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STRICT TOPOLOGIES FOR VECTOR-VALUED FUNCTIONS*: CORRIGENDUM

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Professor D. H. Fremlin has pointed out that the vector-valued integral of Definition 3.10 may fail to exist so that Lemma 3.11 is false without additional hypotheses such as those (suggested by Professor Fremlin) in the restatement of Lemma 3.11 below.

These conditions include the case that the measure m is tight. Hence Proposition 3.12 (except for the "σ-additive" part) and Theorem 3.13, whose collective purpose is a vector-valued measure characterization of tight linear functionals, remain valid.

The author wishes to thank Professor Fremlin for bringing the error to his attention.

The restatements of Lemma 3.11 and Proposition 3.12 follow.

3.11. LEMMA. *Let* $F \in C^*(X:E)$ *and* $m \in M(X:E')$.

(a) *If* $\int_X$ fdm *exists, then*

$$\left|\int_X fdm\right| \leqq \int_X |f|d|m|.$$

(b) *The integral exists (for all* f *as above) if* $|m| \in M_\tau(X)$.

(c) *The integral exists if the range of* f *is measure compact* (see [**12**]) *and* m *is* σ*-additive on the range of* f. (For metric spaces, measure compactness is a set theory problem.)

Proof. (a) See Lemma 3.11 of the article. (b) Let $\epsilon > 0$, $v = |m|$, $a = v(X)$, $b = ||f||$, and cover the range of f by open sets O_β of diameter $\epsilon/2a$. Let $W_\beta = f^{-1}(O_\beta)$. By τ-additivity of $|m|$, there are a finite number of the (cozero sets) $W_1, \ldots, W_n$ whose union W satisfies $v(X\backslash W) < \epsilon/4b$. Partition X by Baire sets $V_1, \ldots, V_{p+1}$, with $p \leqq n + 1$ such that

(1) $V_i \subseteq W_i$ for $i = 1, \ldots, p$;

(2) $V_{p+1} = X\backslash W$ (if nonvoid);

(3) W is the union of the sets $V_1, \ldots, V_p$. Let $x_i \in V_i$ for $i = 1, \ldots, p + 1$. Consider any partition $\{B_j : 1 \leqq j \leqq q\}$ refining $\{V_i\}$ and any choice of points $y_j \in B_j$.

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Let $D_i = \{j : B_j \subseteq V_i\}$ for $1 \leqq i \leqq p + 1$; then

$$\left| \sum_{i=1}^{p+1} m(V_i)f(x_i) - \sum_{j=1}^{q} m(B_j)f(y_i) \right| \leqq \sum_{j \in D_{p+1}} |m(B_j)(f(x_{p+1}) - f(y_i))|$$
$$+ \frac{\epsilon}{2a} \sum_{i=1}^{p} \sum_{j \in D_i} ||m||(B_j) \leqq 2bv(X \backslash W) + \frac{\epsilon}{2a} \cdot a < \epsilon.$$

Thus the integral exists.

(c) This argument is similar to part (b). Assuming that $|m|$ is σ-additive, the measure μ defined by $\mu(E) = |m|(f^{-1}(E))$ is σ-additive (where E is a Baire set of $f(X)$). By the measure compactness assumption, μ is also τ-additive and can be used to produce (by taking inverse images) sets $W_1, \ldots, W_n$ with the same properties as in the proof of part (b). The rest of the proof is the same as in part (b).

3.12. PROPOSITION. *Let $m \in M_\sigma(X:E')$ and*

$$F(f) = \int_X f dm$$

for $f \in C^*(X:E)$. Assume that either

(a) *$m \in M_t(X:E')$, or*

(b) *the range of f is measure compact for all f as above* (this will hold, e.g., if E is measure compact or if X is separable).

Then F is σ-additive and $||F|| = |m|(X)$. If $m \in M_t(X:E')$, then F is tight.

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