ON THE ENERGETICS AND MOMENTUM DISTRIBUTION OF BOW SHOCKS AND COLLIDING WINDS

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Abstract. We discuss recent progress in analytic modeling of stellar wind bow shocks and colliding winds. For thin, radiative shocked layers in steady-state, the shape of the layer as well as its internal flux of mass and momentum are found from the conservation laws of mass, momentum and angular momentum. For the case that the shocked gas is well-mixed, the velocity distribution and mass column density of shocked material are also obtained. These solutions are extended to the problem of a jet bow shock, treated as a non-isotropic "wind" interacting with the ambient medium. We also examine the shell energetics for these simple analytic models. The constraint of conservation of momentum leads to an upper limit to the efficiency of thermalization and radiation of the pre-shock wind kinetic energy. Calculations are presented of this thermalization rate as a function of the input momentum rates of the pre-shock winds.

1. Introduction

Bow shocks are seen in a wide variety of circumstances in astrophysics, and many examples are known in star forming regions. This paper will primarily be concerned with analytic, dynamical models of bow shocks resulting from the collision of two supersonic flows or "winds", which may be compared with a number of different applications. While the models are necessarily

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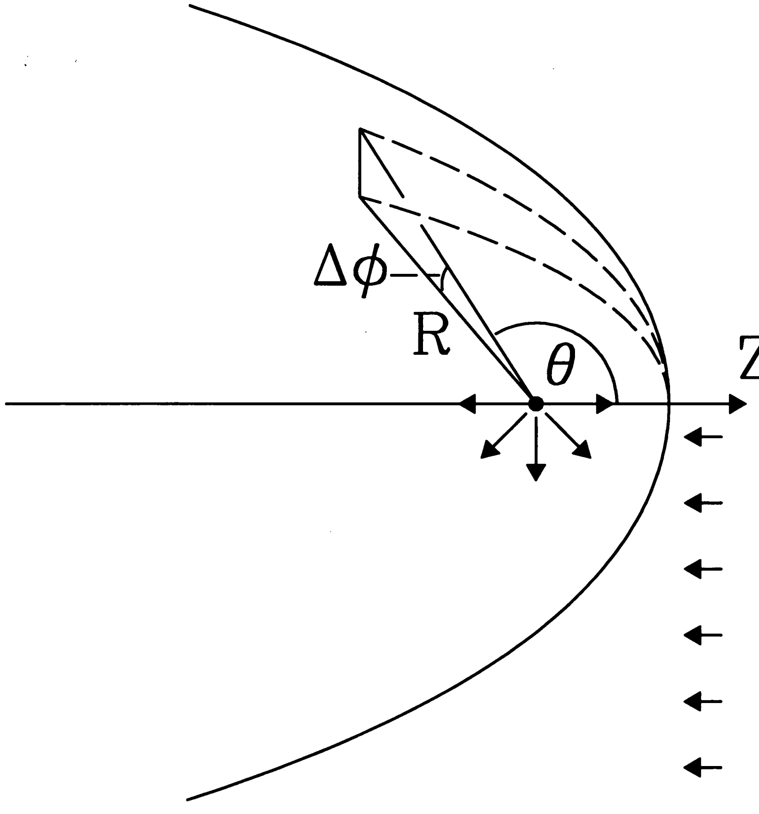

Figure 1. Geometry of the wind/ambient collision. The pre-shock flows are shown in the bottom panel, while the top panel defines the spherical coordinate system with origin at the source of the wind.

simple, in order to be analytically tractable, it is hoped that they will prove useful because they can easily be scaled and may provide insight to guide more detailed numerical studies. Because these calculations include full vector momentum conservation, they represent a step beyond previous analytic works based upon ram pressure balance arguments.

Perhaps the simplest example of a bow shock is that due to the motion of a supersonic, isotropic stellar wind, when the wind-blowing star moves supersonically with respect to the local interstellar medium. If the post-shock cooling is sufficiently rapid, the shocked fluid will lie in a thin layer that takes a steady-state, cometary form in the reference frame of the moving wind source (see Figure 1).

