The Zeldovich Universe:
Genesis and Growth of the Cosmic Web
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Higher-order Lagrangian perturbative theory for the Cosmic Web

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Abstract. Zel'dovich proposed Lagrangian perturbation theory (LPT) for structure formation in the Universe. After this, higher-order perturbative equations have been derived. Recently fourth-order LPT (4LPT) have been derived by two group. We have shown fifth-order LPT (5LPT) In this conference, we notice fourth- and more higher-order perturbative equations. In fourth-order perturbation, because of the difference in handling of spatial derivative, there are two groups of equations. Then we consider the initial conditions for cosmological N-body simulations. Crocce, Pueblas, and Scoccimarro (2007) noticed that second-order perturbation theory (2LPT) is required for accuracy of several percents. We verify the effect of 3LPT initial condition for the simulations. Finally we discuss the way of further improving approach and future applications of LPTs.

Keywords. large-scale structure of universe, methods: analytical

1. Derivation of 5LPT equations and their fitting formulae

The formation of the cosmic web is one of most important problems in modern cosmology. It is considered that the primordial density fluctuation grows by its-self gravitational instability. As one of theoretical approach for the evolution, Lagrangian perturbation has been considered for long time. At first, Zel'dovich (1970) proposed Lagrangian perturbation theory (LPT). Although the perturbation remains linear stage, quasi nonlinear evolution of the density fluctuation is described well. In 1990s, second- and third-order LPTs had been derived (Bouchet *et al.* (1992), Buchert (1992), Buchert & Ehlers (1993), Buchert (1994), Bouchet *et al.* (1995), Sasaki & Kasai (1998)). Recently, fourth-order LPT (4LPT) have been derived (Rampf & Buchert (2012), Tatekawa (2013)). Furthermore, the recursive formula for the derivation of higher-order LPT have been proposed (Rampf (2012)).

In 4LPT, because of difference of treatment for spatial derivative, there are two groups of perturbative equations. In this conference, we have derived perturbative equations for 5LPT based on the procedure by Rampf & Buchert (2012). 5LPT equations consist of 15 longitudinal modes and 11 transverse modes.

In Einstein-de Sitter Universe model, we can derive analytical form for temporal parts up to fifth-order LPT. However in LCDM model, we can derive analytical form for temporal parts only in 1LPT. Therefore we consider fitting formula for temporal parts. Peebles (1984) and Bouchet *et al.* (1995) derived fitting formula up to third-order LPT. We have tried to improve past formulae and derive new formula up to 5LPT. First, we define logarithmic derivative of the temporal parts:

$$f_n \equiv \frac{a}{g_n}\frac{\mathrm{d}g_n}{\mathrm{d}a}\,, \tag{1.1}$$

where a means the scale factor. g_n means the temporal parts in n-th order LPT. Following past formulae, we assume the formula in nLPT as

$$f_n \simeq n\Omega_M^\alpha\,, \tag{1.2}$$

where Ω_M means the density parameter. For derivation, we apply least-squares method for derivation of new formula in $0.1 \leqslant \Omega_M \leqslant 1$. In preliminary results, the formulas are seldom improved. We should consider other approach for the derivation of the fitting formulae.

Before considering application of 5LPT, we should verify two groups of 4LPT equations (Rampf & Buchert (2012), Tatekawa (2013)). After verification, we will consider 3-loop correction for the power spectrum using 5LPT solutions.

2. Initial condition problem for cosmological N-body simulations

For analyses in strongly nonlinear stage, cosmological N-body simulations have been carried out. As initial conditions for the simulations, 1LPT has been applied for long time (for example, Ma & Bertschinger (1995)). Recently, several groups point out that the higher-order LPT requires for the initial condition. Crocce, Pueblas, & Scoccimarro (2006) pointed out if 2LPT is ignored in the initial conditions, the statistical quantities such as non-Gaussianity in the density field at low-z region deviate with several percent.

A new question arises here. Even if the effect of 2LPT seems important for late time, we cannot decide whether 2LPT initial condition is enough or not. If the effect of 3LPT initial condition is negligible, 2LPT initial condition is enough for cosmological N-body simulations. We have analyzed the effect of 3LPT initial conditions (Tatekawa & Mizuno(2007), Tatekawa (2014)).

By the analysis of the evolution for the power spectrum and non-Gaussianity in the density field, we conclude when we require sub percent accuracy, we should consider 3LPT initial conditions. Then the effect of the transverse mode in 3LPT seems negligible.

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