

Parameter Space of Shock Formation in Adiabatic Flows¹

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We study shock formation in a stationary, axisymmetric, adiabatic flow of a perfect fluid in the equatorial plane of a Kerr geometry. For such a flow, there exist two intrinsic constants of motion along a fluid world line, namely the specific total energy, $E = -hu_t$, and the specific angular momentum, $l = -u_\varphi/u_t$, where the u_μ 's are the four velocity components, h is the specific enthalpy, i.e., $h = (P + \varepsilon)/\rho$, with P , ε , and ρ being the pressure, the mass-energy density, and the rest-mass density, respectively.

As shown in Fig. 1 (Fig. 1a is for a Schwarzschild black hole, i.e. the hole's specific angular momentum $a = 0$; Fig. 1b is for a rapid Kerr hole, i.e. $a = 0.99M$, where M is the black-hole mass, and prograde flows; and Fig. 1c is for $a = 0.99M$ and retrograde flows), in the parameter space spanned by E and l there is a strictly defined region bounded by four lines: three characteristic functional curves $l_k(E)$, $l_{max}(E)$, and $l_{min}(E)$, and the vertical line $E = 1$. Only such a flow with parameters located within this region can have two physically realizable sonic points, the inner one r_{in} , and the outer one r_{out} . In between there is still one more, but unrealizable, sonic point, r_{mid} . The region is divided by another characteristic functional curve $l_c(E)$ into two parts: in region I (= Ia + Ib) only r_{out} is realized in a shock-free global solution (i.e., that joining the black-hole horizon to large distances), while in region II (= IIa + IIb) only r_{in} is realized.

There further exists a new strictly defined function, $l_s(E)$, drawn in Fig. 1 as two curves l_{sI} and l_{sII} , for an accretion flow with parameters E and l located within region Ia (bounded by lines l_c , l_{sI} , and $E = 1$), or a wind flow with E and l belonging to region IIa (bounded by l_k , l_{sII} , l_c , and $E = 1$), there are four formal shock locations, denoted by r_{s1} , r_{s2} , r_{s3} , and r_{s4} , which are related to the three formal locations of the sonic point as $r_{s1} < r_{in} < r_{s2} < r_{mid} < r_{s3} < r_{out} < r_{s4}$; while for an accretion flow belonging to region Ib (bounded by l_{sI} , l_{min} , and $E = 1$ in Figs. 1a and 1b, and by l_{sI} , l_{max} , and $E = 1$ in Fig. 1c), or a wind flow belonging to region IIb (bounded by l_k , l_{max} , and l_{sII} in Figs. 1a and 1b, and by l_k , l_{min} , and l_{sII} in Fig. 1c), there are only two formal shock locations, denoted by r_{s1} and r_{s4} , satisfying $r_{s1} < r_{in}$, $r_{out} < r_{s4}$. It is clear that for an accretion flow onto a black hole, both r_{s1} and r_{s4} can be ruled out by employing boundary conditions that the flow must be supersonic when crossing the black-hole horizon and subsonic when starting at a large distance (however,

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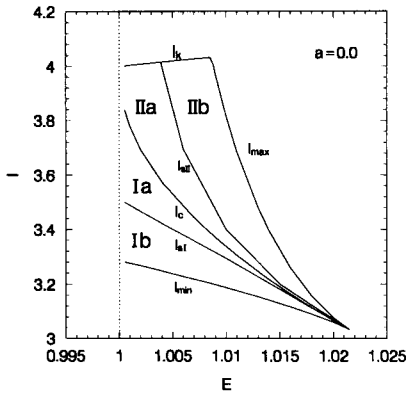


Fig.1a

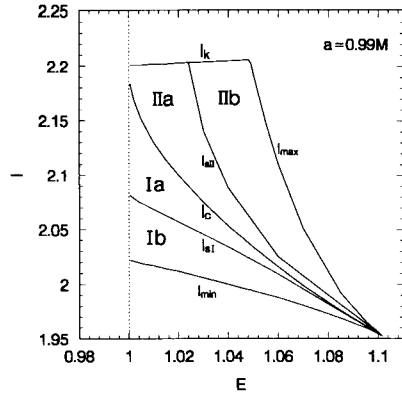


Fig.1b

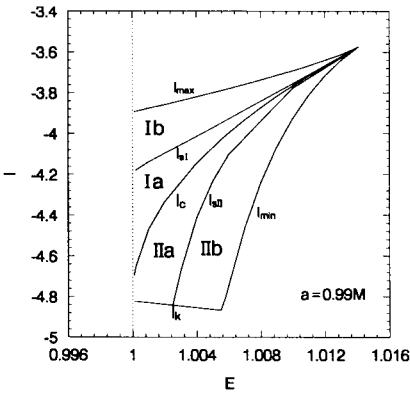


Fig.1c

for accretion onto a neutron star, r_{s1} is a possible shock location); while for a wind flow r_{s1} is not possible either, but r_{s4} is possible if the flow is terminally subsonic. The possibilities of shocks at r_{s2} and r_{s3} cannot be judged in this way, we have therefore made stability analysis and have been able to reach the following conclusion: *for accretion flows the shock at r_{s3} is always stable, while the shock at r_{s2} is unstable except when the value of l is very close to that of l_s ; for wind flows the shock at r_{s3} is always unstable, while the shock at r_{s2} , is stable except when l is very close to l_s .*