Planetary Systems in the Universe - Observation, Formation and Evolution
Proceedings IAU Symposium No. 202, ©2004 IAU
Alan Penny, Pawel Artymowicz, Anne-Marie Lagrange, & Sara Russell, eds.

Frequency Map and Global Dynamics in Planetary systems: Short Period Dynamics

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1. Introduction

Frequency Map Analysis (FMA) (Laskar, 1990, 1999) is a refined numerical method based on Fourier techniques which provide a clear representation of the global dynamics of multi-dimensional systems, which is very effective for systems of 3 degrees of freedom and more, and was applied to a large class of dynamical systems . FMA requires only a very short integration time to obtain a measure of the diffusion of the trajectories, and allows to identify easily the location of the main resonances. Using this method, we have performed a complete analysis of massless particles in the Solar System (Robutel & Laskar 2001), from Mercury to the outer parts of the Kuiper belt (90 AU), for all values of the eccentricities, and several values for the inclinations. This provides a complete dynamical map of the Solar System, which is, in this first step, restricted to mean motion resonances. The dynamics of a planetary system which all the bodies have no zero mass can be studied with the same methods: an application to the Jupiter-Saturn system can be fined in (Robutel & Laskar 2002). We present here the application of this method to the understanding of the dynamics of the newly discovered υ-Andromedae system.

2. Frequency map analysis

For an Hamiltonian system on $\mathbf{R}^n \times \mathbf{T}^n$, FMA constructs numerically a map which associates the n-dimensional frequency vector to the action-like variables (see Laskar, 1999 for details). The dynamical behavior of the Hamiltonian system is obtained from the study of the regularity of this frequency map. The construction of the frequency map requires only a very short integration time and allows to get a measure of the diffusion of the trajectories. This diffusion, corresponding to the variation of the fundamental frequencies with respect to time is computed in the following way: a first determination of the frequencies is done on the time interval $[0, T]$ and a second one on $[T, 2T]$. For a quasiperiodic solution, the two frequency vectors are equal, and if it is not the case, their difference gives an estimate of the chaotic diffusion of the trajectory over the time T.

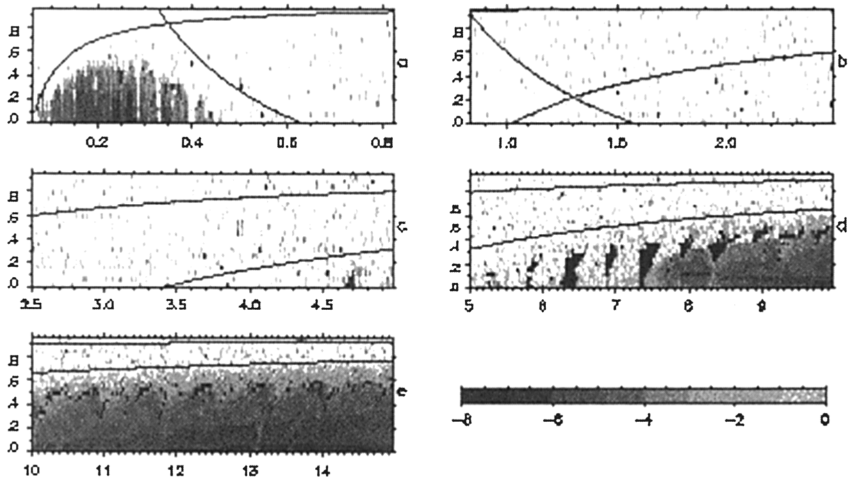

Figure 1. Massless bodies in the υ-Andromedae system: Frequency map in the (a_0, e_0) plane for $M_0 = \omega_0 = 0$. The gray code is for $\log_{10} \sigma$, measured on a time span of 4500 Ma. The black lines are the collision curves with the planets.

