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HOW MAY SEISMOLOGICAL MEASUREMENTS CONSTRAIN PARAMETERS OF STELLAR STRUCTURE?

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ABSTRACT Pulsation frequency separations, combined with other observations, can constrain the structural parameters of Sun-like stars. As an example, we treat a hypothetical visual binary system resembling α Cen.

INTRODUCTION

Information about a star's structure is contained in its two characteristic pulsation *frequency separations* $\Delta\nu_0 \equiv \nu_{n+1,0} - \nu_{n,0}$, and $\delta_0 \equiv \nu_{n+1,0} - \nu_{n,2}$, where $\nu_{n,l}$ is the frequency of a p-mode with radial order n and angular degree l (Ulrich 1986, Gough 1987, Christensen-Dalsgaard 1988). Unfortunately, the information is somewhat confused by the lack of accurate knowledge about other parameters, especially the abundances of helium and of heavy elements. Thus we need to understand how oscillation information may be used in concert with other observable stellar properties. Here we describe single and binary systems with models which relate variable *parameters*, such as the stellar masses, initial composition, and age of the system, to larger sets of *observables*, including photometric, spectroscopic, and astrometric indices, as well as oscillation frequencies. After linearization of these relations, we employ a singular value decomposition (SVD) to calculate the errors in estimated parameters resulting from plausible errors in the observed quantities. To simplify the analysis, errors arising from inaccurate or incomplete physics in the model are ignored.

PARAMETERS, OBSERVABLES, ERRORS, AND MODELS

Here we treat two systems: a field star similar to α Cen A, and a visual binary similar to the α Cen system. The systems are considered to be observed at distances of both 1.3 and 100 pc. The model parameters and observables for these

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systems are shown in Table I. The standard error for the frequency separations was estimated by assuming that one-half of the modes in the range considered have sufficiently large amplitudes during a given observing run to allow their

then follow from the least-squares fit used to determine $\Delta\nu_0$ and δ_0.

TABLE I Parameters and observables for field star (top section), and for visual binary (both sections combined). M_1 and M_2 are the masses of the stars, Y and Z the abundances of helium and heavy elements, α the mixing-length parameter, τ the age of the system, D its distance, and a the separation of the binary. For the observables, m_{Vi} and $(B\text{-}V)_i$ are apparent magnitudes and color indices, π' is the parallax, a' the apparent separation of the binary, M_1/M_2 the mass ratio inferred from the orbit, and $P_{\rm orbit}$ the orbital period; σ is the estimated standard error in each observable.

Model Parameter	Value(s)	Observable	σ
$\log M_1$ ($M_\odot$)	0.0374	m_{V1}	0.01 mag
$\log Y$	−0.5510	$(B\text{-}V)_1$	0.01 mag
$\log Z$	−1.7000	$\log Z$	0.17
$\log \alpha$	0.40	π'	0.004 ″
$\log \tau$ (yr)	9.544	$(\delta_0)_1$	0.82 μHz
$\log D$ (pc)	0.114, 2.	$(\Delta\nu_0)_1$	0.06 μHz
$\log M_2$ ($M_\odot$)	−.0458	m_{V2}	0.01 mag
$\log a$ (AU)	1.301	a'	0.05 ″
		$(B\text{-}V)_2$	0.01 mag
		$\log(M_1/M_2)$	0.0004
		$\log P_{\rm orbit}$	0.0001
		$(\delta_0)_2$	0.82 μHz
		$(\Delta\nu_0)_2$	0.06 μHz

The reference stellar models were computed by means of a "standard" evolution calculation with the parameters listed in Table I, chosen to obtain a pair of models roughly similar to α Cen. Measures of the average separations $\Delta\nu_0$ and $\delta\nu_0$ were found from a least-squares fit (Christensen-Dalsgaard 1988) to frequencies computed by solving the equations of adiabatic oscillation. The fit included modes of degree $l = 0 - 3$ and with $17 \leq n + l/2 \leq 29$. To compute the necessary derivative matrix, we varied each of the parameters M_1, M_2, Y, Z, α, and τ separately, by suitable small amounts. Each derivative was computed from differences centered on the reference values given in Table I.

RESULTS AND CONCLUSIONS

The results of the calculations for a few interesting cases are shown in Fig. 1. Evidently the oscillation frequency separation data can be important in constraining the model parameters, but which parameters are constrained depends

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strongly on the circumstances of the stellar system under study and on the available observables. Without oscillation data, the properties of isolated field stars (especially their ages) are essentially unconstrained. With such data, it becomes possible to make useful estimates of the stellar ages, and to improve significantly the estimates of the mixing length and (for nearby stars) the mass, or (for stars at large distances) the distance itself. For visual binaries similar to α Cen, the effect of adding oscillation information depends strongly on the distance to the system. For nearby systems (where astrometric data is very good), one learns mostly about Y, Z, and α, with lesser improvement in the estimate of the system's age. For distant systems, all parameter estimates improve significantly, with the most notable improvements in Y, the masses, the distance, and the semimajor axis of the orbit. Particularly for the nearby case, all parameters are well enough determined that detailed comparisons with stellar evolution theory seem feasible.

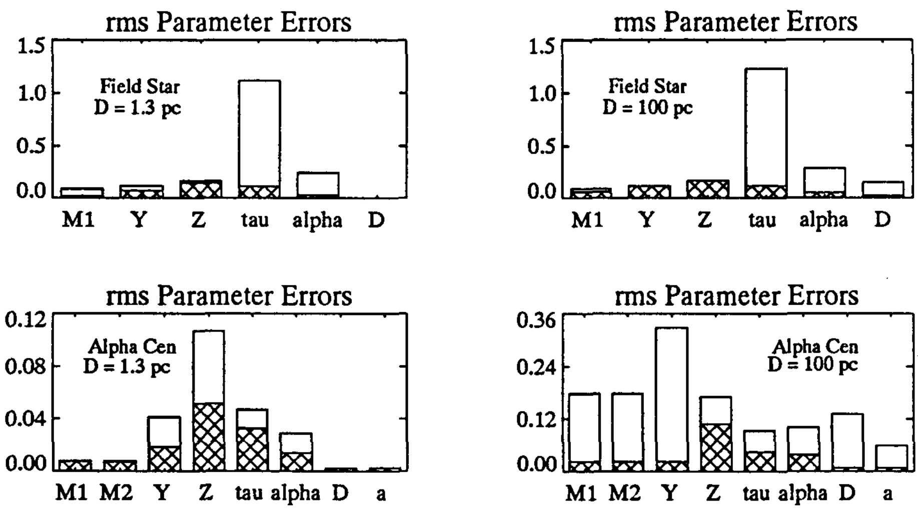

Fig. 1. RMS errors in $\log_{10}$ of the inferred structure parameters for field star (top) and visual binary (bottom), at distances of 1.3 and 100 pc. Results without oscillation data are shown as empty bars, with such data as cross-hatched bars.

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