This model is sufficiently generic that it has been applied to many circumstances, and has been solved numerically by several authors. It was initially proposed by Baranov, Krasnobaev and Kulikovskii (1971, hereafter BKK) to describe the interaction of the solar wind with the very local interstellar medium. More recently, Van Buren *et al.* (1990) and Mac Low *et al.* (1991) have applied such a bow shock model to the confinement of cometary, ultracompact HII regions surrounding moving O stars, while Aldcroft, Romani & Cordes (1992) have applied it to pulsar wind bow shocks. The source of the wind need not be a stellar object, and similar models have been calculated, using ram pressure arguments, for a photoevaporating clump embedded in a moving medium (Dyson 1975), and for comets (Hoopis & Mendis 1980).

A new solution method for thin shell bow shocks in steady-state was given by Wilkin (1996), which yields the exact solution analytically to the above-described problem, including the shape of the bow shock and the mass column density and velocity of flow of the shocked material, assuming a mixed layer. This solution method is based upon exact, vector momentum conservation in the shocked layer, and will be described in §2. A very similar picture has been given for the bow shock due to a propagating jet, in which shocked material is imagined to be sprayed forward of the jet shock, mimicking a moving wind which interacts with the surrounding medium. The analytic solution for these bow shocks is given in §3.

A closely related problem is that of the collision of two spherical winds, which yields a family of possible bow shocks depending on the relative strengths of the two winds. An extention of the solution method (Cantó, Raga & Wilkin 1996) to consider angular momentum conservation allows the solution of this more complicated problem, discussed in §4.

The extention of these models to non-axisymmetry is obviously a necessity due to the many asymmetric observed bow shocks. Non-axisymmetric, ram pressure balance models of "proplyds", described as photoevaporating disks embedded in a stellar wind, have recently been developed by Henney *et al.* (1996); Henney (1996), see also Henney & Arthur, this volume. The extention of the analytic method to non-axisymmetric situations is given by Wilkin (1997a,b), and will not be discussed further here.

2. The Stellar Wind Bow Shock Model

The stellar wind drives a shock into the ambient medium, while the supersonic wind is abruptly decelerated, leading to two layers of shocked gas. These layers are assumed to mix, and postshock cooling is assumed to be

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efficient so that the dense shell has negligible thickness. The star moves with speed V_* in a uniform medium of density ρ_a. The isotropic stellar wind has mass loss rate $\dot{M}_w$ and constant speed V_w, yielding a cometary structure with the stellar trajectory as symmetry axis. The flow is assumed hypersonic, so that pressure forces are neglected. In this idealized model, the thin shell is fully described by three quantities: the shell radius $R(\theta)$, mass surface density $\sigma(\theta)$, and the tangential speed $v_t(\theta)$ of shocked material flowing along the shell.

Let the z axis be the axis of symmetry of the shell, with the stellar motion in the $\hat{\mathbf{z}}$ direction (to the right in Figure 1). In the frame of the star, the ambient medium appears as a uniform wind in the $-\hat{\mathbf{z}}$ direction. The stellar wind and the ambient medium collide head-on at $\theta = 0$, and the radius of this starting point of the shell is found by balancing the ram pressures of the wind and ambient medium, $\rho_w V_w^2 = \rho_a V_*^2$, which yields

$$R_0 = \sqrt{\frac{\dot{M}_w V_w}{4\pi\rho_a V_*^2}}. \tag{1}$$

This standoff distance sets the length scale of the shell. The shape of the shell is a universal function, which is scaled according to equation (1) to accommodate all values of the four dimensional parameters $(\dot{M}_w, V_w, \rho_a, V_*)$.

The fluxes of mass and momentum crossing an annulus of the shell, $2\pi\Phi_m(\theta)$, and $2\pi\Phi_t(\theta)$, are given, respectively, by

$$\Phi_m = \varpi\sigma v_t, \qquad \text{and} \qquad \Phi_t = \varpi\sigma v_t^2, \tag{2}$$

where the cylindrical radius ϖ is $R\sin\theta$. In steady-state, the mass traversing a ring of the shell at polar angle θ from the standoff point is given by the mass flux from the stellar wind intercepted by the solid angle of the forward part of the shell plus the contribution from the ambient medium striking the circular area of the projected cross section of the shell:

$$2\pi\Phi_m = \dot{M}_w \frac{\Omega}{4\pi} + \pi\varpi^2\rho_a V_*. \tag{3}$$

Here $\Omega = 2\pi(1 - \cos\theta)$ is the solid angle from the axis to the annulus at θ.