3. Short time dynamics of test particles in the υ-Andromedae

In our experiment, we assume that the system is planar and that the masses of the three planets are equal to their lower limit. The other initial conditions are chosen in the Lick data taken from (Butler et al., 1999). Considering that this planetary system is stable, at least for several millions of years (Lissauer, 1999), we have only to study the global stability of test particles orbiting between these planets. As the motion of the massless body does not affect the planetary motions, the planet frequencies are fixed, and it is sufficient to study the frequency map in a 2-dimensional space of action-like variables. For any particle, these 2 action variables are the usual elliptical elements (a, e) (semi-major axis and eccentricity), while the 2 angle variables are the associated angles (M, ω) (mean anomaly, argument of perihelion). We fix all the initial angles of the particle (M_0, ω_0) (see Laskar, 1999), and construct numerically the frequency map which associates the numerically determined (n, g) frequencies (associated to mean longitude and perihelion) to the initial actions (a_0, e_0). We are only interested here in the short time dynamics, so only the mean motion frequency n will be determined (the two others are associated to the secular motion). This frequency will be called the "proper mean motion", in the sense that if the motion is quasiperiodic, this quantity is a integral of the motion. We thus consider only the map $(a_0, e_0) \longrightarrow n$. If the motion is not quasiperiodic, the two values of $n^{(1)}$ and $n^{(2)}$ obtained over the consecutive intervals $[0, T]$ and $[T, 2T]$ will not be equal in general, and the quantity $\sigma = 1 - n^{(2)}/n^{(1)}$ is computed to provide a measure of the diffusion rate of this trajectory over a time T. We can consider that in this problem, a diffusion rate σ greater than 10^{-5} corresponds to

https://doi.org/10.1017/S0074180900217968 Published online by Cambridge University Press

an unstable motion (the trajectories for which $\sigma > 10^{-2}$ are strongly chaotic), while orbits with $\sigma < 10^{-6}$ could be stable (except in presence of secular chaos, not detected here). The planetary system with test particles is integrated over two consecutive time intervals of 9000 years. This apparently short time (more than 1 400 000 revolutions of the first planet, 27 000 of the second one, and 5 180 revolutions of the third planet), is in fact long enough to allow us to obtain a full vision of the the stability regions and the location of the mean motion resonances in the υ-Andromedae system (Fig.1).

The initial phase angles of the test particles are fixed to $M_0 = \omega_0 = 0$, the initial eccentricity e_0 is sampled for 0 to 0.95 with a resolution of 0.05. Figures 1a is the frequency map for the region between the two first ($0.06AU \leq a_0 \leq 0.826AU$ with a step size of $0.0038AU$). The two small vertical gaps at 0.094 AU and 0.015 AU mark the presence of the (1:-2) and (1:-4) resonances with the inner planet. The three other gaps at respectively 0.283, 0.328 and 0.4 AU correspond to the (5:-1), (4:-1) and (3:-1) mean motion resonances with the second planet. The (5:-2) resonance with the second planet marks the beginning of the region where almost all test particles are rapidly ejected after close encounters with planets. In the region between the first two planets, we can see on Fig.1a, that there is only a small spot, ($a_0 \approx 0.25AU, e_0 \leq 0.15$) of apparently regular initial conditions where particles could survive for a long time, although for very long times, one should also check for secular resonances and chaos in this area. Figure 1b ($a_0 \in [0.83, 2.5]$ AU with a step size of 0.00835 AU) shows clearly that no particle can survive there, and that we are in presence of a very strong chaotic behavior due to resonance overlap and close encounters with the planets. The situation is the same in Fig.1c ($a_0 \in [2.5, 5]$ AU with a step size of 0.0125 AU). As we move away from the third planet, we find more stable areas (Fig.1d-e). The region between 5 and 7.5 AU is still dominated by resonances and there should not be stable regions there apart from the resonant islands. Beyond 7.5 AU, the diffusion rate decreases, and we have a stable zone similar to the zone observed beyond Neptune in our Solar System (Robutel & Laskar 2001). This should thus be the place to look for additional bodies. The present study was done with a massless particle, but it should be noted that for a massive planet, the chaotic regions will even be larger, and the regular regions will be smaller than the ones found here.

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