

The impression was gained that this extensive revision of a well established book was carried out by an author still very much absorbed in his subject.

J. FULTON

KOLMOGOROV, A. N., AND FOMIN, S. V., *Elements of the Theory of Functions and Functional Analysis*, vol. i: *Metric and Normed Spaces*, translated by LEO F. BORON (Graylock Press, Rochester, N.Y., 1957), 129 pp., 32s.

This is an excellent introduction to the ideas and methods of functional analysis. The original version was based on lectures given by the author in Moscow, and the translator has produced a very readable English text. Although nothing beyond elementary analysis is presupposed, the principal abstract concepts are illustrated by an ample variety of examples, and the theory is elegantly applied to problems of considerable practical interest. The approach is essentially "classical": all limits are sequential, and no use is made of any principle of transfinite induction. This is a weakness from the point of view of the serious student of modern analysis, but it makes substantial parts of the subject easily accessible to many others.

There are four chapters. The first is concerned with some elementary facts of abstract set theory (with no mention of the Axiom of Choice). Chapter II, the longest, is devoted to metric spaces, with a brief reference to general topological spaces. The main themes here are completeness and compactness. The fixed-point theorem for contraction mappings in a complete metric space is thoroughly exploited in a discussion of iterative methods for solving equations, including differential and integral equations, for which several existence theorems are proved. The fundamental properties of compact sets in metric spaces are established (the word "compact" is used in the relative sense, which is now unusual), and there is a careful account of the relations between compactness and equicontinuity in spaces of continuous functions on compacta. The chapter ends with a useful discussion of rectifiable curves. Questions of category are not considered, and there is no mention of local compactness.

Chapter III, on normed vector spaces, is concerned largely with continuous linear functionals. Conjugate spaces are determined in some simple cases, and there are brief discussions of reflexivity and weak convergence. A proof of the Hahn-Banach extension theorem, and an account of weak* compactness, are restricted to separable spaces (of necessity, since transfinite induction is not available). Continuous linear operators on Banach spaces are also considered, and there is a proof of Banach's theorem on the continuity of inverse operators. The Banach-Steinhaus theorem is, surprisingly, omitted. An addendum to this chapter gives some of the main facts about "generalized functions" (distributions).

In Chapter IV, the idea of the spectrum of a linear operator is introduced, and the main theorems on completely continuous operators are proved, giving the Fredholm theory of integral equations.

There is a good index, and the translator has added a bibliography. The authors promise further volumes, in which they propose to discuss, among other things, Lebesgue integration and Hilbert-space theory.

J. D. WESTON

PONTRYAGIN, L. S., *Foundations of Combinatorial Topology* (Graylock Press, Rochester, N.Y., 1952), 99 pp., \$3.00.

This is a translation of the first (1947) Russian edition of a book which the author says is "essentially a semester course in combinatorial topology which I have given several times at Moscow National University".

There are three chapters. In Chapter I the Betti (homology) groups are defined for polyhedra; in Chapter II the topological invariance of the groups is proved;