Following Wilkin (1996), we may calculate the rate at which vector momentum is imparted to the shell by the stellar wind, by considering a wedge of small, constant width in the azimuthal angle $\Delta\phi$ about the symmetry axis (Fig. 1). The surface integral of the wind vector momentum flux onto the shell does not depend on the detailed shape of the shell, because the coasting wind is momentum-conserving. We perform the integral over a spherical surface, using $\hat{\mathbf{r}} = \hat{\varpi}\sin\theta + \hat{\mathbf{z}}\cos\theta$:

$$\mathbf{\Phi}_w \Delta\phi \;=\; \int_{\text{wedge}} \rho_w \mathbf{V}_w (\mathbf{V}_w \cdot \hat{\mathbf{n}})\, dA$$

$$\mathbf{\Phi}_w \Delta\phi = \dot{\mathrm{M}}_w \mathrm{V}_w \left[(\theta - \sin\theta\cos\theta)\ \hat{\varpi} + \sin^2\theta\ \hat{\mathbf{z}}\right]\Delta\phi/8\pi. \tag{4}$$

The momentum deposited by the ambient medium is in the $-\hat{\mathbf{z}}$ direction, and depends only upon the circular cross section:

$$\mathbf{\Phi}_a \Delta\phi = -\varpi^2 \rho_a \mathrm{V}_*^2\ \hat{\mathbf{z}}\,\Delta\phi/2. \tag{5}$$

The total momentum flux onto the $\Delta\phi$ wedge of the shell is the sum of the wind and ambient contributions. To conserve momentum in steady-state, the (tangential) momentum flux $\Phi_t\,\hat{\mathbf{t}}\Delta\phi$ traversing a $\Delta\phi$ azimuthal width of an annulus of the shell must equal the momentum flux $(\mathbf{\Phi}_w + \mathbf{\Phi}_a)\Delta\phi$ received by the shell surfaces between the standoff point and the annulus:

$$\mathbf{\Phi}_t = \frac{\dot{\mathrm{M}}_w \mathrm{V}_w}{8\pi}\left[(\theta - \sin\theta\cos\theta)\ \hat{\varpi} + \sin^2\theta\ \hat{\mathbf{z}}\right] - \frac{\varpi^2}{2}\rho_a \mathrm{V}_*^2\,\hat{\mathbf{z}}, \tag{6}$$

where $\mathbf{\Phi}_t = \Phi_t\,\hat{\mathbf{t}}$ is the vector momentum flux in the shell, and $\hat{\mathbf{t}}$ is a tangential unit vector at constant ϕ. The momentum flux has magnitude

$$2\pi\Phi_t = \pi \mathrm{R}_0^2 \rho_a \mathrm{V}_*^2 \sqrt{(\theta - \sin\theta\cos\theta)^2 + (\tilde{\varpi}^2 - \sin^2\theta)^2}, \tag{7}$$

where a tilde indicates a length in units of R_0. We now know the *vector* momentum flux at any point in the shell. The direction of flow is that of this momentum flux, so the shell shape is given by the *trajectory equation*

$$\frac{d\tilde{z}}{d\tilde{\varpi}} = \frac{\mathrm{v}_z}{\mathrm{v}_\varpi} = \frac{\Phi_{t,z}}{\Phi_{t,\varpi}} = \frac{-\tilde{\varpi}^2 + \sin^2\theta}{\theta - \sin\theta\cos\theta}. \tag{8}$$

It can be shown (Wilkin 1996) that the exact integral to this equation is

$$\mathrm{R}(\theta) = \mathrm{R}_0 \csc\theta\sqrt{3(1 - \theta\cot\theta)}. \tag{9}$$

This formula for $\mathrm{R}(\theta)$, together with the momentum flux $\Phi_t(\theta)$ of equation (7), is the solution of the equations of BKK with the desired initial conditions $\mathrm{R}(0) = \mathrm{R}_0$, and $\mathrm{R}'(0) = 0$. With the previously known mass integral, we obtain the remaining shell properties. Referring to equations (2), the tangential velocity in the shell is $\mathrm{v}_t = \Phi_t/\Phi_m$, while the mass surface density is given by $\sigma = \Phi_m^2/\varpi\Phi_t$. Both the mass surface density and the tangential velocity depend upon the nondimensional parameter $\alpha = \mathrm{V}_*/\mathrm{V}_w$.

