mathbf{e}_\varphi\right)\left[\mathbf{b}\mathbf{e}_2 + \mathbf{e}_2 \mathbf{b}\right] + \left(\mathbf{e}_1 \cdot \mathbf{e}_\varphi\right)\left[\mathbf{b}\mathbf{e}_1 + \mathbf{e}_1 \mathbf{b}\right]\right).\end{aligned}$$

The following comments about the solution are in order:

- The total tensor pressure $\underline{\underline{\Pi}}_s^{tot}$ is symmetric in the system defined by the vectors $(\mathbf{b}, \mathbf{e}_1, \mathbf{e}_2)$.
- The leading-order pressure tensor $\underline{\underline{\Pi}}_s$ calculated in this approximation is diagonal but non-isotropic. We notice that the source of this anisotropy in Eq. (2.1) is just the temperature anisotropy. In the limit of isotropic temperature, the leading-order pressure tensor becomes diagonal and isotropic, as can be easily verified.
- The first-order correction $\Delta\underline{\underline{\Pi}}_s$ is non-diagonal and non-isotropic. In particular, non-diagonal terms are carried by the tensor $\Delta\underline{\underline{\Pi}}_s^3$ given in Eq. (2.5). Two main properties of the solution contribute to generating the non-isotropic feature of $\Delta\underline{\underline{\Pi}}_s$. The first is the temperature anisotropy, while the second is the existence of the *diamagnetic part* of the KDF obtained from the Taylor expansion of $\widehat{f_{*s}}$ and depending on the thermodynamic forces associated with the gradients of the fluid fields (Cremaschini *et al.* 2010). Indeed, we notice that taking the limit of isotropic temperature is not enough to make the tensor $\Delta\underline{\underline{\Pi}}_s$ isotropic as well. In fact, even if the parallel and perpendicular temperatures are equal, since the plasma is magnetized and collisionless the KDF will not be perfectly Maxwellian and deviations carried by the diamagnetic part act as a source of anisotropy.

3. Conclusions

In this paper a kinetic solution to the problem concerning the determination of closure conditions for fluid MHD equations has been presented. The theory has been developed within the framework of the Vlasov-Maxwell description and is applicable to non-relativistic axisymmetric collisionless AD plasmas. The specific case of asymptotic stationary configurations has been considered, in which the magnetic field is assumed to admit locally a family of closed nested magnetic surfaces. Some important features of collisionless magnetized plasmas have been included in the treatment. These concern the anisotropies induced by the existence of the magnetic field, like the temperature anisotropy, and the deviation of the equilibrium KDF away from the exact Maxwellian case, expressed through the diamagnetic part of the KDF. In particular, in this study, the calculation of the tensor pressure and the discussion of its physical properties have been considered.

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https://doi.org/10.1017/S1743921311007010 Published online by Cambridge University Press