SPIEGEL, E. and O'DONNELL, C. J. *Incidence Algebras* (Pure and Applied Mathematics Vol. 206, Marcel Dekker, 1997), ix + 335 pp., 0 8247 0036 8, US$125.

This is a monograph on a rather specialised topic. In Chapter 1 the basic definitions are given. An incidence algebra $I(X, R)$ is defined over a locally finite partially ordered set X and a commutative ring R as the set of mappings from $X \times X$ to R with the restriction that $f(x, y) = 0$ if x is not less than or equal to y; the usual matrix operations of addition, multiplication and multiplication by scalars in R are used to make this set into an associative algebra over R. Examples are given to show where these structures can arise in combinatorial or geometric settings as well as in algebra.

Chapter 2 gives a good account of the Möbius Algebra leading to the Polya-de Bruijn Theorem. Chapter 3 leads from generating functions and incidence coalgebras to incidence Hopf algebras. In Chapter 4 radicals of the incidence algebra are considered. The Jacobson radical is related to that for R as expected. It is shown that the upper and lower nil radicals coincide. Chapters 5 and 6 are devoted respectively to maximal ideals and to prime ideals. In Chapter 7 the isomorphism problem is considered. If F is a field then $I(X, F)$ and $I(Y, F)$ are isomorphic if and only if X and Y are isomorphic. For a ring R somewhat weaker conclusions are drawn but the full result is shown to hold in certain cases. Further ring theoretic properties are considered in the final chapter, with special attention being given to rings of quotients and polynomial identities.

The monograph is well written. The definitions and results are clearly presented. There is a full list of references and at the end of each chapter there is a brief history of the material and a clear attribution to the original developers. Personally I found the combinatorial applications rather than the algebra to be of the greatest interest.

A. D. SANDS

KISSIN, E. and SHULMAN, V. *Representations on Krein spaces and derivations of C^*-algebras* (Pitman Monographs and Surveys in Pure and Applied Mathematics Vol. 89, Longman, 1997), iii + 602 pp., 0 582 23157 4, £85.

Krein spaces first appeared in physics papers in the early 1940s, when Dirac and Pauli proposed their use in quantum field theory. However, the first mathematical paper about operators on Krein spaces, written by Pontryagin and published in 1944, was based on a suggestion of Sobolev arising from a problem in classical mechanics. Subsequently the theory was developed by M. G. Krein, Naimark, Phillips and others, and it has found assorted applications to topics in quantum field theory, differential equations, complex function theory and operator theory.

A Krein space is a particular type of (infinite-dimensional) indefinite inner product space. Any orthogonal decomposition $H = H_- \oplus H_+$ of a Hilbert space H produces a Krein space, and every Krein space arises in this way. However, this correspondence is not bijective, because the same Krein space arises from many different choices of definite inner product and decomposition of H. The theory of Krein spaces is concerned with aspects of the indefinite inner product which are independent of these choices; for example, the dimensions of H_- and H_+ satisfy this. An operator which is symmetric with respect to the indefinite inner product on a Krein space is said to be "J-symmetric". The theory of J-symmetric operators has some common features with the usual theory of self-adjoint operators, but it is considerably more intricate and some of its aspects are completely different from the self-adjoint case. These aspects of Krein spaces have been described in earlier books, for example *Indefinite inner product spaces* by J. Bognár (Springer-Verlag, 1974) and *Linear operators in spaces with indefinite metric* by T. Y. Azizov and I. S. Iokhvidov (Wiley, 1989).

https://doi.org/10.1017/S0013091500020368 Published online by Cambridge University Press