The total amount of incident kinetic energy thermalized in the bow shock per unit time is given by the sum of the incident kinetic energy fluxes in the center of mass frame. This must be the ambient frame, since in any other frame, the ambient medium deposits an infinite amount of

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momentum per unit time, because the cylindrical radius of the bow shock formally diverges far back in the tail. Letting primes indicate velocities in the ambient frame, so that $\mathbf{V}'_w = V_w\,\hat{\mathbf{r}} + V_*\,\hat{\mathbf{z}}$, the rate of thermalization of incident kinetic energy is given by (Wilkin, Cantó & Raga 1997)

$$\begin{aligned} \dot{E}_{th} &= \int_{\rm shell} \frac{1}{2}\,\rho_w |\mathbf{V}'_w|^2 (\mathbf{V}'_w - V_*\,\hat{\mathbf{z}}) \cdot \hat{\mathbf{n}}\,{\rm d}A \\ &= \frac{1}{2}\,\dot{\rm M}_w (V_w^2 + V_*^2) \end{aligned} \tag{10}$$

This sets an upper limit to the rate at which energy can be radiated by the bow shock in steady-state, in the absence of other energy sources (*i.e.* radiation from the wind source). It is interesting that this very simple result was not noticed in the previous numerical calculations of the bow shock.

One would also like to describe the bow shock kinematics in the ambient, rather than stellar, frame. At any angle θ, the ratio of sideways (V'_ϖ) to forward (V'_z) velocity, and therefore sideways to forward momentum, is given by

$$\frac{V'_\varpi}{V'_z} = \frac{\theta - \sin\theta\cos\theta}{\sin^2\theta + 2\alpha(1-\cos\theta)}, \tag{11}$$

which has the limit of $\pi/4\alpha$ as $\theta \to \pi$. Clearly any ratio of sideways to forward momentum is possible, for some value of α, even for large θ, in the tail of the bow shock (See Figure 2). This may be relevant to the application of bow shock models to the ends of molecular outflows, because it is known that most of the momentum in such outflows is in the forward direction. Lada & Fich (1996) argued that this constraint excluded bow shock models, because they only considered the immediate post-shock velocity, rather than the tangential velocity in the shell after radiative relaxation and mixing with previously shocked material. We see that for values of α greater than unity, the momentum will be primarily in the forward direction.

3. Application to Jet Bow Shocks and Molecular Outflows?

The thin shell bow shock model has been applied in modified form to the problem of jet-driven molecular outflows in star forming regions. Raga & Cabrit (1993) considered internal working surfaces in a jet due to a time-dependent jet velocity, while Zhang & Zheng (1997) treated the terminal bow shock at the end of the jet. In these studies, the shocked fluid is assumed to be sprayed forward of the jet shock as a "wind" which interacts with the surrounding material. This "wind" is no longer taken to be isotropic in the reference frame of the bow shock, but is assumed to extend from the symmetry axis only to an angle $\theta_0 < \pi$ (Zhang & Zheng), or in the case of Raga & Cabrit, to extend from $\theta = \pi/4$ to $3\pi/4$. Zhang &

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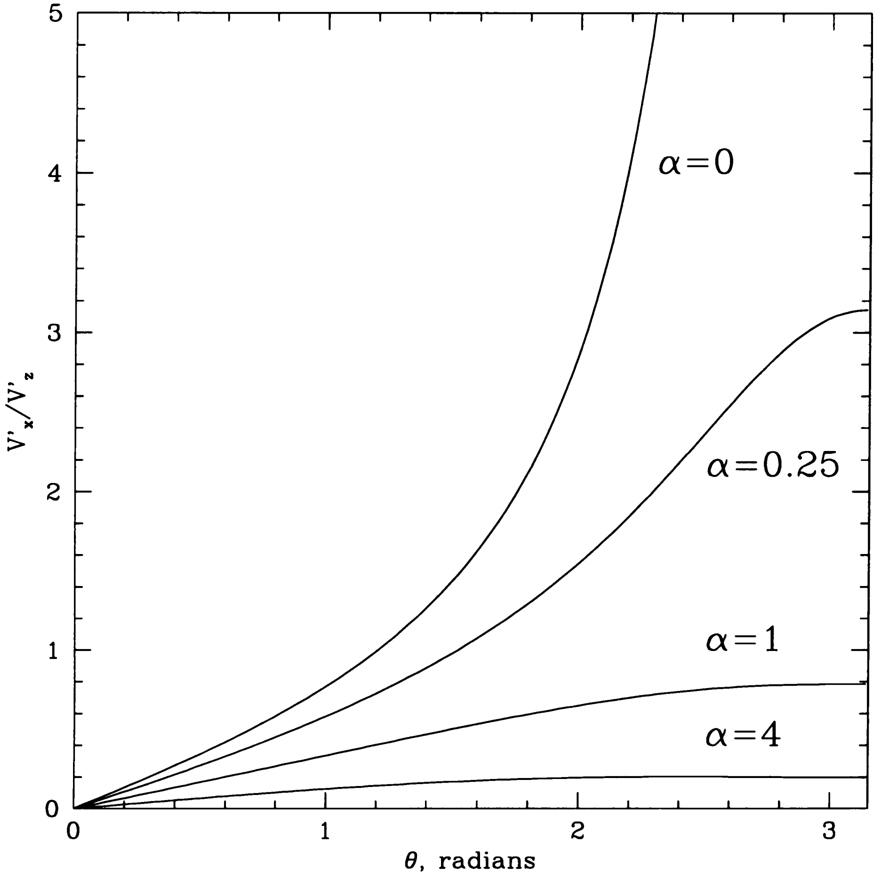

