LETTER TO THE EDITOR

Dear Editor,

Characterization of the exponential distribution by the relevation transform

Some readers of our paper (Lau and Rao (1990)) have brought to our attention the papers by Westcott (1981), Rao and Shanbhag (1986) and the lecture notes by Kakosyan et al. (1984) which dealt with the same problem. The authors would like to thank them through the pages of this journal.

Westcott (1981) has given a solution to the problem by probabilistic arguments assuming that $\bar{G}$ is continuous (notations as in our paper) with $\bar{G}(0) = 1$ and that the equation

$$\int_0^x \bar{G}(x-t)d\bar{F}(t) = \int_0^x \frac{\bar{G}(x)}{\bar{G}(t)}\, d\bar{F}(t) \tag{1}$$

holds for all survival functions $\bar{F}$ improving the earlier work in Grosswald et al. (1980). Rao and Shanbhag (1986), p. 668 and Kakosyan et al. (1984) gave stronger results. Our result is close to Theorem 4.3.1 given in Kakosyan et al. (1984). Following the notation of our paper, we now state an improved version of Theorem 3.1 in our paper as suggested by a reader.

Theorem. Suppose $\bar{F}$ and $\bar{G}$ are survival functions on $[0, \infty)$ such that $\lim_{t \to 0_+} G(t)/t$ exists and $\bar{G}(0) > 0$. Further suppose that $\bar{G}$ is continuous and zero is a cluster point of supp(F). If $\bar{G}$ satisfies Equation (1) for all x with $\bar{G}(x) > 0$, then $\bar{G}(x) = \exp(-\alpha x)$ for some $\alpha > 0$.

The result follows essentially via the argument produced by us for proving our theorem, or a somewhat simplified version of it without involving our Proposition 2.2.

We take the opportunity to correct the following errors in the statement and proof of Theorem 3.1.

Page 729, Line 19 from below: '$\bar{F}$ has a point of increase' should be 'F has a point of increase'

Page 729, Line 17 from below: '$\alpha \geqq 0$' should be '$\alpha > 0$'

Page 729, Line 13 from below: '$l(t) = \bar{F}(t)$' should be '$l(t) = F(t)$'

Page 729, Line 10 from below: '$h'_+(0) \geqq 0$' should be '$h'_+(0) > 0$.'

The authors thank, the anonymous readers for pointing out these corrections and for their remarks.

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Yours sincerely,

K. A. SING LAU

Department of Mathematics
University of Pittsburgh
Pittsburgh PA 15260
USA

B. L. S. PRAKASA RAO

Indian Statistical Institute
Delhi Centre
7 S.J.S. Sansanwal Marg
New Delhi 110 016
India