But the second line of (35) is the residue at the pole of $1/\sin\pi(\zeta - t)$. Hence the sum of the four similar expressions

$$= -\text{residue at pole of } 1/\sin\pi(\zeta - c)$$

$$= -\frac{\sin\pi c}{\sin\pi(t-c) \text{ do. } x, y, z}.$$

Thus for the sum of the series in (33) we have

$$S = \frac{\pi^3 \sin\pi c}{\sin\pi(t-c) \text{ do. } x, y, z} \times$$
$$\frac{\Pi(t+x+y+z-2c)}{\Pi(y+z-c)\,\Pi(z+x-c)\,\Pi(x+y-c)\,\Pi(t+x-c) \text{ do. } y, z}, \quad (36)$$

where $R(t+x+y+z-2c) > -1$.

To exhibit the result as the summation of a series of rational terms, multiply both sides of (36) by

$$\frac{\Pi t\,\Pi x\,\Pi y\,\Pi z}{\Pi(c-1-t) \text{ do. } x, y, z}.$$

Then

$$c + (c+2)\;\frac{c-t}{t+1}\;\text{ do. } x, y, z, \;+ \ldots + (c+2n)\;\frac{(c-t)^{(n)}}{(t+1)^{(n)}}\;\text{ do. } x, y\, z, \;+ \ldots$$

$$+ (c-2)\;\frac{t}{c-t-1}\text{ do. } x, y, z, \;+ \ldots + (c-2n)\;\frac{t^{(-n)}}{(c-t-1)^{(-n)}}\text{ do. } x, y, z, \;+ \ldots$$

$$= \frac{\sin\pi c}{\pi}\,\frac{\Pi t\,\Pi x\,\Pi y\,\Pi z\,\Pi(t-c)\,\Pi(x-c)\,\Pi(y-c)\,\Pi(z-c)\,\Pi(t+x+y+z-2c)}{\Pi(y+z-c)\Pi(z+x-c)\Pi(x+y-c)\Pi(t+x-c)\Pi(t+y-c)\,\Pi(t+z-c)}. \quad (37)$$

For $t = 0$, this is equivalent to (9).

The result may be put in somewhat more striking form by writing $2a$ for c, and then $t+a$, $x+a$, $y+a$, $z+a$ for t, x, y, z.

Of special cases of (37), those obtained by writing $t = c/2$, $t = \infty$, $t = (c-1)/2$ may be mentioned.

On the Resolution of Integral Algebraic Expressions into Factors.

By R. F. Muirhead, M.A., D.Sc.

On Arithmetical Approximations.

By R. F. Davis, M.A.

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