Figure 2. Ratio of sideways (V'_ϖ) to forward (V'_z) velocity for shocked gas in the ambient reference frame for the moving bow shock.

Zheng solved numerically for the shape of the bow shock and the kinematics of shocked material, pointing out that such bow shocks can have a large amount of momentum in the forward direction.

The formalism discussed in §2 can also be applied to this problem as well, although we only give the solution for the case considered by Zhang & Zheng. The solution is unchanged for $\theta \leq \theta_0$, except that $\dot{M}_w$ must be replaced by $4\pi\dot{M}_w/\Omega_0$, where $\Omega_0 = 2\pi(1 - \cos\theta_0)$ is the solid angle of the shocked jet "wind". For larger angles, the trajectory equation is the same as equation (8), except that θ is replaced by θ_0, a constant. Integration

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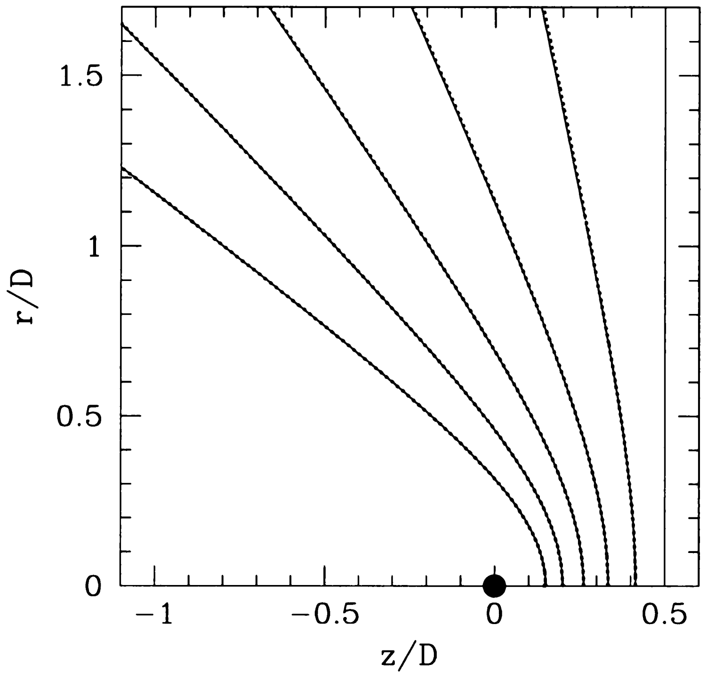

Figure 3. Solutions to the two-wind collision problem. The two wind sources are separated by a distance D, so that the weaker source is at the origin, while the stronger source is off the right edge of the graph at $z/D = 1$. Moving from right to left, the solutions correspond to $\beta = 1$, 0.5, 0.25, 0.125, 0.0625, 0.03125.

then yields the shape

$$R = R_0\sqrt{3[\sin^2\theta_0 + \cot\theta(\sin\theta_0\cos\theta_0 - \theta_0)]}. \qquad (12)$$

Zhang & Zheng found that the ratio of forward to sideways velocity in the ambient frame approached a constant for large θ. Indeed, this velocity ratio is given by equation (11) with θ replaced by θ_0 if $\theta > \theta_0$. Once the shocked fluid element is beyond the maximum wind angle, its trajectory

