so that

$$\log_{10}\frac{m+1}{10}+\log_{10}\left(1+\frac{y}{(m+1)10^m}\right)=\frac{y}{10^m}$$

which is easily solved by successive approximations.

But it is simpler, and forms a capital exercise, to find, say to six places, the greater root, by mere inspection of a good Table of Logarithms.

Thus we find, for instance,

m	x
17	$182,615.10^{13}$
18	$192,852.10^{14}$
96	$979,911.10^{92}$
97	$989,956.10^{93}$

Geometrical Proof of the Tangency of the Inscribed and Nine-Point Circles.

By William Harvey, B.A.

S (fig. 85) is the circumscribed centre, and O the orthocentre of the triangle ABC; AX the perpendicular from A on BC, and P the middle point of BC.

SP produced bisects the arc BC in V, and I, the centre of the inscribed circle, lies on AV, and is so situated that $AI.IV=2Rr$. (*See Note*). Also the angle XAV = angle AVS = angle SAV.

N, the centre of the nine-point circle, bisects the distance OS, and the circumference passes through P, X and L, the middle point of AO. Hence N bisects both LP and OS, and

$$SP=OL=AL;$$

therefore LP is parallel to AS.

NHM is a radius of the nine-point circle, bisecting the chord XP in H, and the arc XP in M; ID is a radius of the inscribed circle.

Since the chord XMP is bisected at M,

$$\begin{aligned}\text{the angle XPM} &= \frac{1}{2}\text{angle XLP},\\ &= \frac{1}{2}\text{ angle XAS},\\ &= \text{angle XAV or AVS.}\end{aligned}$$

Hence, if through I we draw a straight line (not shown in the figure) parallel to BC to meet AX and SV, the segments of this line are respectively equal to XD and PD, and we have by similar triangles

$$\mathrm{XD} : \mathrm{IA} = \mathrm{HM} : \mathrm{MP};$$

$$\mathrm{PD} : \mathrm{IV} = \mathrm{HM} : \mathrm{MP};$$

and therefore

$$\mathrm{XD.PD} : \mathrm{IA.IV} = \mathrm{HM}^2 : \mathrm{MP}^2.$$

But $\quad \mathrm{IA.IV} = 2\mathrm{R}r$, and $\mathrm{MP}^2 = \mathrm{R.HM}$.

Hence $\quad \mathrm{XD.PD} = 2r.\mathrm{HM}$.

But $\quad \mathrm{HP}^2 - \mathrm{HD}^2 = \mathrm{XD.PD} = 2r.\mathrm{HM}$.

Now

$$\begin{aligned} \mathrm{IN}^2 &= (\mathrm{NH} - r)^2 + \mathrm{HD}^2, \\ &= (\mathrm{NH} - r)^2 + \mathrm{HP}^2 - 2r.\mathrm{HM}, \\ &= \mathrm{NH}^2 + \mathrm{HP}^2 - 2r.(\mathrm{NH} + \mathrm{HM}) + r^2, \\ &= \frac{\mathrm{R}^2}{4} - r\mathrm{R} + r^2; \end{aligned}$$

Or $\quad \mathrm{IN} = \dfrac{\mathrm{R}}{2} - r.$

Whence the tangency of the circles is evident.

Note.—The following is a simple proof of the theorem $\mathrm{AI.IV} = 2\mathrm{R}r$, assumed in the foregoing proof :—

Draw IE perpendicular to AC, and ST perpendicular to VC.

Since angle AVC = angle B = 2 angle IBC, I lies on the circumference of a circle whose centre is V and radius VB or VC.

Hence $\quad \mathrm{IV} = \mathrm{VC}$.

Again, the triangles AIE, SVT are clearly similar;

$$\therefore \; \mathrm{AI} : \mathrm{IE} = \mathrm{SV} : \mathrm{VT}.$$

But $\quad \mathrm{VT} = \dfrac{1}{2}\mathrm{VC} = \dfrac{1}{2}\mathrm{IV};$

$$\therefore \; \mathrm{AI.IV} = 2\mathrm{R}r.$$