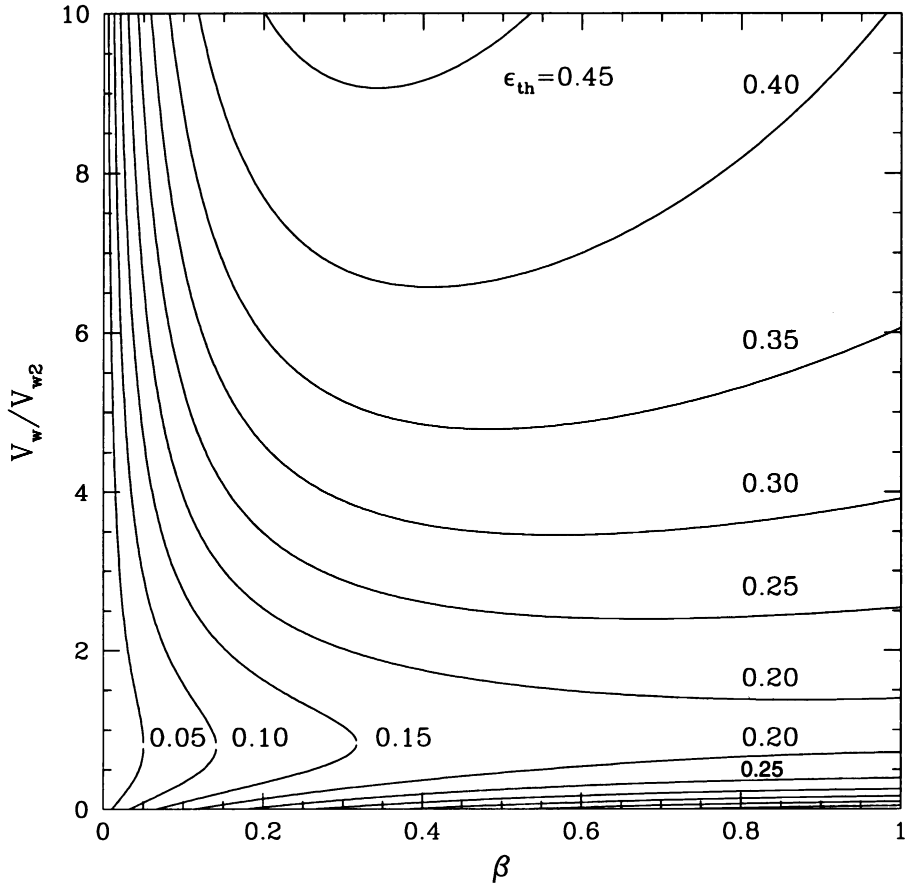

Figure 4. Efficiency of thermalization of pre-shock wind kinetic energy. Contours of equal thermalization efficiency ϵ_{th} are shown in terms of the ratio of wind momentum loss rates, β, and the ratio of wind speeds.

is a straight line, and the solution is a momentum-conserving snowplow in which the gas decelerates as it sweeps up the surrounding material.

4. Two Wind Collision in Binaries

The problem of the collision of two spherical winds has been solved by the same approach, for the case of a radiative, thin shell (Cantó, Raga, & Wilkin 1996). The ratio of momentum loss rates of the two stars is the fundamental parameter determining the shape of the bow shock, and several examples are shown in Figure 3. The solution method has been extended by including the consideration of angular momentum (about the origin). Although this may seem redundant with conservation of vector momentum, the benefit is that it provides a general, although implicit, integral to the trajectory equation (8) for more general wind collisions. If we define an angular momentum flux $\Phi_J = \Phi_m \, R \, v_\theta$, the radius of the shell is given implicitly by

$$\Phi_J = \Phi_\varpi z - \Phi_z \varpi, \tag{13}$$

where the flux functions refer to the flow in the shell, so they are determined by adding the contributions from the two winds.

Conservation of momentum implies that not all of the wind pre-shock kinetic energy can be thermalized and radiated. The maximum amount that may be thermalized and is available for radiation is given by the kinetic energy in the center of mass frame. We have calculated the rate at which energy is thermalized in the shocks and post-shock mixing. This is displayed in Figure 4, as the efficiency by which the total of the two wind kinetic energies are intercepted by the shell and thermalized. This represents an upper limit to the steady-state radiative luminosity of the bow shock.

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